

Strong and weak rarity value in Small Fish Wars

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Abstract

We analyze the effects of rarity value on the economic and ecological sustainability of a natural resource. Rarity value means that under extreme scarcity of the resource unit profits increase ‘explosively’. We focus on equilibrium behavior of very patient agents in a Small Fish War. In such a setting, agents interacting on a body of water have two options: they can fish with restraint or without. Fishing with restraint allows the fish stock to recover; fishing without yields higher immediate but lower future catches.

We make a distinction between weak and strong rarity value. In the weak variant, equilibrium behavior induces high sustainable fish stocks and long-run yields. In the strong one, very high equilibrium rewards may be obtained by almost exhausting the resource, but more moderate equilibrium rewards are feasible without endangering sustainability.

Keywords: common pool resource games, rarity value

1 Introduction

We analyze strategic interaction in a fishery under rarity value (Courchamp *et al.* [2006]). Rarity value is a price-scarcity relationship in which unit prices of a commodity increase sharply as it becomes less and less available. In the case of a natural resource, this may induce the following scenario. Once a species becomes rare, its value may increase, this may induce greater incentives to exploit the resource, leading to even greater rarity, hence a higher value etc. As a result an Allee effect may occur, i.e., the population size or density may be pushed below a threshold beyond which only negative growth rates exist. Courchamp *et al.* [2006] list an impressive number of real-world cases showing that rarity value inducing an Anthropogenic Allee Effect (*AAE*) is not an armchair-scientist’s oddity.

We augment the *Small Fish War*¹ of Joosten [2007] with an *AAE* and with a mechanism inducing rarity value. In a *Small Fish War*, two agents possess the fishing rights to a body of water, and they have essentially two options, to fish with or without restraint. Restraint in practice may take various forms, e.g., on catching seasons, on quantities caught, on technologies, e.g., boats, nets. Essential is that unrestrained fishing yields a higher immediate catch, but continued unrestrained fishing may lead to decreasing future catches; restrained fishing by both agents is sustainable.

In a *Small Fish War* agents wish to maximize their average catches over an infinite time-horizon. In such a setting, a ‘tragedy of the commons’ is not inevitable, as Pareto-efficient outcomes can be sustained by subgame perfect equilibria inducing rather high fish stocks. In a wide range of the parameter space of the model, the more the catches deteriorate due to over-fishing, the greater the gap between Pareto-efficient and the ‘never restraint’ outcomes. Even equilibria which are most harmful sustain stocks well above ‘never-restraint’ ones. On the other hand, ‘perfect restraint’ is never Pareto-efficient *in the same parameter range*.

Courchamp *et al.* [2006] disregard several subtleties which we address here. For instance, the analysis largely relies on considering unit profits and agents discount the future so heavily that the infinite stream of future profits evaluated at each point in time, is sufficiently similar to the prevailing one-shot situation. Also, the influence of one agent on the resource and on the other agents is negligible, but collectively they can really harm the resource and each other. Moreover, to reach the scarcity region where prices indeed explode to levels dwarfing search costs relatively, a region may have to be crossed in which profits are very low or even negative. Courchamp *et al.* [2006] can not explain how this region is crossed in a consistent manner.

To deal with the subtleties mentioned, we analyze *total profits*, i.e., unit profits times quantities, *averaged over an infinite period* by very *patient* agents with (possibly) *bounded catching capacities* in an *interactive decision making* framework with both short and long term strategic externalities in our newly designed *Small Fish War*. A subtlety discovered while examining this model was that it is crucial to make a distinction between strong and weak rarity value, and we propose an operational criterion to separate one from the other.

Our analysis shows that a *Small Fish War* under strong rarity value may very well exhibit the environmental and economic effects sketched by Courchamp *et al.* [2006], i.e., ‘no restraint’ is the Pareto-efficient equilibrium, or if that would bring about the *AAE*, then behavior inducing stocks just above the Allee threshold is consistent with Pareto-efficiency. It is also immediate and without contradiction that the agents (may be willing and

¹The *Great Fish War* of Levhari & Mirman [1980] reveals that various regimes of strategic interaction induce a ‘tragedy of the commons’ (Hardin [1968]).

able to) achieve the necessary scarcity levels to exploit strong rarity value. Contrastingly, weak rarity value leads to ‘almost perfect restraint’ being Pareto-efficient equilibria, to high fish stocks and to rewards slightly higher than ‘perfect restraint’ rewards. Errors in assessment of the nature of rarity value may lead to a ‘tragedy of the commons’, i.e., deterioration of both the economic and the resource systems, or to a ‘tragedy of the herdsmen’, i.e., missed opportunities for the economic system.

Both variants of rarity value leave ample room to serve economic and ecological goals simultaneously as many equilibria yield sustainable rewards above ‘perfect restraint’. This suggests a basis for *effective* policies for management and conservation of natural resources as Holden [1994] attributes *ineffectiveness* of for instance the common fishery policies of the European community to their biological focus instead of an economic one (see also e.g., Brooks *et al.* [2008], BenDor *et al.* [2008], Sanchirico *et al.* [2007]). The immense dangers of strong rarity value can be controlled if the resource is managed diligently and strictly, e.g., according to the Precautionary Approach of the International Council for the Exploration of the Sea (cf., e.g., ICES [2005]), and its economic opportunities can be quite considerable.

Next, we review the *Small Fish War* and then add an Allee effect under the assumption that unit profits are fixed. In Section 3 we examine the combined effects of rarity value and the Anthropogenic Allee Effect. Section 4 concludes with a discussion.

2 Small Fish Wars and impacts of Allee effects

A *Small Fish War* is played by players A and B at discrete moments in time called stages. Each player has two actions and each stage players choose an action independently and simultaneously. We denote the action set of player A (B) by $J^A = \{0, 1\}$ ($= J^B$) and $J \equiv J^A \times J^B$. Action 1 for either player denotes the action without restraint, e.g., catching with fine-mazed net or catching a high quantity; the other one is the action with a restriction. The payoffs at stage $t' \in \mathbb{N}$ of the play depend on the choices of the players at that stage, and on the relative frequencies with which all actions were actually chosen until then.

Crucial is the *current rate of overfishing*, i.e., how often the agents have caught without restraint until then. Let j_t^A (j_t^B) be the action chosen by player A (B) at stage $t \geq 2$, and let $q \geq 0$. Define the **current rate of overfishing** ρ_t recursively by

$$\rho_1 = \rho \in [0, 1], \text{ and } \rho_t = \frac{q + t - 1}{q + t} \rho_{t-1} + \frac{1}{q + t} \left(\frac{j_{t-1}^A + j_{t-1}^B}{2} \right). \quad (1)$$

So, ρ is the rate of overfishing taken at the start of the period analyzed. A completely untouched system has $\rho = 0$; a system where overfishing has gone

on for prolonged periods of time has $\rho \approx 1$. Parameter q helps to moderate early effects, as an early decision has a very jumpy effect on the rate of overfishing compared to the same decision later on. Our analysis focuses on long term horizons for which the values of ρ and q become irrelevant.

The *fish stocks* depend on the current rate of overfishing ρ_t . Let

$$\mu_t \equiv 1 + (1 - \underline{m}) \left[\frac{n_2}{n_1 - n_2} \rho_t^{n_1} - \frac{n_1}{n_1 - n_2} \rho_t^{n_2} \right]. \quad (2)$$

Here, μ_t is the **fish stock** (normalized to the unit interval), where $\mu_t = 0$ ($\mu_t = 1$) indicates that the fish stock is depleted (at full capacity). The parameter $\underline{m} \in [0, 1]$ represents the **minimal stock due to overexploitation** by the agents, i.e., if steady overexploitation leads to $\rho_t \rightarrow 1$, then the fish stock will be at \underline{m} . So, (2) determines how the fish stock deteriorates from its maximum due to fishing without restraint. The parameters $n_1 > n_2 > 1$ determine the shape of the function connecting the normalized fish stocks to the rate of overfishing. Both n_1, n_2 are determined by biological and ecological features, and one obtains the familiar inverted S-curves for large ranges of these parameters. The minimal fish stock \underline{m} depends partly on ecological factors, but, for the sake of discussion, should be seen as predominantly depending on technology, and on the relative size of the agents with respect to the body of water they operate on.

At each stage a bi-matrix game is played, and the choices of the players at that stage determine their stage payoffs. Let the **stage payoffs at stage** $t \in \mathbb{N}$ be represented by

$$\begin{bmatrix} a\mu_t, a\mu_t & b\mu_t, c\mu_t \\ c\mu_t, b\mu_t & d\mu_t, d\mu_t \end{bmatrix}. \quad (3)$$

If player A chooses action 0 and B chooses action 1, A 's stage payoff is $b\mu_t$ and B 's is $c\mu_t$. We assume that fishing without restraint yields a higher catch in any current stage than fishing with restraint, hence $a < c$, $b < d$. We assume that two-sided catching without restraint yields higher immediate payoffs than catching with restraint, i.e., $a < d$. The unique stage-game equilibrium is the strategy pair in which both players use action 1. Observe that $\underline{m} = 1$ yields a standard repeated game.

If both agents never show restraint, then the associated long run stage payoffs are $d\underline{m}$; if they show perfect restraint, these payoffs are a . In the remainder we make assumptions which guarantee that the problem at hand falls into the category of social dilemmas (see e.g., Komorita & Parks [1996]).

Remark 1 *'Never restraint' gives at most half the long-run stage payoffs associated with 'perfect restraint', i.e., $d\underline{m} \leq \frac{a}{2}$; the sharpest decline of the stock occurs for rates of overfishing between 0.25 and 0.75.*

2.1 Strategies and rewards

The sets of all strategies for A (B) is denoted by \mathcal{X}^A (\mathcal{X}^B). The payoff to player k , $k = A, B$, at stage t , is stochastic and depends on the strategy-pair $(\pi, \sigma) \in \mathcal{X}^A \times \mathcal{X}^B$; the **expected stage payoff** is denoted by $R_t^k(\pi, \sigma)$.

The players receive an infinite stream of stage payoffs during the play, and they are assumed to wish to maximize their average rewards. For a given pair of strategies (π, σ) , player k 's **average reward**, $k = A, B$, is given by $\gamma^k(\pi, \sigma) = \liminf_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T R_t^k(\pi, \sigma)$; $\gamma(\pi, \sigma) \equiv (\gamma^A(\pi, \sigma), \gamma^B(\pi, \sigma))$. First, we focus on rewards from strategies which are pure and jointly convergent. Then, we extend our analysis to obtain larger sets of feasible rewards.

A strategy is **pure**, if at *each* stage a **pure action** is chosen, i.e., an action is chosen with probability 1. The set of pure strategies for player k is \mathcal{P}^k , and $\mathcal{P} \equiv \mathcal{P}^A \times \mathcal{P}^B$. The strategy pair $(\pi, \sigma) \in \mathcal{X}^A \times \mathcal{X}^B$ is **jointly convergent** if and only if $z \in \Delta^{m \times n}$ exists such that for all $\varepsilon > 0$, $(i, j) \in J$:

$$\limsup_{t \rightarrow \infty} \Pr_{\pi, \sigma} \left[\left| \frac{\#\{j_u^A=i \text{ and } j_u^B=j \mid 1 \leq u \leq t\}}{t} - z_{i+1, j+1} \right| \geq \varepsilon \right] = 0,$$

where $\Delta^{m \times n}$ denotes the set of all nonnegative $m \times n$ -matrices such that the entries add up to 1; $\Pr_{\pi, \sigma}$ denotes the probability under strategy-pair (π, σ) . \mathcal{JC} denotes the set of jointly-convergent strategy pairs. The **set of jointly-convergent pure-strategy rewards** $P^{\mathcal{JC}}$ is then the set of pairs of rewards each of which can be obtained by using a pair of jointly-convergent strategies. Under such a pair of strategies, the relative frequency of action pair $(i, j) \in J$ converges to a fixed number (with probability 1 to $z_{i+1, j+1}$ in the terminology of Billingsley [1986, p.274]).

With respect to jointly-convergent strategies, Eq. (2) and the arguments presented imply that

$$\lim_{t \rightarrow \infty} \mu_t = 1 + (1 - \underline{m}) \left(\frac{n_2}{n_1 - n_2} \rho^{n_1} - \frac{n_1}{n_1 - n_2} \rho^{n_2} \right),$$

where $\rho \equiv z_{22} + \frac{1}{2}(z_{12} + z_{21})$. So, the expected long-term fish stock converges to a fixed number as well, hence the bi-matrices representing the stage payoffs in Eq. (3) ‘converge’ in the long run, too.

To compute the rewards connected to a pair of jointly-convergent strategies is then a matter of simple ‘book keeping’. Let

$$\varphi(z) \equiv \lim_{t \rightarrow \infty} \mu_t \sum_{(i, j) \in J} z_{i+1, j+1} (a_{i+1, j+1}, b_{i+1, j+1}).$$

Here, $(a_{i+1, j+1}, b_{i+1, j+1})$ is the entry in Eq. (3) corresponding to action pair $(i, j) \in J$. The interpretation of $\varphi(z)$ is that under jointly-convergent strategy pair (π, σ) the relative frequency of action pair $(i, j) \in J$ being chosen is $z_{i+1, j+1}$ and each time this occurs the players receive $\lim_{t \rightarrow \infty} \mu_t$

times the associated entry in Eq. (3) in the long run. Hence, the players receive an average amount of $\varphi(z)$. So, $\gamma(\pi, \sigma) = \varphi(z)$. Figure 1 gives an illustration of the set of rewards from jointly-convergent strategies.

2.2 Threats and equilibria

The strategy pair (π^*, σ^*) is an **equilibrium**, if no player can improve by unilateral deviation. An equilibrium is called **subgame perfect** if for each possible state and possible history (even unreached states and histories) the subsequent play corresponds to an equilibrium, i.e., no player can improve by deviating unilaterally from then on. In the construction of equilibria for repeated games, ‘threats’ play an important role. A threat specifies the conditions under which one player will punish the other, as well as the subsequent measures. More details are given in e.g., Joosten *et al.* [2003].

We call $v = (v^A, v^B)$ the **threat point**, where $v^A = \min_{\sigma \in \mathcal{X}^B} \max_{\pi \in \mathcal{X}^A} \gamma^A(\pi, \sigma)$, and $v^B = \min_{\pi \in \mathcal{X}^A} \max_{\sigma \in \mathcal{X}^B} \gamma^B(\pi, \sigma)$. So, v^A is the highest amount A can get if B tries to minimize his average payoffs. Under a pair of **individually rational** (feasible) rewards each player receives at least the threat-point reward. We can now present the major result of Joosten [2007].

Theorem 1 *Each pair of rewards in the convex hull of all jointly-convergent pure-strategy rewards giving each player strictly more than the threat-point reward, can be supported by a subgame-perfect equilibrium.*

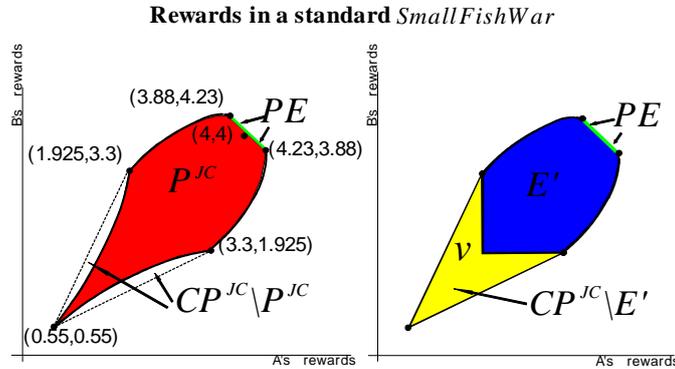


Figure 1: Here, $\underline{m} = 0.1$, $n_1 = 3$, $n_2 = 2$, $a = 4$, $b = 3.5$, $c = 6$, $d = 5.5$. PE denotes Pareto efficient rewards. Left: the red area denotes P^{JC} , the set of jointly-convergent pure-strategy rewards; CP^{JC} is its convex hull. Right: the blue area E' represents equilibrium rewards; $v \approx (1.925, 1.925)$.

The following consequence of Theorem 1 is illustrated in Figure 1.

Corollary 2 Let $E' = \{(x, y) \in P^{\mathcal{J}C} \mid (x, y) > v\}$, then each pair of rewards in the convex hull of $cl E'$ can be supported by an equilibrium. Moreover, all rewards in E' can be supported by a subgame-perfect equilibrium.

Remark 2 We introduce the **relative improvement over perfect restraint** Δ defined as the total symmetric Pareto-efficient rewards minus the total ‘perfect restraint’ rewards relative to the total ‘perfect restraint’ rewards. In the example used for Figure 1,

$$\Delta \approx \frac{8.11 - 8}{8} = 0.01375.$$

Pareto-efficient equilibria yield similar combined improvements, and yield more than seven times the combined ‘never restraint’ rewards (0.55, 0.55). Furthermore, these equilibria induce play in which both players simultaneously show restraint for about 85.6% of the stages; otherwise precisely one shows restraint.

2.3 A Small Fish War with an Allee effect

We now introduce an Allee effect into the *Small Fish War*. The following quote may be found in Bercé *et al.* [2006]: ‘Allee effects occur whenever fitness of an individual in a small or sparse population decreases as the population size or density declines’. Courchamp *et al.* [2006] explain: ‘Populations suffering from Allee effects may exhibit negative growth rates at low densities, which drives them to even lower densities and ultimately to extinction’. Bercé *et al.* [2006] also define an Allee threshold as the ‘critical population size or density below which the per capita population growth rate becomes negative’.

Let therefore Th denote an Allee threshold measured in the same dimension as the fish stock. We formalize the explanations above by

$$\begin{aligned} \mu_t &= 1 + (1 - \underline{m}) \left[\frac{n_2}{n_1 - n_2} \rho_t^{n_1} - \frac{n_1}{n_1 - n_2} \rho_t^{n_2} \right] \text{ if } \mu_s \geq Th \text{ for all } s \leq t, \\ \mu_t &\leq (1 - \theta)^{\gamma(t-s')} \text{ for all } t \geq s' \text{ if } \mu_{s'} < Th \text{ where } \gamma \geq 1, \theta \in (0, 1). \end{aligned} \quad (4)$$

The second part of (4) captures the Allee effect in a rather general manner implying that the population decreases (at least) *exponentially*. Hence, if under strategy pair (π, σ) the fish stock at any point in time drops below the Allee threshold, then $\lim_{t \rightarrow \infty} \mu_t = 0$; we normalize the associated rewards to $\gamma(\pi, \sigma) = (0, 0)$ and call them Collapse Rewards, since the resource system as well as the economic system depending on it break down.

Figure 2 visualizes sets of rewards under the Allee effect; without this effect, $P^{\mathcal{J}C}$ would have been pointed in the South-West as in Figure 1. Equilibrium rewards are quite far removed from the Collapse Rewards. This means that self-interested rational agents will behave in the interest of the

Rewards in a *SmallFishWar* with an AlleeEffect

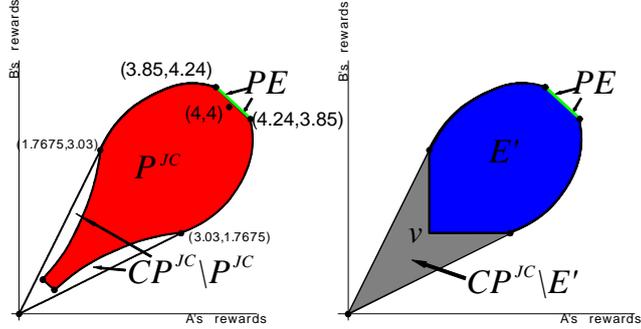


Figure 2: P^{JC} (red), CP^{JC} , PE (green) and E' (blue) for $\underline{m} = 0.01$ and $Th = 0.1$, all other parameters remain as in Figure 1. Collapse Rewards are $(0, 0)$, equilibrium rewards are to the North-East of $v \approx (1.7675, 1.7675)$.

environmental system in order to guarantee high fish stocks staying far above the Allee threshold. Here, $\Delta \approx 0.011$. The Allee threshold influences the set of *equilibrium rewards only* if it is rather high, the set of *equilibrium strategies* is obviously reduced for any level of the threshold.

3 Rarity value

The *Small Fish War* and its extension presented implicitly model a situation in which agents sell their catches at a competitive market while incurring fixed unit search costs, at least fixed with respect to the scarcity of the resource in their fishing environment. Alternatively, if neither prices on the market, nor search costs are fixed, then one can regard the model as pertaining to a situation in which unit prices go up approximately in the fashion as the unit search costs do for increasing scarcity.

In some cases, fishermen extract less and less in quantities, nevertheless obtain higher and higher revenues. An ‘ongoing’ real-world example of such an anomaly might be the market for bluefin tuna (*Thunnus Thynnus*). Stocks seem to have decreased by 80 percent in the five years prior to 2007, and prices have sky-rocketed in 2007 especially in the Far East (e.g., Veldkamp [2007]). In economics a range of similar anomalies are known as Veblen and status goods (cf., e.g., Leibenstein [1950]), but these are hardly ever linked to exhaustible resources, let alone animal species facing extinction. Courchamp *et al.* [2006] model and analyze the latter aspect, and cite real-world cases in which prices for certain rare animals increase more than

search costs leading to the extinction of these endangered species.

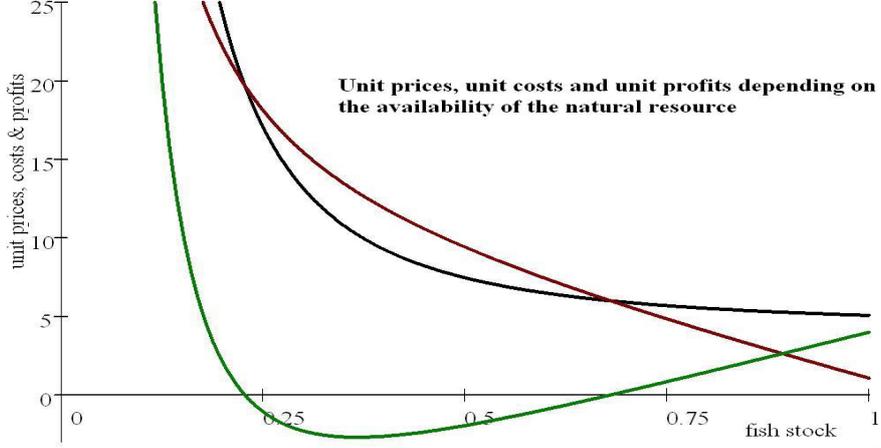


Figure 3: The unit costs (red), unit prices (black) and unit profits (green) as functions of the level of the fish stock.

Figure 3 is an adaptation of a poignant figure from Courchamp *et al.* [2006]. The unit costs of catching depend on availability, to be captured by the fish stock μ in our model (on the horizontal axis). Unit costs increase as the species becomes rarer, i.e., μ becomes smaller. Unit prices remain nearly constant between $\mu = 1$ and $\mu = 0.6$, but for lower availability of the fish stock they go up sharply. The formula for the unit profit curve is

$$\pi(\mu) = p(\mu) - c(\mu) = \frac{4}{3.75} \left(\left(4 + 0.75 \frac{1}{\mu^2} \right) - \left(12 - 12\mu + \frac{1}{\mu^{1.5}} \right) \right). \quad (5)$$

where $p(\mu)$ is the unit price to be obtained on the market and $c(\mu)$ is the unit costs given the (normalized) fish stock μ . Note that that $\pi(1) = 4$ and that $\lim_{\mu \downarrow 0} \pi(\mu)$ does not exist.²

So, unit profits decrease as fish stocks decrease from maximal level, because the unit price remains almost constant, but unit search costs increase steadily. If the fish stock continues to fall below approximately 0.675, unit profits become negative, i.e., the agents would incur losses by catching fish. However, if the fish stock would fall below approximately 0.228, then the unit price driven by scarcity exceeds unit costs again. Moreover, increasing scarcity causes the unit price to increase more than the unit costs from then on. According to Courchamp *et al.* [2006] such a profit-scarcity relationship spells doom for the survival of the animal species.

²To allow comparison numbers have been scaled, i.e., unit profit is 4 if $\mu = 1$.

Figure 4 illustrates the effects of adding Eq. (5) to the model used for expository purposes. Since a range with negative unit profits exists, we find negative average rewards whereas in the original game all rewards were positive. For fish stocks with $\mu < 0.228$ unit profits increase steadily as fish stocks decline. The Pareto-dominant rewards are the ‘no-restraint’ rewards. Note that the latter rewards are located in the South-West in Figure 1, but in the North-East in Figure 4. ‘Perfect restraint’ provides relatively high rewards of (4, 4), but belongs to the Pareto-efficient set of jointly-convergent pure-strategy rewards in neither figure.

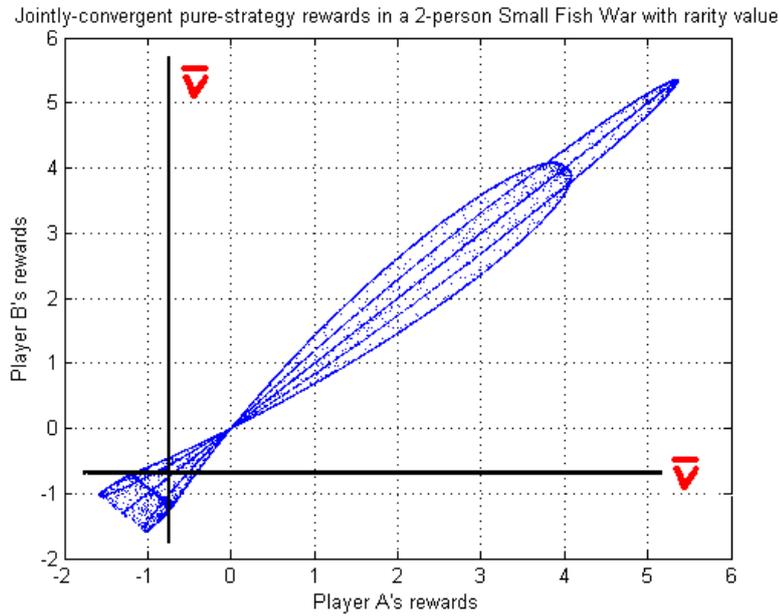


Figure 4: $P^{\mathcal{J}\mathcal{C}}$ for the parameters as before, but with $Th_{AAE} = \underline{m} = 0.1$ and π . Matlab generated 250,000 pairs of rewards, unevenly distributed, hence clusters occur (the heavy ‘lines’). In reality, the set is dense. The system displays strong rarity value as $\Delta/\Delta_{Base} \approx 0.34/0.01375 > 1$. Equilibrium rewards are to the ‘North-East’ of (\bar{v}, \bar{v}) .

We have the following result pertaining to the threat point $v = (v^A, v^B)$, the proof is straightforward, therefore omitted.

Lemma 3 *For the model introduced, we have $v^A, v^B \leq \bar{v} = \min\{\frac{a}{4}, \frac{b}{4}\} \times (\frac{1}{2}(1 + \underline{m})) \pi_t (\frac{1}{2}(1 + \underline{m}))$.*

Given $v \leq (\bar{v}, \bar{v})$, the following is trivially valid in view of Theorem 1.

Corollary 4 *Let $E' = \{(x, y) \in P^{\mathcal{J}^C} \mid (x, y) > (\bar{v}, \bar{v})\}$, then each pair of rewards in the convex hull of $cl E'$ can be supported by an equilibrium. Moreover, all rewards in E' can be supported by a subgame-perfect equilibrium.*

The Pareto optimal equilibrium in Figure 4 is ‘no restraint’ which yields approximately (5.36, 5.36) and induces minimal fish stock. Note that this implies that $\Delta \approx 0.34$, a 34% relative improvement over ‘perfect restraint’. Clearly, the price effect more than compensates the effect of low catches.

Remark 3 *To distinguish **weak** and **strong rarity value**, we propose the following criterion. Let Δ_{Base} be the relative improvement over perfect restraint if unit profits were fixed at the ‘base’ level, i.e., for maximal availability of the resource. Then, we say that the system displays **weak rarity value** if $\Delta/\Delta_{Base} \leq 1$, and **strong rarity value** otherwise. Alternatively stated, the system displays strong rarity value if and only if the symmetric Pareto-efficient rewards in the model with rarity value exceed those in the model without. In the model leading up to Figure 4, $\Delta/\Delta_{Base} \approx 0.34/0.01375 > 1$.*

3.1 Rewards, rarity value and parameter choices

Here, we identify two factors decisive for the sustainability of the resource system. The first one is related to the ‘actual harm’ caused by persistent unrestricted catching. The second is that under the evaluation criterion chosen unit profits are not the real issue, but long-term average total profits.

3.1.1 The influence of minimal fish stock

In Figure 5, as in the original Small Fish War, Pareto-optimal equilibria give rewards quite close to and slightly above 4 and a large proportion of the catches must be restrained. Several equilibrium rewards may be obtained in two different ways. One way is to obtain the equilibria by both agents being fairly modest in the propensity to catch without restraint, the other is by both agents catching without restraint quite frequently. This can be seen in Figure 5, as those rewards situated in the beak in Figure 4, have withdrawn into the ‘body of the fish shape’. They are visible as the heavier lines in the interior of the ‘body’. Here, $\Delta/\Delta_{Base} \approx 0.007/0.0121$, so this system displays weak rarity value.

Rarity value effects prevail as in the case related to Figure 4 with the ‘beak’ becoming larger for smaller \underline{m} . Hence, if the catching capacity increases, the induced unique Pareto-efficient equilibrium yields increasing rewards. This need not imply exhaustion of the resource, but that the agents may bring about the lowest possible fish stock deliberately. For $\underline{m} = 0.06$, $Th_{AAE} = 0$, and all other parameters as before, we found that the Pareto-efficient equilibrium yields (11.705, 11.705), and $\Delta/\Delta_{Base} \approx 1.926/0.0116$ which implies strong rarity value.

A 2-person Small Fish War with rarity value for a high value of the minimal fish stock:
Jointly-convergent pure-strategy rewards

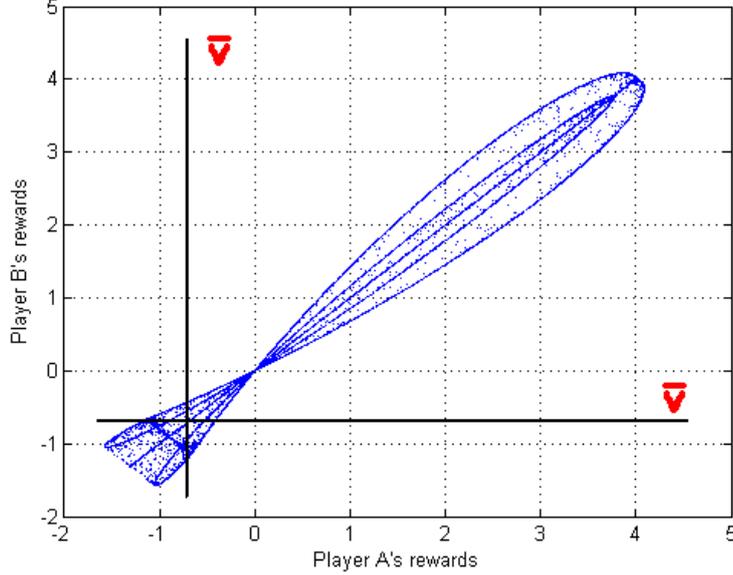


Figure 5: $P^{\mathcal{J}C}$ for $Th_{AAE} = 0.1$, $\underline{m} = 0.12$. Here, $\Delta/\Delta_{Base} \approx 0.007/0.0121 < 1$, so this system displays weak rarity value. Again, all rewards to the North-East of (\bar{v}, \bar{v}) can be obtained by an equilibrium.

3.1.2 The influence of the unit profit function

Unit profits going to infinity is not a **necessary** condition to induce strong rarity value. This stems from the insight that it is quite arbitrary how the profit function behaves below the technically feasible minimal fish stock level, or the Allee threshold for that matter. The system will either not get there anyway, or the system collapses inducing lower rewards. This strengthens the result mentioned, in our point of view, as unit profits going to infinity seem quite artificial.

We will now show that unit profits going to infinity is not **sufficient** to obtain strong rarity value, either. Now, let the unit profits be given by

$$\pi'(\mu) = p(\mu) - c(\mu) = \frac{4}{3.75} \left[\left(4 + 0.75 \frac{1}{\mu} \right) - \left(12 - 12\mu + \frac{1}{\mu^{0.5}} \right) \right].$$

Qualitative features of π and π' are similar; $\pi'(1) = 4$ and that $\lim_{\mu \downarrow 0} \pi'(\mu)$ does not exist. Unit profits decrease for lower stock levels, become negative for even lower ones. If however, fish stocks become very low, rarity value drives unit profits to infinity as in the previous case. The most significant difference for the analysis is that even if the catching capacities of the fish-

ermen were unlimited, the price effects do not dominate the quantity effects sufficiently to obtain sufficiently high rewards. For $\underline{m} = Th_{AAE} = 0$ and π' we generated a graph qualitatively equivalent to Figure 5 with $\Delta \approx 0.00067$ whereas $\Delta_{Base} \approx 0.0107$. So, for this system displaying weak rarity value, even highest scarcity is insufficient to obtain rewards inducing effects leading to the scenario as suggested by Courchamp *et al.* [2006].

3.2 The Anthropogenic Allee Effect

As we have seen in the *Small Fish War* with constant prices to which an Allee effect was added, a subset of the jointly-convergent pure-strategy rewards is cut off. Since Allee effects only occur if the fish stock drops below a certain threshold, only lower left-hand-side rewards in $P^{\mathcal{J}C}$ are affected there. To visualize this effect, the reader may compare Figures 1 and 2. However, under ‘rarity value’ an Allee effect may be expected to cut off rewards in the upper right-hand corner as the rewards associated with ‘no restraint’ move considerably to the North-East.

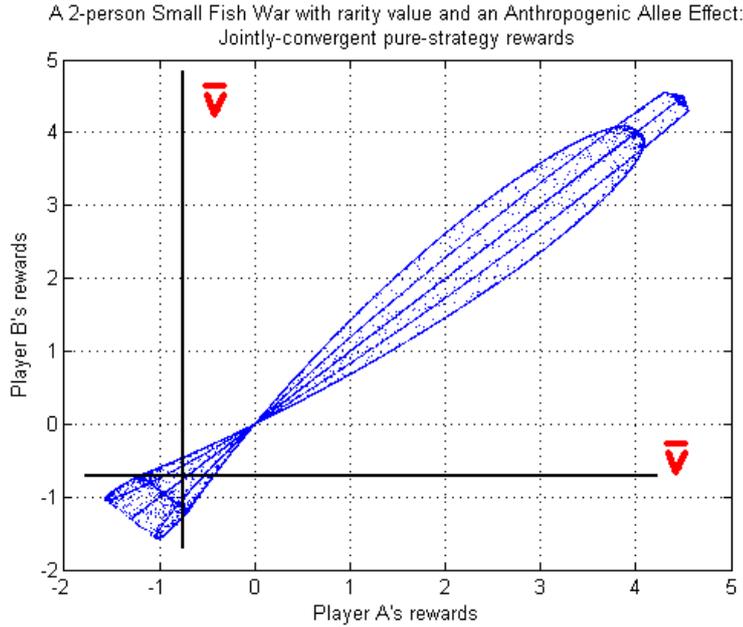


Figure 6: Here, $\underline{m} = 0.1$ and $Th_{AAE} = 0.11$, other parameters were taken as before. $P^{\mathcal{J}C}$ is cut off in the ‘beak’. Here, $\Delta/\Delta_{Base} \approx 0.125/0.01375 > 1$. Equilibrium rewards are to the North-East of (\bar{v}, \bar{v}) again.

Indeed, an analysis of the set of jointly-convergent pure-strategy rewards confirms this intuition. Comparing Figures 6 and 4 reveals that part of the

‘beak’ in Figure 4 has disappeared as anticipated, i.e., the Anthropogenic Allee Effect reduces the set of rewards. Since all rewards to the ‘North-East’ of (\bar{v}, \bar{v}) in Figure 6 are equilibrium rewards, the *AAE* eliminates a set of rewards being considerable Pareto-improvements over ‘perfect restraint’ compared to the model without *AAE*.

Note that $\Delta/\Delta_{Base} \approx 0.125/0.01375$. So, despite the *AAE* removing a set of high-reward equilibria, the Allee threshold is not high enough to turn strong rarity value into the weak variant. Moreover, a 10% increase relative to perfect restraint is feasible without reaching the Allee threshold. Computations show that for $Th_{AAE} > 0.1163$, the system displays weak rarity value, i.e., the remainder of the ‘beak’ disappears completely.

3.2.1 Hazards and the ‘Precautionary Approach’

What is to make of Pareto-efficient equilibrium rewards obtained by reaching the Allee threshold? We treated the Allee threshold as deterministic and known. There might however, be a stochastic component to such a threshold if known. Alternatively, the threshold might not be known at all, or it might have been estimated carefully but still with a stochastic residual. Bercé *et al.* [2007] claim that Allee effects may work in combinations, hence an external change may push the system over the threshold.

Furthermore, externalities from other fisheries may push the fish stock targeted downward irrespective of the behavior of the agents. For instance, Döring *et al.* [2005] give an overview of negative externalities on own and other fisheries by different types of gear used in the Baltic Sea. Furthermore, Kim *et al.* [2008] build a framework to assess damages by marine sand mining on fisheries in Korea, demonstrating that phenomena completely unrelated to any fishery may affect the fish stock targeted.

These considerations spell doom if the Allee threshold is approached in the long run. So, there seems to be some wisdom in the Precautionary Approach of the International Council for the Exploration of the Sea (ICES) establishing limits on stock levels in order to manage fisheries in a ‘safe’ way (see ICES [2005]). For this approach two limits are relevant to our considerations: the biomass limit and the precautionary biomass limit.³ The former is considered to be a stock level below which the probability of total breakdown is very high and reproductive capacity is reduced. Two variants seem to be possible, one is similar to the scenario pictured with respect to the Allee effect, the second one is similar to the poaching pit (cf., e.g., Bulte [2003]), in which a ban on fishing may not even be sufficient to guarantee recovery and the species may remain vulnerable to extinction (see also Hall *et al.* [2008]). The latter limit is a level such that if the stocks should fall below it, short-term measures to reduce fishing should suffice for recovery.

³See e.g., Döring & Egelkraut [2008] and Maroto & Moran [2008] for studies using the same limits guiding management strategies in fisheries.

4 Conclusion and discussion

The main purpose of this paper was to examine the consequences of rarity value in a *Small Fish War* (Joosten [2007]) extensively.⁴ A second goal was to incorporate an Allee effect, i.e., once the population size or density of the resource falls below an Allee threshold, only negative growth rates occur. Without rarity value, high sustainable yields can only be accomplished if the agents preserve the resource at stock levels well above minimum. An Allee effect does not influence the set of equilibrium rewards unless its threshold is rather high. *So, self-interested rationality and sustainability of the resource in absence of rarity value may go together very well.*

Rarity value brings about a very diverse picture. For instance, we showed that even if unit profits go to infinity under maximum scarcity, it is by no means guaranteed that the associated rewards constitute even a Pareto-improvement over the ‘perfect restraint’ equilibrium. We found it useful to distinguish strong and weak rarity value, and developed an operational criterion to separate one from the other. Under weak rarity value, we have a similar result as in the fixed-unit-profits variant: *self-interest and sustainability provide no tensions.* However, if rewards associated with maximum scarcity are sufficiently high, the economic system and the resource system have ‘conflicting interests’. *Highest sustainable equilibrium rewards under strong rarity value can only be accomplished by reaching the lowest possible sustainable fish stock.*

However, *equilibrium behavior need not necessarily imply ruthlessness.* Many equilibria induce rewards above the ‘perfect restraint’ rewards for both instances of rarity value staying away from the Allee threshold. So, there is room for compromise between ecological and economic maximalistic goals, overcoming the one-sidedness of management policies for natural resources as noted by e.g., Holden [1994], Brooks *et al.* [2008], thus improving chances of success cf., e.g., BenDor *et al.* [2008], Sanchirico *et al.* [2007].

Our model explains how the agents achieve the necessary scarcity levels to exploit strong rarity value. Courchamp *et al.* [2006] assume that they can, but it remains unclear how the agents do it. For instance, if unit profits become negative, myopic agents choose restraint, the fish stock recovers slightly inducing positive profits, and behavior must oscillate from then on between catching without and with restraint. Sufficiently patient agents may not be deterred by temporarily negative unit profits on their way to induce the necessary scarcity levels for huge profits due to strong rarity value. So, this leads to the surprising insight that *myopia may be better than farsightedness* for sustainability under strong rarity value.

The following matrix gives an overview of the consequences of the com-

⁴Rarity value applies to other systems of renewable resources as well (Courchamp *et al.* [2006]). An anonymous referee suggested we emphasize this point.

plex interplay of factors from economics, biology and technology:

	$\underline{m} < Th_{AAE}$	$\underline{m} \geq Th_{AAE}$
$\Delta/\Delta_{Base} > 1$	$\mu_\infty \downarrow Th_{AAE}$	$\mu_\infty = \underline{m}$
$\Delta/\Delta_{Base} \leq 1$	$\mu_\infty \approx 1$	$\mu_\infty \approx 1$

Here, \underline{m} is the minimal fish stock brought about by prolonged overexploitation of the fishery; Th_{AAE} is the threshold of the Anthropogenic Allee Effect (AAE); $\Delta/\Delta_{Base} > 1$ ($\Delta/\Delta_{Base} \leq 1$) denotes strong (weak) rarity value and μ_∞ is the long-term fish stock under the interplay of the parameters if sufficiently patient agents strive for Pareto-efficient equilibria. Especially in the strong rarity value case, such behavior might imply *flirting with disaster*, as the slightest mistake in actions or estimations of the parameters involved by the agents, or an unforeseen change in environmental, ecological or climatic conditions might bring about the AAE. If the agents were to agree upon managing the resource according to the Precautionary Approach (see e.g., ICES 2005), the following overview is appropriate:

	$\underline{m} < Th_{AAE}$	$\underline{m} \geq Th_{AAE}$
$\Delta/\Delta_{Base} > 1$	$\mu_\infty = b_{pl}$	$\mu_\infty = \max\{\underline{m}, b_{pl}\}$
$\Delta/\Delta_{Base} \leq 1$	$\mu_\infty \approx 1$	$\mu_\infty \approx 1$

Here, we assumed that b_{pl} , the precautionary biomass limit, is rather low, i.e., $b_{pl} \ll 1$, yet sufficiently higher than $\max\{b_l, Th_{AAE}\}$, i.e., higher than both the biomass limit and the threshold of the AAE. The precautionary biomass limit is a level such that if the stocks should fall below it, short-term measures to reduce fishing should suffice for recovery.

Rarity value is similar in spirit but unrelated to increasing marginal returns as studied by Maroto & Moran [2008] for instance. The authors show in a standard model using net present value optimization that increasing marginal returns and weak dependence of marginal costs on stock induce collapse of schooling fisheries of species with high reproduction rates even if managed by a very patient single agent ('owner'). Their arguments work in two steps. First, they bring to the fore empirical work showing that for several schooling species limited dependence of marginal costs on stock levels exists, cf., e.g., MacCall [1976], Csirke [1989]. Hence, in the case of schooling fisheries marginal returns independent from fish stocks as in e.g., Bjørndal [1988], might be more justified than boundlessly increasing marginal harvesting costs for decreasing stock levels as analyzed by Gordon [1954]. Then, they cite several papers giving a rationale for increasing marginal returns on catching efforts due to positive externalities and technological advances, e.g., Bjørndal & Conrad [1987], Bjørndal *et al.* [1993], Hanneson [1975], Sterner [2007]. Maroto and Moran proceed however, with constant unit prices of harvest and constant variable cost independent from stock levels. We have a strong intuition that rarity value increases the effects pictured by

Maroto & Moran [2008]. The ongoing tragedy of the bluefin tuna (*Thunnus Thynnus*) brings, in all likelihood, together strong rarity value, increasing marginal returns as tuna is a schooling species, low reproduction rates, and a multitude of myopic agents (instead of one (or few) very patient owner(s)).

Our analysis allows only qualitative interpretations. We regard the model in its present state of development as a vehicle of communication to get across ideas and messages about complex ecological-economical systems. To enable more tangible conclusions, predictions and recommendations, future research should yield quantitative expressions, preferably empirically established, e.g., on the precise ramifications of rarity value, the levels of the threshold for the *AAE*, and the (precautionary) biomass limit. The quality of information matters, as weak rarity value mistaken for a strong version induces a ‘tragedy of the commons’, i.e., resource and exploiters suffer both; a strong version mistaken for a weak one induces a ‘tragedy of the herdsman’, i.e., the exploiters forfeit considerable income. Classifying rarity value correctly induces ‘no tragedy’ for the weak variant, i.e., resource and exploiters flourish, and a ‘tragedy of the herd’ for the strong one, i.e., the resource suffers. Related, but logically independent, is the issue of imperfect information. For instance, if in a system at the cutting edge of weak and strong rarity value agents wish to achieve a symmetric Pareto-efficient equilibrium, they must have the right information about the system, and perfect information regarding the behavior of the other agent(s).

The type of agent that we regard as being modeled, is not the individual fisherman, but rather countries, regions, villages or cooperatives of fishermen. It is debatable whether the latter types care for the future sufficiently to induce sustainability (see e.g., Ostrom [1990], Ostrom *et al.* [1994] for optimistic views), but individual fishermen’s preferences seem too myopic (cf., e.g., Hillis & Wheelan [1994], Bjørndal [1988], Maroto & Moran [2008]). As we did not invoke an authority, we have not dealt with such an institution’s policies to enhance sustainability. Two intrinsically interrelated research lines may lead to desirable further developments. One is to examine the precise role of patience and especially intermediate ranges of patience of the agents in this complex system, and possibly reinforcing effect of the scarcity of a resource on the impatience of the agents (as mentioned in e.g., Hillis & Wheelan [1994]). The other is the positioning of an authority, if options to deal with the ‘commons’ as in e.g., Ostrom [1990], Ostrom *et al.* [1994] are lacking, with regard to policy in view of results from the former line of research.

The number of agents throughout this paper was two, the lowest number to model strategic interaction. We did not take more agents because complexities in notation and representation arise, hardly justified by an added value. Figure 7 provides a glimpse of an example of a three-agent *Small Fish War*. Nothing in our approach would prevent us from introducing more agents, but only two- or three agent models can be visualized.

Jointly-convergent pure-strategy rewards in a 3-person Small Fish War with rarity value

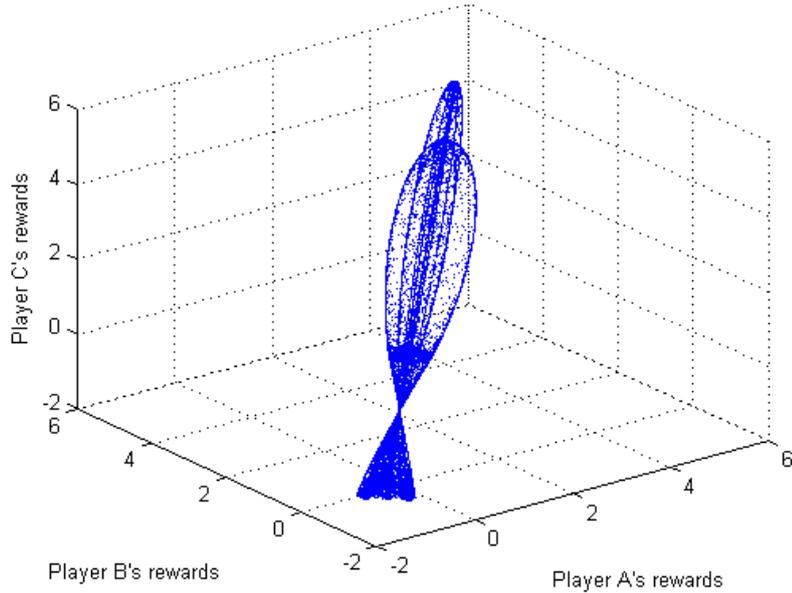


Figure 7: Rewards obtained by jointly-convergent pure strategies for an extension of the model underlying Figure 4 to three players.

Another self-imposed limitation was the number of stage-game actions. For applied work, more levels or dimensions of restraining measures may be necessary which requires a larger number of actions. For instance, when considering three types of nets, two types of boats, two options for the duration of fishing seasons combined with three levels of quota as measures for sustainable management of the resource, then 36 actions would be a logical consequence. In case mixing between certain actions is not possible for practical reasons, e.g., it is not easy to switch boats easily, a tree structure in the decisions may be required, yet its subtrees are subject to the present analysis. Adding actions changes nothing to our approach conceptually. We refrained from doing so to economize on notations.

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