An acoustic finite element including viscothermal effects

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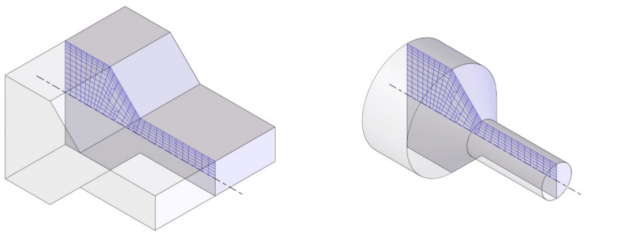
Introduction

In acoustics, it is generally assumed that viscothermal effects can be neglected. However, for geometries that include small confinements of air or thin air layers (found in for instance hearing aids or acoustical resonators), this assumption does not yield valid results.

Objective

Special analytical models that include viscothermal effects are available, but only for a limit number of geometries. To overcome this limitation, an acoustic finite element that includes viscothermal effects is developed.



2D layer elements   Axial symmetric elements

Fig.1) **Finite element discretization of a layer geometry and axial symmetric geometry**

Methods

The finite element is based on three conservation laws supplemented by a constitutive relation:

- Full linearized Navier Stokes equations (conservation of impulse)
- Equation of continuity (conservation of mass)
- Energy equation (conservation of energy)
- Equation of state for an ideal gas

This system of equations is linearized and rewritten yielding a so called mixed formulation of three equations with velocity (v), pressure (p) and temperature (T) as degrees of freedom.

The method of weighed residuals is used to discretize the mixed formulation. Two dimensional bi-linear quadrilateral and triangular elements are developed for layer geometries and axial symmetric geometries (figure 1).

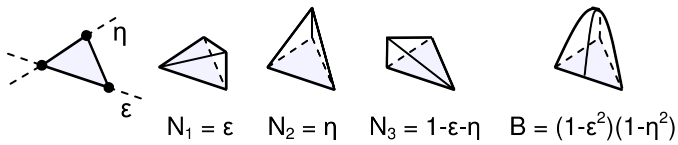


Fig.2) **Lagrange shape functions (N_i) supplemented by the bubble function (B) for a linear triangle element**

All variables are approximated by Lagrange shape functions (figure 2). In order to obtain a numerical stable solution, these shape functions are supplemented by a bubble function for the particle velocity in both directions.

Numerical Validation

The results of the new finite elements are compared with an analytical solution. For simple geometries (layers and cylinders with constant thickness or radius) the analytical solution is provided by the Low Reduced Frequency (LRF) model.

Results

The FEM results converge quickly to the LRF solution for bi-linear quadrilaterals and triangles in both the Cartesian and axial symmetrical case.

An example of velocity results for a layer geometry of non-constant thickness is given in figure 3.

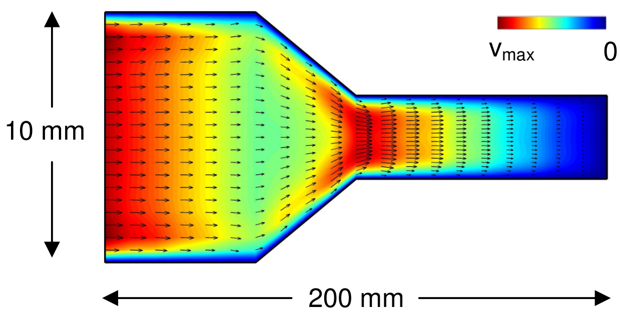


Fig.3) **Velocity (real part) for a layer with a non-constant thickness**

Future Work

In order to describe 3D wave propagation in arbitrary geometries, tetraeder and brick elements are developed.