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The minimal model of a continuous, physical system, and more

Norbert E. Ligterink, Peter C. Breedveld
Control Engineering Group, University Twente
P.O. box 217, 7500AE Enschede,
The Netherlands
n.e.ligterink@utwente.nl,
p.c.breedveld@utwente.nl

Arjan J. van der Schaft
System and Control Theory, University Twente
P.O. box 217, 7500AE Enschede,
The Netherlands
and
Applied Analysis Group, University Groningen
a.j.vanderschaft@utwente.nl

1 Abstract

Infinite dimensional, or continuous systems, and their finite element method (FEM) representations have, for many applications in control and optimization, too many degrees of freedom to be of practical use. Model order reduction is an essential part of modelling such systems. What features to retain is determined by the dynamics of interest, which follow from the input to the system and the specifications of the model. The input is necessary to bring the system into a nontrivial dynamical state, the output does not always follow from the specifications.

The appropriate output to an input, in the port-based modelling, is the response to an input, which product is the power transfer in or out of the system. The input and output are therefore collocated in a single "port". In continuous systems the variables of the input are usually not the same as the internal variables, or the state. For example, the input to an elastic system can be force or velocity, while the corresponding internal variables are the stress and strain tensors. The relation among boundary variables and internal variables can be formalized in the port-Hamiltonian H'. This extension of the Hamiltonian H of a physical system automatically dictates the appropriate boundary variables and the correct boundary conditions to be set, even in the case of an open system, where energy flows in and out of the system through the boundaries. The port-Hamiltonian extension of a Hamiltonian is dictated by the differential operator D, which is chosen appropriately for the system at hand:

$$H(q(z), p(z)) = H'(Dq(z), p(z)) = H'(q'(z), p(z)) \quad ,$$

where $q(z)$ and $p(z)$ are the canonical position and momenta, fields of the reference, or geometrical coordinate z. The gradient $D = \nabla_z$ is often used for second-order PDE's. The directional position $Dq = q'$ includes geometrical information in the field variable $q'(z)$, such that an associated energy flux $\mathbf{S}(z)$, across any boundary, can be defined:

$$\mathbf{S} = \delta_{q'} H' \theta \delta_p H' \quad ,$$

where $\delta_{q'}$ and δ_p are the variational derivatives, and θ the bilinear boundary operator. Isolating boundaries require $\mathbf{S} = 0$, which leads to appropriate boundary conditions, such as Neumann and Dirichlet boundary conditions for isolated systems.

The conservation laws and balance laws of systems clearly have a central role in the port-Hamiltonian formulation. Retaining these laws in the reduced model leads to a greater model competence, and serve as guideline for understanding the model in detail.

A model consists of a part which is the direct consequence of particular input. This part must always be present in a model to balance forces or flows between the different boundary inputs. This model is called the minimal model. On top of that, a model of a continuous system may have some internal states, which might lead to vibrational modes, if the model is excited through particular input. Such internal states lead to initial conditions which specify the state at a certain, starting time. Appropriately, the internal states, or modes, are excited by the input only through the minimal model. The separation of the total system into the minimal model; the direct result of boundary input, and the internal states, driving by the time-variations of the minimal model lead to a higher stability of the simulation than a direct FEM model based on the combined system. Furthermore, such composite model might lead to inconsistent initial and boundary values.

In this contribution, we will explain the ideas of the port-Hamiltonian and the minimal model using some simple examples from structural and chemical engineering. If time permits the advantages for nonlinear systems are explained. In particular the advantages lie in the possibility to use the boundary, or input, variables throughout the system, which avoids the nonlinear conversions and inversions. Furthermore, it yields a natural nonlinear state, arising from the minimal model, around which linearized, and reduced, models for the internal states can explain the configuration dependence of the vibrational frequencies.

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