Proceedings of 25th Conference
on Decision and Control
Athens, Greece • December 1986

L_∞ - APPROXIMATIONS OF NONRATIONAL TRANSFER FUNCTION : AN EXAMPLE.

R.F. Curtain and H.J. Zwart

Institute of Mathematics, University of
Groningen, P.O. Box 800, 9700 AV Groningen,
The Netherlands.

1. Summary

In this paper we shall show that delay transfer functions can be well approximated by a finite dimensional system.

2. Theory

In Curtain and Zwart [2,3] it was shown how one could obtain low order L_∞-approximations of scalar delay systems of the general form

$$G(s) = \frac{\sum_{i=0}^{n_1} p_i(s)e^{-\gamma_i s}}{\sum_{j=1}^{n_2} q_j(s)e^{-\alpha_j s}}, \qquad (2.1)$$

where $0=\gamma_0<\gamma_1<\ldots<\gamma_{n_1}, 0\leq\alpha_1<\ldots<\alpha_{n_2}$ and p_i, q_j are polynomials.

The key lies in first obtaining a partial fraction expansion for the transfer of the form

$$G(s) = \sum_{j=1}^{\infty} \frac{\mathrm{Res}(G;z_j)}{s-z_j}, \qquad (2.2)$$

where z_j are the poles of G and $\mathrm{Res}(G;z_j)$ the residue at pole z_j.

If $\sum_{j=1}^{\infty} \left|\frac{\mathrm{Res}(G;z_j)}{\mathrm{Re} z_j}\right| < \infty$, the transfer function is of nuclear type and the L_∞-approximation techniques of Glover, Curtain and Partington [4] apply. In fact what one does is to first make a modal approximation of (high) order N

$$G^N(s) = \sum_{j=1}^{N} \frac{\mathrm{Res}(G;z_j)}{s-z_j} \qquad (2.3)$$

for which we know that

$$\| G(s) - G^N(s) \|_\infty \leq \sum_{j=N+1}^{\infty} \left| \frac{\mathrm{Res}(G;z_j)}{\mathrm{Re}\, z_j} \right| \qquad (2.4)$$

Then we can apply finite dimensional methods to find a low order L_∞-approximation for $G^N(s)$. (cf Curtain and Glover [1]) Here we apply this approach to the following delay system.

3. Example.

$$\dot{x}_1(t) = -x_1(t) + e^{-2}.x_1(t+1) + x_2(t)$$

$$\dot{x}_2(t) = -2x_2(t) + u(t)$$

$$y(t) = x_1(t)$$

This system has transfer function

$$G(s) = \frac{1}{(s+2)(s+1-e^{-2-s})}$$ and the poles are given by,

(Ito [5])

No.	Poles
1	-0.721535
2	-2
3,4	-3.588317 ± 4.155305 i
5,6	-4.417631 ± 10.68603 i
7,8	-4.861502 ± 17.05611 i
9,10	-5.167754 ± 23.38558 i
11,12	-5.401945 ± 29.69798 i
13,14	-5.591627 ± 36.00146 i
15,16	-5.751047 ± 42.29965 i
17,18	-5.888543 ± 48.59442 i
19,20	-6.009422 ± 54.88686 i
21,22	-6.117267 ± 61.17761 i
23,24	-6.214618 ± 67.46710 i

From the 24 th order approximation $G^{24}(s)$, of G(s) we make a second order approximation of G(s), by taking the second order balanced approximation, $G_2^{24}(s)$ of $G^{24}(s)$

The singular values of $G^{24}(s)$ are given by:

3.3244E-001	6.0623E-006
4.2580E-002	5.2561E-006
1.3698E-003	3.4373E-006
7.4542E-004	3.3142E-006
1.8456E-004	2.1781E-006
8.5399E-005	2.0217E-006
4.4582E-005	1.3747E-006
2.9246E-005	1.3374E-006
2.3230E-005	9.1356E-007
2.2511E-005	9.0001E-007
1.2450E-005	6.1742E-007
9.7036E-006	6.0394E-007

The second order approximation $G_2^{24}(s) = C(s-A)^{-1}B$ is given by

$$A = \begin{bmatrix} -0.36858 & 0.84906 \\ -0.84906 & -2.9023 \end{bmatrix} \quad ; B = \begin{bmatrix} 0.49504 \\ 0.49715 \end{bmatrix}$$

$$C = (0.49504 \quad -0.49715).$$

We have that $\| G(s) - G^{24}(s) \|_\infty \approx 6.10^{-5}$, and the next figures will show how well G(s) is approximated by $G_2^{24}(s)$.

Figure 1: Nyquist plot of G(s)
Figure 2: Nyquist plot of $G(s) - G_2^{24}(s)$.
Figure 3: Step response of G(s).
Figure 4: Difference between the step response of G(s) and $G_2^{24}(s)$.

4. References

1. R.F. CURTAIN and K. GLOVER, Controller Design for Distributed Systems based on Hankel-norm Approximation, 1986, IEEE Trans, AC-31, pp. 173-176.

2. R.F. CURTAIN and H.J. ZWART, L_∞-approximations for Delay Systems, submitted to Systems and Control Letters, 1986.

3. R.F. CURTAIN and H.J. ZWART, Spectral Realisations for Delay Systems, presented at the International Conference on Control of Distributed Parameter Systems, July 6-12, 1986, Vorau.

4. K. GLOVER, R.F. CURTAIN and J. PARTINGTON, Realisation and Approximation of Linear Infinite Dimensional Systems with Error Bounds, Report CUED/F-CAMS/TR.258, Cambridge University, 1986.

5. K. ITO, On the Approximation of Eigenvalues associated with Functional Differential Equations, ICASE Report No. 82-29, September 23, 1982.

FIGURE 1

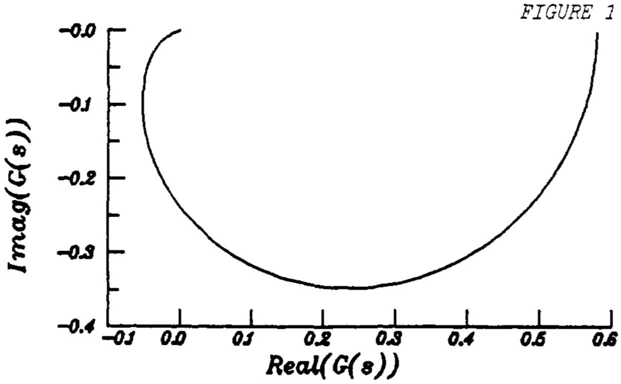


FIGURE 2

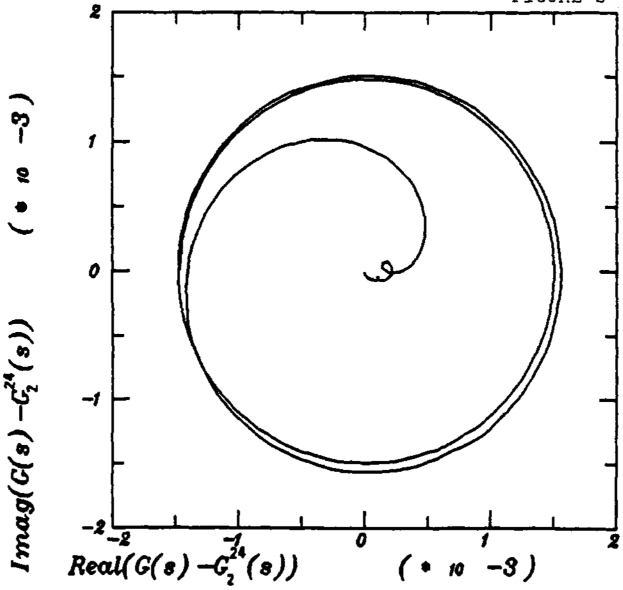


FIGURE 3

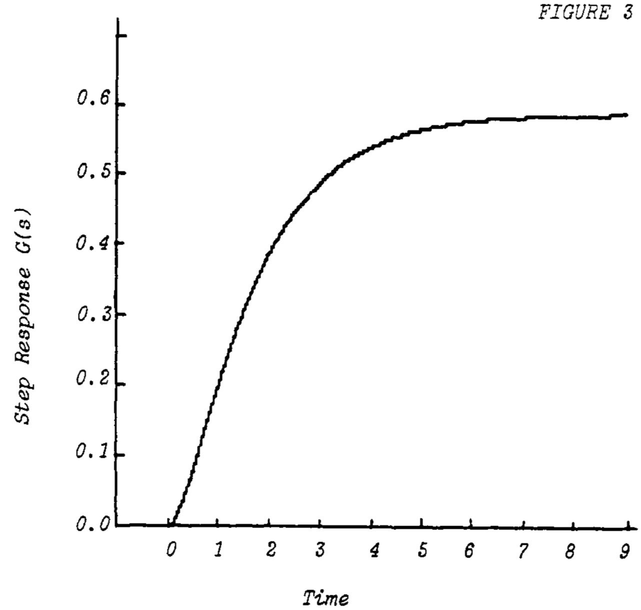
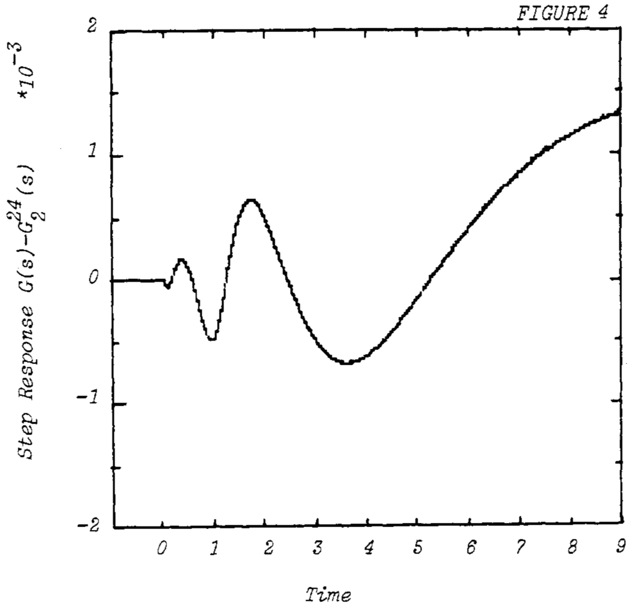


FIGURE 4



Remark concerning figure 4:

The maximal difference between the step response of G(s) and $G_2^{24}(s)$ is $1.4*10^{-3}$ and this is already reached at time is 9.