Momentum diffusion for coupled atom-cavity oscillators

K. Murr,[1,*] P. Maunz,[1] P. W. H. Pinkse,[1] T. Puppe,[1] I. Schuster,[1] D. Vitali,[2] and G. Rempe[1,†]

[1]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, D-85748 Garching, Germany*
[2]*Dipartimento di Fisica, Università di Camerino, I-62032 Camerino, Italy*

It is shown that the momentum diffusion of free-space laser cooling has a natural correspondence in optical cavities when the internal state of the atom is treated as a harmonic oscillator. We derive a general expression for the momentum diffusion, which is valid for most configurations of interest: The atom or the cavity or both can be probed by lasers, with or without the presence of traps inducing local atomic frequency shifts. It is shown that, albeit the (possibly strong) coupling between atom and cavity, it is sufficient for deriving the momentum diffusion to consider that the atom couples to a mean cavity field, which gives a first contribution, and that the cavity mode couples to a mean atomic dipole, giving a second contribution. Both contributions have an intuitive form and present a clear symmetry. The total diffusion is the sum of these two contributions plus the diffusion originating from the fluctuations of the forces due to the coupling to the vacuum modes other than the cavity mode (the so-called spontaneous emission term). Examples are given that help to evaluate the heating rates induced by an optical cavity for experiments operating at low atomic saturation. We also point out intriguing situations where the atom is heated although it cannot scatter light.

PACS number(s): 32.80.Lg, 42.50.Lc, 42.50.Ct

I. INTRODUCTION

A single atom coupled to a single mode of an optical cavity is the archetype model of dissipative electrodynamics. This system provides a matter-light interface for fundamental studies as well as applications. It has two well-known characteristics (see Fig. 1). First, the cavity mirrors confine the light, which can lead to a strong influence of the atom on the cavity field and vice versa. Second, in addition to the atom emitting photons into free space, the cavity mirrors now offer an extra loss channel. The spatially directed light can easily be detected to observe the properties of the system. The new dissipative channel is also responsible for a new dissipative force, which can efficiently dampen atomic motion [1–15]. However, dissipation inevitably leads to fluctuations, in particular, light-force fluctuations, which heat the atom.

Indeed, several experiments in the regime of strong coupling and low atomic excitation reported excessively large heating rates [16–20], up to two orders of magnitude larger than for an atom in free-space laser fields. The physical origin of this is obscured: Expressions that were calculated for the heating rates are restricted to specific geometries. They are not accessible to an intuitive interpretation as they mix those two effects of confinement and dissipation, and contain correction terms to the heating that could be negative.

This paper derives a general expression for the heating rate, which also gives insight into the physics of force fluctuations in a cavity: By effectively decoupling the atom-cavity system, we first show that the equation for the momentum diffusion can be written as the sum of positive contributions, where we clearly identify the fluctuations of the atomic dipole and the fluctuations of the cavity field. Second, this form is intuitive as it allows us to perceive the

heating in free space [21,22] and in confined space alike. While our expression generalizes previous ones [4,5,7–11], a powerful new feature is its invariance: It applies, regardless of whether the atom and/or the cavity is probed, regardless of the spatial structure of the involved light fields, with or without the inclusion of Stark shifts due to an additional dipole trap, and could be naturally extended to an arbitrary number of atoms and modes, linear or ring cavities. We explain that such an invariance is a signature of harmonic oscillators, a description which applies to the experimentally most relevant regime of vanishing atomic excitation. Lastly, we point out some conceptually interesting situations, such as the persistence of heating induced by cavity-field fluctuations even if the cavity field is in the vacuum state $|0\rangle$.

II. FREE-SPACE MOMENTUM DIFFUSION AT LOW SATURATION

From the theory of Brownian motion, the random character of a force leads to a spread of the particle's momentum $\mathbf{p}$. This heating mechanism is characterized by the (momentum) diffusion coefficient $2D = d/dt\langle(\Delta\mathbf{p})^2\rangle$. For a particle of mass m the heating rate is then given by D/m (energy per unit time). For a two-state atom at rest and coupled to a single-mode laser, the diffusion in the harmonic description, which requires a low excited state $|e\rangle$ occupation probability P_e, reads [22,23]

$$2D = (\hbar k)^2 2\gamma P_e + |\hbar \boldsymbol{\nabla} \langle\sigma\rangle|^2 2\gamma. \tag{1}$$

The first term is the familiar contribution arising from the random direction of the spontaneously emitted photons (with momentum norm $\hbar k$), occurring at a rate $2\gamma P_e$, where $1/2\gamma$ is the lifetime of the upper state of the atom. The second term stems from the fluctuations of the laser force, and is determined by the gradient of the atomic mean coherence $\langle\sigma\rangle$ (defining the dipole moment). At low atomic saturation P_e

*Electronic address: karim.murr@mpq.mpg.de

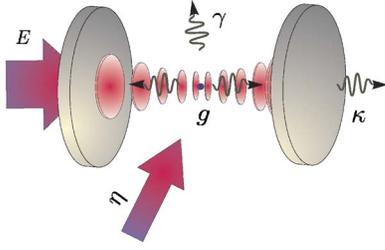†Electronic address: gerhard.rempe@mpq.mpg.de

FIG. 1. (Color online) An atom placed between two mirrors is excited from the side (laser-atom Rabi frequency 2η). The cavity mode is also coherently pumped by a laser source (coupling strength E). The mirrors dissipate the field at a rate κ, which is the analogue of the relaxation rate γ of the atom's dipole moment. The atom-cavity vacuum Rabi frequency is denoted by $2g$. Forces exist mainly along the cavity axis (due to E and/or η), and along the side-laser beam.

$\approx |\langle\sigma\rangle|^2 \ll 1$, the mean coherence is given by $\langle\sigma\rangle = \eta(\mathbf{r})/(\Delta_a - i\gamma)$, where $2|\eta(\mathbf{r})|$ is the laser-atom Rabi frequency at the position $\mathbf{r}$ of the center of mass of the atom and $\Delta_a = \omega_{eg} - \omega_L$ is the detuning between the transition frequency ω_{eg} of the atom and the laser frequency ω_L.

For a laser running wave $\eta(\mathbf{r}) = \eta_0 e^{i\mathbf{k}_L\cdot\mathbf{r}}$ representing a stream of photons all propagating with the same momentum $\hbar\mathbf{k}_L$ ($\omega_L = c|\mathbf{k}_L|$), that diffusion originates from the random character of the absorption process [21,22]. Per unit time, the atom absorbs a random number of photons, and hence acquires a random momentum, along the laser beam. Despite all laser photons having the same momentum, the atom thus heats, as long as there is light. For a laser standing wave $\eta(\mathbf{r}) = \eta_0 \cos(\mathbf{k}_L\cdot\mathbf{r})$, the momentum diffusion can be simply understood in terms of the absorption and emission of photons only when the diffusion is averaged over a spatial period. In this case the standing wave is seen as two independent running waves of half intensity, and the diffusion is just the sum of each contribution [22]. Otherwise, interpreting the momentum diffusion in terms of "recoil kicks" for any location in a standing wave is not trivial. A known intriguing example, which is discussed in detail in Ref. [22] (see also Ref. [23]), is that of an atom at a node. At a node, there is no light, $\eta(\mathbf{r}) = 0$, meaning that the atom does not emit photons, $P_e = 0$, but the diffusion is finite $D \neq 0$.

Besides, we recall that the diffusion coefficient derived within the dressed-state picture [24] is not the one discussed here [Eq. (1)]. More precisely, the term derived from the harmonic description $|\hbar\boldsymbol{\nabla}\langle\sigma\rangle|^2 2\gamma$, and the term obtained from the dressed-state picture [24] are distinct limits of the full expression for the diffusion [23]. Quantitatively, Eq. (1) dominates the dressed-state one for $P_e \ll 1$ and $2P_e \ll (\gamma/\Delta_a)^{2/3}$ [23]. Those inequalities also show that for large detuning Δ_a and sufficiently high intensity the dressed-state term [24] can be dominant at low atomic saturation. For example, for $\Delta_a = 10^3\gamma$ and $P_e = 0.01$, one has $P_e \ll 1$ but $2P_e > (\gamma/\Delta_a)^{2/3}$. However, for most situations of interest the limit of low saturation coincides with the harmonic limit considered here.

III. MOMENTUM DIFFUSION IN THE PRESENCE OF A CAVITY

We show that in a cavity, and independently from which way the system is excited, the diffusion for coupled atom-cavity oscillators can be written in the following invariant form:

$$2D = (\hbar k)^2 2\gamma P_e + |\hbar\boldsymbol{\nabla}\langle\sigma\rangle|^2 2\gamma + |\hbar\boldsymbol{\nabla}\langle a\rangle|^2 2\kappa. \quad (2)$$

The mean coherence $\langle\sigma\rangle$ ($P_e \approx |\langle\sigma\rangle|^2$) is now evaluated within the cavity setting. The additional term $|\hbar\boldsymbol{\nabla}\langle a\rangle|^2 2\kappa$ comes from the fluctuations of the cavity mode. It depends only on the gradient of the steady-state amplitude of the cavity field $\langle a\rangle$, and is proportional to the decay rate 2κ at which photons escape through the mirrors. The momentum diffusion is completely determined from the expectation values $\langle\sigma\rangle$ and $\langle a\rangle$, which depend on the position of the atom and which contain all the information on the coupling between the atom and cavity [see Eqs. (6), below].

Let us now derive Eq. (2) and analyze and interpret the relative contributions of the different terms. The derivation of Eq. (2) is obtained via two unconventional ways. The first way is to start from the well-known driven Jaynes-Cummings Hamiltonian and to perform a particular mathematical operation, which corresponds to transferring the information on the dynamics into the components of a new force operator. The second way shows that Eq. (2) is also the momentum diffusion obtained from two decoupled Hamiltonians, one describing the interaction of an atom coupled to a mean cavity field, and the other one describing the interaction of a cavity mode coupled to a mean atomic dipole [12]. The second way gives us a natural interpretation and also shows that one can truncate the Jaynes-Cummings Hamiltonian without modifying the result for the diffusion coefficient.

A. Derivation of Eq. (2) as a solution from the extended Jaynes-Cummings Hamiltonian

One first starts from a general driven atom-cavity system, described by the extended Jaynes-Cummings Hamiltonian (labeled "*JC*"),

$$\begin{aligned} H_{JC}/\hbar = {} & \Delta_a\sigma^\dagger\sigma - \eta(\mathbf{r})\sigma^\dagger - \eta^*(\mathbf{r})\sigma + \Delta_c a^\dagger a + Ea^\dagger + E^* a \\ & + g(\mathbf{r})a\sigma^\dagger + g^*(\mathbf{r})a^\dagger\sigma, \quad (3) \end{aligned}$$

where the atom, with lowering operator σ, is dipole coupled (last line) to a single cavity mode of creation operator $a^\dagger$. Here $2g$ is the atom-cavity mode vacuum Rabi frequency. We omit writing the kinetic term $\mathbf{p}^2/2m$ because the position $\mathbf{r}$ is (ultimately) treated classically and we focus on the lowest order in the velocity $\mathbf{v}$, $k_L v \ll (\gamma,\kappa)$ (atom almost at rest). The atom and mode are coherently excited by two different near-resonant classical laser sources that have the same frequency ω_L; E is the coupling strength of the axial laser and we recall that $2|\eta|$ is the side laser-atom Rabi frequency. The strength E is defined such that $|E|^2/(\Delta_c^2 + \kappa^2)$ is the empty cavity photon number, where $\Delta_c = \omega_{cav} - \omega_L$ is the detuning between a cavity resonance frequency ω_{cav} and the laser(s).

We assumed the rotating-wave approximation and wrote the Hamiltonian in the interaction picture with respect to the laser frequency ω_L. The evolution of the reduced density matrix $\rho(t)$ for the atom-cavity system must account for the loss mechanisms (atom and cavity decay): $\dot{\rho} = -i[H_{JC}, \rho]/\hbar + \kappa L_a \rho + \gamma L_\sigma \rho$, where $L_a \rho = 2a\rho a^\dagger - \rho a^\dagger a - a^\dagger a \rho$, and for the atom $L_\sigma \rho = 2 \int d^2 \hat{\mathbf{k}} N(\hat{\mathbf{k}}) e^{-i\mathbf{k}\cdot\mathbf{r}} \sigma \rho \sigma^\dagger e^{+i\mathbf{k}\cdot\mathbf{r}} - \rho \sigma^\dagger \sigma - \sigma^\dagger \sigma \rho$, where $\hat{\mathbf{k}} = \mathbf{k}/k$ ($k = \omega_{eg}/c$) is the direction of spontaneously emitted photons, with the angular distribution $N(\hat{\mathbf{k}})$ (dipole pattern), $\int d^2 \hat{\mathbf{k}} N(\hat{k}) = 1$.

We now show that Eq. (2) is the correct diffusion obtained from Eq. (3) if the internal state of the atom is treated as a harmonic oscillator $[\sigma, \sigma^\dagger] \rightarrow 1$ (no saturation effects). The conventional procedure [5,10,11,22,23] leads to a first expression, which we write in a compact form $2D = (\hbar k)^2 2\gamma P_e - \mathbf{u}^\dagger (M^{-1} + M^{-1\dagger}) \mathbf{u}$. The 2×2 matrix $M = P - iG$ determines the dynamics of coupled oscillators, where $P \neq P^\dagger$ is the diagonal matrix for decoupled oscillators $[P_{11} = -i(\Delta_a - i\gamma), P_{22} = -i(\Delta_c - i\kappa)]$, and $G = G^\dagger$ is the coherent coupling off-diagonal Hermitian matrix ($G_{12} = g$). Already in limiting cases, this expression is complicated [4,5,10,11]. Our first clarification is based on the following observation: The vector $\mathbf{u}^\dagger$ is the coordinate of the force fluctuation $\delta \mathbf{F}_{JC} = \mathbf{u}^\dagger w + w^\dagger \mathbf{u} + [\mathcal{O}]$ in the subspace $w = (\delta\sigma, \delta a)^T$ of the fluctuations $\delta\sigma = \sigma - \langle\sigma\rangle$ and $\delta a = a - \langle a\rangle$. The $(\mathbf{u}^\dagger, \mathbf{u})$ are the only coordinates of $\delta \mathbf{F}_{JC}$ that contribute to the diffusion in the harmonic limit. Writing $\mathbf{F}_{JC} = -\boldsymbol{\nabla}H_{JC}$, and then expressing (σ, a) in terms of $(\delta\sigma, \delta a)$, one obtains $\mathbf{u} = \hbar[\boldsymbol{\nabla}I - i(\boldsymbol{\nabla}M)S]$, where $I = (\eta, -E)^T$ and $S = (\langle\sigma\rangle, \langle a\rangle)^T$. Now, in steady state one has $I - iMS = 0$, which by differentiation gives $\mathbf{u} = i\hbar M \boldsymbol{\nabla}S$: Hence $2D = (\hbar k)^2 2\gamma P_e - \hbar \boldsymbol{\nabla}S^\dagger (P + P^\dagger) \hbar \boldsymbol{\nabla}S$. The matrix $P^\dagger + P$ is diagonal, with -2γ and -2κ as elements. This is Eq. (2). The diffusion tensor in three dimensions exhibits a similar structure as Eq. (2) provided it is defined symmetric with respect to the spatial coordinates. A closer inspection also shows that Eq. (2) still holds if any of the frequencies $(\Delta_a, \eta, \Delta_c, E, g)$ depend on the atomic position. In particular, one can include the presence of a dipole trap through local Stark shifts $\Delta_a \rightarrow \Delta_a(\mathbf{r})$.

B. Derivation of Eq. (2) as a solution from decoupled Hamiltonians

Equation (2) can be derived in a more direct way by establishing a correspondence to the "mean-field" Hamiltonian $H = H_{atom} + H_{mode} + U$ [12],

$$H_{atom}/\hbar = \Delta_a \sigma^\dagger \sigma - ([\eta - g\langle a\rangle]\sigma^\dagger + \text{H.c.}), \quad (4a)$$

$$H_{mode}/\hbar = \Delta_c a^\dagger a + ([E + g^*\langle\sigma\rangle]a^\dagger + \text{H.c.}), \quad (4b)$$

with the mean interaction energy $U = -\hbar g\langle a\rangle\langle a\rangle^* + \text{H.c.}$ Note that, since the expectation values are dynamical variables, derivation of the friction force requires U. However, for our purposes the atom is (almost) at rest, i.e., the mean values $\langle a\rangle$ and $\langle\sigma\rangle$ are fixed and the Hamiltonian H [Eq. (4)] decouples. Our Hamiltonian and the Jaynes-Cummings one lead to the same expression for the momentum diffusion, Eq. (2). In-

deed, provided that one defines the force as $\mathbf{F} = -\boldsymbol{\nabla}H$, Eq. (2) is again obtained, but now straightforwardly: Eq. (4a) gives

$$2D_{atom} = |\hbar \boldsymbol{\nabla}\langle\sigma\rangle|^2 2\gamma, \quad (5a)$$

and Eq. (4b) gives

$$2D_{mode} = |\hbar \boldsymbol{\nabla}\langle a\rangle|^2 2\kappa, \quad (5b)$$

and therefore Eq. (2), $2D = (\hbar k)^2 2\gamma P_e + 2D_{atom} + 2D_{mode}$.

The interpretation of Eq. (2) follows immediately from that of the mean-field Hamiltonian, Eqs. (4). In the first Hamiltonian (4a), the atom "sees" a total field, which is the incident source field η and a mean cavity field $\langle a\rangle$. Thus it experiences an effective coupling $\Omega_{atom}(\mathbf{r}) = \eta(\mathbf{r}) - g\langle a\rangle(\mathbf{r})$, similar to free space. Symmetrically, the cavity field "sees" a source field of strength E and a mean atomic dipole $\langle\sigma\rangle$. Thus, the cavity mode is driven with an effective strength $\Omega_{mode}(\mathbf{r}) = E + g^*\langle\sigma\rangle(\mathbf{r})$. One thus obtains a general picture where each object is coupled to a classical-averaged partner, and fluctuates. We discuss this picture in a specific example in Sec. V.

The simple form of Eq. (2) may suggest that the result could have been guessed from the beginning, the argument being that the driven Jaynes-Cumming Hamiltonian for an atomic oscillator is already symmetric if one exchanges atom and cavity variables so that every physical quantity must reflect this symmetry $(\Delta_a, \gamma, -\eta, g, \sigma) \leftrightarrow (\Delta_c, \kappa, E, g^*, a)$. This argument is not sufficient because terms of the form $\propto \Re[(\hbar \boldsymbol{\nabla}\langle\sigma\rangle)^*)(\hbar \boldsymbol{\nabla}\langle a\rangle)]$ or $\propto (\Delta_c \gamma + \Delta_a \kappa)$ do all satisfy the symmetry of coupled oscillators, but do not appear in Eq. (2). It is already not obvious that the diffusion coefficient is a function of only $\langle\sigma\rangle$ and $\langle a\rangle$ (and κ, γ). Also, cross terms of the form $\propto \Re[(\hbar \boldsymbol{\nabla}\langle\sigma\rangle^*)(\hbar \boldsymbol{\nabla}\langle a\rangle)]$ could have been expected because of the form of the dipole-coupling Hamiltonian, $ga\sigma^\dagger + \text{H.c.}$ It is possible to show that the form of Eq. (2) follows from a general property of integrals of time-symmetric autocorrelation functions for dipole-coupled oscillators (which is not a property satisfied by all observables). This property will not be detailed here, rather we express it through the following subtle point: The original Hamiltonian (3) can be expressed as the second one (4), plus a term, $H_{JC} = H + (g\delta a\delta\sigma^\dagger + \text{H.c.})$. The additional term $g\delta a\delta\sigma^\dagger + \text{H.c.}$ has an essential role in the dynamics as it gives large contributions to the diffusion derived from the couple $(H_{JC}, \mathbf{F}_{JC})$. However, this additional term can be omitted, yielding H, but one must then redefine the force operator as $\mathbf{F} = -\boldsymbol{\nabla}H$ such that the couple $(H, \mathbf{F})$ still gives Eq. (2). In other words, Eq. (2) can be obtained if one truncates Eq. (3), which means that atom and mode are dynamically decoupled, but the force must be modified too; it now depends on the position $\mathbf{r}$ also through $\langle\sigma\rangle$ and $\langle a\rangle$. This is the main reason why the identification of our result with respect to existing expressions [4,5,9–12] is not immediate.

IV. EVALUATING THE DIFFUSION

The momentum diffusion is completely determined given the expectation values $\langle\sigma\rangle = \Omega_{atom}/\tilde{\omega}_a$ (with $\tilde{\omega}_a = \Delta_a - i\gamma$), and $\langle a\rangle = -\Omega_{mode}/\tilde{\omega}_c$ (with $\tilde{\omega}_c = \Delta_c - i\kappa$), which gives

$$\langle \sigma \rangle = \frac{1}{1 - \nu(\mathbf{r})} \left[\eta(\mathbf{r}) + g(\mathbf{r}) \frac{E}{\widetilde{\omega}_c} \right] \bigg/ \widetilde{\omega}_a, \qquad (6a)$$

$$\langle a \rangle = -\frac{1}{1 - \nu(\mathbf{r})} \left[E + g^*(\mathbf{r}) \frac{\eta(\mathbf{r})}{\widetilde{\omega}_a} \right] \bigg/ \widetilde{\omega}_c, \qquad (6b)$$

where $\nu(\mathbf{r})$ is the generalized cooperativity parameter $\nu = |g|^2/(\widetilde{\omega}_a \widetilde{\omega}_c)$ [12]. For coupled atom-cavity oscillators, those expectation values are solutions to both Hamiltonians (3) and (4) [12]. To obtain the momentum diffusion one calculates the gradients $\nabla\langle\sigma\rangle$ and $\nabla\langle a \rangle$, the position dependence being in $g(\mathbf{r})$, $\nu(\mathbf{r})$, $\eta(\mathbf{r})$, and possibly $\Delta_a(\mathbf{r})$.

A parameter regime, which is often considered theoretically as well as experimentally, is that of large atomic detuning Δ_a and low cavity detuning Δ_c. For large $|\Delta_a|$ the atomic excitation is suppressed so that diffusion due to photon emission into free space can be neglected, whereas a low value for $|\Delta_c|$ would enhance the scattering into the cavity. While in this regime a strong friction force exists, there is also a strong heating rate, which we now estimate. Let us first consider the situation where the cavity is pumped ($E \neq 0, \eta = 0$) and we focus on a linear cavity $g(\mathbf{r}) = g_0 \cos(\mathbf{k}_{cav} \cdot \mathbf{r})$. Assuming $\Delta_c = 0$ and $|\Delta_a| \gg (|g|^2/\kappa, \gamma)$ (such that $|\nu| \rightarrow 0$), one obtains to lowest order in $1/\Delta_a$, $\nabla\langle a\rangle = (\nabla\nu)E/i\kappa + \mathcal{O}(1/\Delta_a^2)$ and $\nabla\langle\sigma\rangle = [-(\nabla g)E/i\kappa][1/(\Delta_a - i\gamma)] + \mathcal{O}(1/\Delta_a^2)$. This gives $D_{mode} \approx 8C D_{atom}$, where $C = |g|^2/2\kappa\gamma$ is the cooperativity parameter. We then notice that for $|\Delta_a| \gg (|g|^2/\kappa, \gamma)$, the value of the diffusion D_{atom} is close to that in free space for a Rabi frequency equal to $2gE/\kappa$ (recall that $|E|^2/\kappa^2$ is the number of photons in the cavity without the atom). When spatially averaged along the cavity axis, the spontaneous emission term is also equal to $\overline{D_{atom}}$. One therefore sees that for an equivalent excitation probability the averaged heating rate for an atom in a cavity is $1 + C_0$ times larger than the total free-space value, where $C_0 = |g_0|^2/2\kappa\gamma$ is C taken for maximum coupling $g = g_0$ (antinode) [10]. For current optical cavities this factor can reach $C_0 \approx 10^2$. That the momentum diffusion for an intracavity atom is C times larger than the free-space value is a quite general rule in the regime of large $|\Delta_a|$ and low $|\Delta_c|$, with numerical factors depending on how the system is probed and on whether spatial averaging is invoked. For example, if the atom is probed directly ($E = 0, \eta \neq 0$), the gradient that dominates along the cavity axis is $\nabla\langle a\rangle = [(\nabla g)/i\kappa][\eta/(\Delta_a - i\gamma)] + \mathcal{O}(1/\Delta_a^2)$, while $\nabla\langle\sigma\rangle$ is at least of order $\mathcal{O}(1/\Delta_a^2)$. Therefore, the term $D_{mode} \approx C_0(\hbar k)^2 \gamma P_e$ is C_0 times larger than the spontaneous emission term.

V. INTERPRETATIONS AND DISCUSSION

Typically, the dynamics of two coupled oscillators is dominated by the one which is closer to resonance with the laser (e.g., $D_{mode} \gg D_{atom}$ for $|\Delta_c| \ll |\Delta_a|$). This is illustrated in Fig. 2 where the normal-mode spectrum appears from the heating rate (at the normal modes we have $\nu \rightarrow 1$, i.e., $\Delta_a\Delta_c \approx |g|^2$). On the right peak, we have a fluctuating cavity field coupled to a mean atomic dipole, while on the left peak the diffusion is dominated by the fluctuations of an atomic
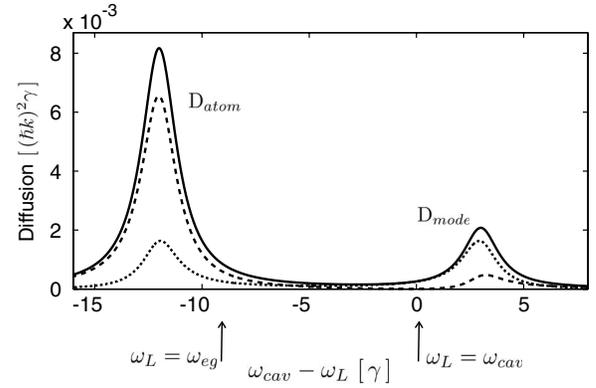

FIG. 2. Diffusion $D_{atom} + D_{mode}$ (solid) along the laser axis when the atom is excited from the side, for a running wave laser, as a function of the laser frequency ω_L ($E = 0, |\eta| = 0.1\gamma, \kappa = \gamma, g_0 = 6\gamma, \omega_{eg} - \omega_{cav} = 9\gamma$). Diffusion due to spontaneous emission is equal to D_{atom}. The peaks display the normal modes, D_{atom} (dashed) dominating for $\omega_L \approx \omega_{eg}$ and far from the cavity resonance ω_{cav} (fluctuating atom), and vice versa for D_{mode} (dotted) dominating for $\omega_L \approx \omega_{cav}$ (fluctuating mode).

dipole coupled to a mean cavity field. It is instructive to compare Fig. 2 with Fig. 4 of Ref. [20], where the normal-mode spectrum (or vacuum Rabi splitting) appeared from the measurement of (the inverse of) the time a trapped atom remained in the cavity. The dramatic reduction of the storage time in this experiment has been attributed to the large value of the diffusion $D_{atom} + D_{mode}$ on the peaks. As we saw, those two terms have relative contributions depending on the detunings and hence give insight into the origin of the appearance of the peaks. Notice that in Ref. [20] the cavity is probed whereas the plot here is for atom pumping. In this case the height of the two peaks in Fig. 2 should be interchanged and one would focus on the diffusion along the cavity axis. This does not change the conclusions regarding the distribution of D_{atom} and D_{mode} on their respective peaks.

For a running wave laser in free space $\eta(\mathbf{r}) = \eta_0 e^{i\mathbf{k}_L \cdot \mathbf{r}}$, the diffusion stems from the random character of the absorption process [21,22]. If N_{abs} is the number of photons absorbed during a time $t \gg 1/2\gamma$, the atom gains the momentum $\mathbf{p} = \hbar\mathbf{k}_L N_{abs}$ (ignoring spontaneous and stimulated emission). In steady state, $\langle N_{abs} \rangle = 2\gamma P_e t$ is given by the spontaneous emission rate $2\gamma P_e$. Since N_{abs} is a random variable, the momentum spreads as $\Delta p^2 = (\hbar\mathbf{k}_L)^2 \Delta N_{abs}^2$. Now, in the harmonic description, the photon statistics follow the Poisson law $\langle \Delta N_{abs}^2 \rangle = \langle N_{abs} \rangle$. Hence the second term of Eq. (1) $|\hbar\nabla\langle\sigma\rangle|^2 2\gamma = (\hbar\mathbf{k}_L)^2 2\gamma P_e$. For comparison, we look at the diffusion along the laser axis, orthogonal to the cavity. Defining $\mathbf{F}_{(a,m)} = -\langle\nabla H_{(atom,mode)}\rangle$ (here $U = \text{const}$) gives $\mathbf{F}_a = \hbar\mathbf{k}_L 2\gamma P_e$ and $\mathbf{F}_m = \hbar\mathbf{k}_L 2\kappa N_{cav}$, where $N_{cav} = \langle a^\dagger a\rangle$ is the cavity photon number. Thus the radiation pressure force acting on the atom $\langle\mathbf{F}\rangle = \hbar\mathbf{k}_L(2\gamma P_e + 2\kappa N_{cav})$ is proportional to the rate at which photons are removed from the laser $d\langle N_{abs}\rangle/dt = 2\gamma P_e + 2\kappa N_{cav}$. The diffusion along the side laser is due to the statistics of photons that are removed from the beam, in short, absorbed by the atom-cavity system. That diffusion is plotted in Figs. 2 and 3. As shown in Fig. 3, for
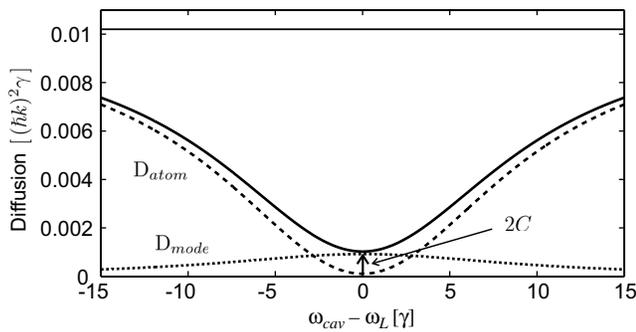
FIG. 3. Diffusion $D_{atom}+D_{mode}$ (solid) along the side laser, resonant to the atom $\omega_L=\omega_{eg}$, vs the cavity frequency ω_{cav} ($E=0$, $|\eta|=0.1\gamma$, $\kappa=\gamma$, $g_0=3\gamma$). The diffusion is well suppressed compared to free space for the same intensity (horizontal line). D_{mode} (dotted) reaches a maximum for $\omega_{cav}=\omega_L$, where one has $D_{mode}=2CD_{atom}\gg D_{atom}$ (dashed), and decreases when the light hardly builds up inside the cavity $\omega_{cav}\neq\omega_L$.

the same laser intensity, the diffusion is suppressed with respect to free space. This is due to the suppression of the atomic fluorescence ($|1-\nu|^2\gg1$). Otherwise, as discussed previously, for a given excitation probability P_e the ratio between the heating in a cavity and that in free space scales like the cooperativity parameter C.

While this diffusion can be well understood in terms of recoil kicks, we now put the atom at a node $g=0$ and consider diffusion along the cavity axis $\nabla g\neq0$. Surprisingly, although the quantum state of the cavity mode is the vacuum $|0\rangle$ (recall that $E=0$), the atom is still "kicked" along the cavity axis $|\hbar\nabla\langle a\rangle|^2 2\kappa\neq0$. Moreover, the heating is proportional to the rate 2κ at which photons are removed from the cavity, but there are no photons to leak out. Another intriguing situation occurs if instead of the cavity the atom is in the ground state $|g\rangle$. For $g\neq0$, this is achieved when in addition to the side-laser running wave a coherent field is injected into the cavity (see Fig. 1). For $\eta=-gE/\tilde{\omega}_c$ one can show that the steady state from Eqs. (3) and (4) is pure $\rho_s=|g\rangle\langle g|\otimes|-E/\tilde{\omega}_c\rangle\langle-E/\tilde{\omega}_c|$, where $|-E/\tilde{\omega}_c\rangle$ is a coherent state of the cavity mode. This suppression of fluorescence occurs under a configuration which is more general than that of Ref. [25] since it accounts for $E\neq0$, and, in particular, it holds true for a real lossy cavity $\kappa\neq0$. The atom is invisible in the cavity

transmission, but it is coupled. Here, no radiation pressure acts on the atom, at rest, although the side laser is switched on. The diffusion, however, is finite.

Such cases resemble the very intriguing situation of an atom at a node of a standing wave, where the intensity locally vanishes $P_e=0$, meaning that there are no fluorescence photons, but $D\neq0$ [22]. The origin of the finite diffusion even when the oscillators are in their ground state is due to the fluctuations ($\delta\sigma,\delta a$) of the atomic coherence σ and mode operator a, which in fact are independent of the intensity. As noted above, they are the only fluctuations that contribute to the diffusion in the harmonic limit, and this is also true for any other location in space (where the oscillators are no more in their ground state). Following a terminology known from free space [23], one would say that the momentum diffusion in the experimentally relevant limit is governed by zero-point fluctuations.

We end this paper with some remarks, which allow us to perceive the physics from a broader perspective. When one looks at the history of interpretations of the forces and their fluctuations in free space (e.g. Ref. [22] and references therein), it is clear that the interpretations crucially depend on the spatial modulation of the laser light. This is justified when saturation effects are considered, because then the form of the diffusion coefficient depends upon the structure of the laser light. But in the harmonic limit, the diffusion reduces to the invariant form (1). Adding the cavity, we found the invariant form (2), which includes Eq. (1) and which is valid regardless of the way the atom-cavity system is excited. Therefore, even if we interpreted the particular case of radiation pressure, and communicated possible links to the problem of the atom at the node of a standing wave, it is appealing to interpret Eq. (2), as much as Eq. (1) in a way that reflects such an invariance. A general interpretation of Eq. (2) is that of a fluctuating atomic dipole coupled to a mean field, $|\hbar\nabla\langle\sigma\rangle|^2 2\gamma$, and a fluctuating cavity field coupled to a mean atomic dipole, $|\hbar\nabla\langle a\rangle|^2 2\kappa$.

ACKNOWLEDGMENTS

[1] T. W. Mossberg, M. Lewenstein, and D. J. Gauthier, Phys. Rev. Lett. **67**, 1723 (1991).

[2] T. Zaugg, P. Meystre, G. Lenz, and M. Wilkens, Phys. Rev. A **49**, 3011 (1994).

[3] A. C. Doherty, A. S. Parkins, S. M. Tan, and D. F. Walls, Phys. Rev. A **56**, 833 (1997).

[4] P. Horak, G. Hechenblaikner, K. M. Gheri, H. Stecher, and H. Ritsch, Phys. Rev. Lett. **79**, 4974 (1997).

[5] G. Hechenblaikner, M. Gangl, P. Horak, and H. Ritsch, Phys. Rev. A **58**, 3030 (1998).

[6] V. Vuletić and S. Chu, Phys. Rev. Lett. **84**, 3787 (2000).

[7] S. J. van Enk, J. McKeever, H. J. Kimble, and J. Ye, Phys. Rev. A **64**, 013407 (2001).

[8] V. Vuletić, H. W. Chan, and A. T. Black, Phys. Rev. A **64**, 033405 (2001).

[9] P. Domokos, P. Horak, and H. Ritsch, J. Phys. B **34**, 187 (2001).

[10] T. Fischer, P. Maunz, T. Puppe, P. W. H. Pinkse, and G. Rempe, New J. Phys. **3**, 11.1 (2001).

[11] P. Domokos, T. Salzburger, and H. Ritsch, Phys. Rev. A **66**, 043406 (2002).

[12] K. Murr, J. Phys. B **36**, 2515 (2003).

[13] P. Domokos, A. Vukics, and H. Ritsch, Phys. Rev. Lett. **92**, 103601 (2004).

[14] P. Maunz, T. Puppe, I. Schuster, N. Syassen, P. W. H. Pinkse, and G. Rempe, Nature (London) **428**, 50 (2004).

[15] S. Nußmann, K. Murr, M. Hijlkema, B. Weber, A. Kuhn, and G. Rempe, Nat. Phys. **1**, 122 (2005).

[16] P. Münstermann, T. Fischer, P. Maunz, P. W. H. Pinkse, and G. Rempe, Phys. Rev. Lett. **82**, 3791 (1999).

[17] J. Ye, D. W. Vernooy, and H. J. Kimble, Phys. Rev. Lett. **83**, 4987 (1999).

[18] J. McKeever, J. R. Buck, A. D. Boozer, A. Kuzmich, H.-C. Nägerl, D. M. Stamper-Kurn, and H. J. Kimble, Phys. Rev. Lett. **90**, 133602 (2003).

[19] A. Boca, R. Miller, K. M. Birnbaum, A. D. Boozer, J. McK-

eever, and H. J. Kimble, Phys. Rev. Lett. **93**, 233603 (2004).

[20] P. Maunz, T. Puppe, I. Schuster, N. Syassen, P. W. H. Pinkse, and G. Rempe, Phys. Rev. Lett. **94**, 033002 (2005).

[21] R. J. Cook, Phys. Rev. A **22**, 1078 (1980).

[22] C. Cohen-Tannoudji, in *Fundamental Systems in Quantum Optics*, edited by J. Dalibard, J. M. Raimond, and J. Zinn-Justin, Les Houches, Session LIII, 1990 (Elsevier Science, North-Holland, Amsterdam, 1992), p. 1.

[23] J. P. Gordon and A. Ashkin, Phys. Rev. A **21**, 1606 (1980).

[24] J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **2**, 1707 (1985).

[25] P. M. Alsing, D. A. Cardimona, and H. J. Carmichael, Phys. Rev. A **45**, 1793 (1992).