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SIGNAL RECOVERY AND SYNTHESIS WITH INCOMPLETE INFORMATION AND PARTIAL CONSTRAINTS

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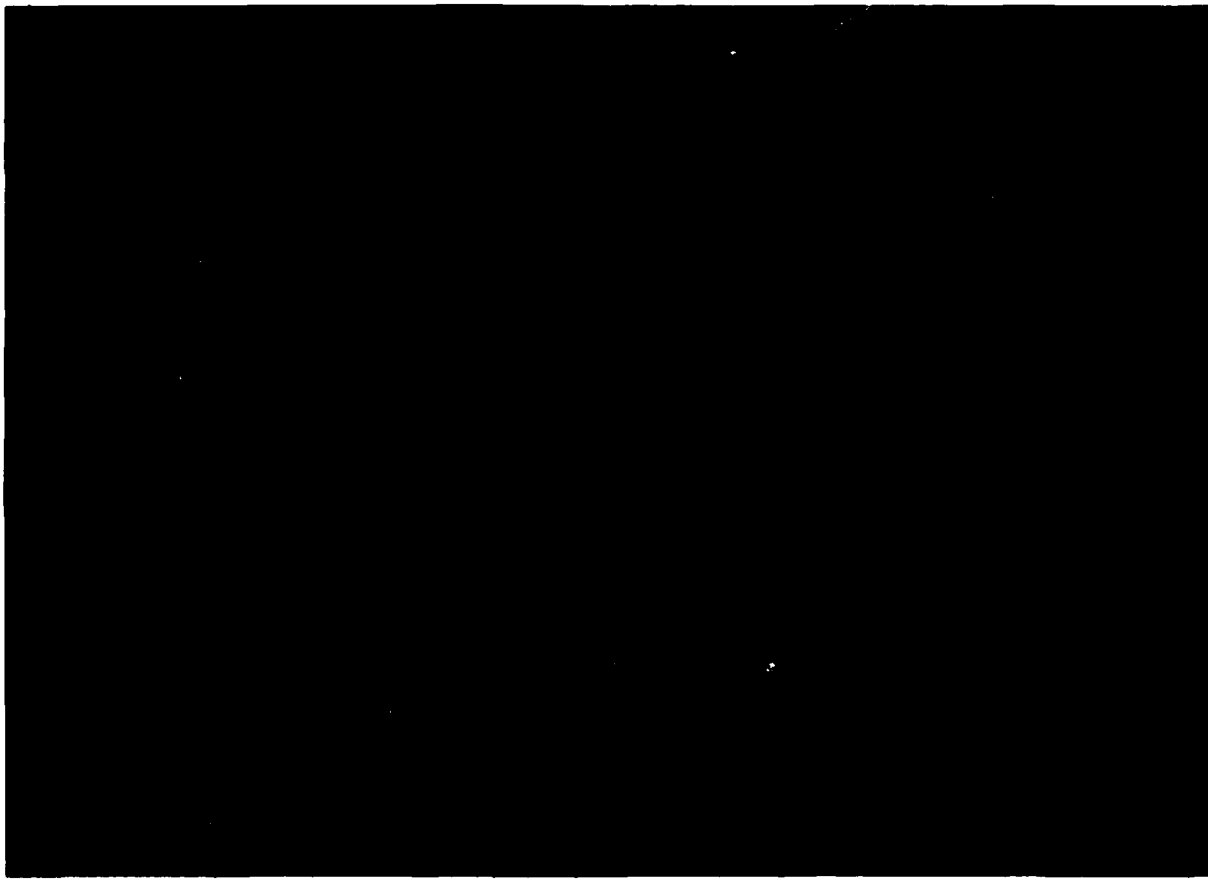

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TECHNICAL DIGEST

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TOPICAL MEETING ON SIGNAL RECOVERY AND SYNTHESIS WITH INCOMPLETE INFORMATION AND PARTIAL CONSTRAINTS

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Phase Retrieval from Intensity Data degraded by Shot-Noise.

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1. Introduction. In the paper by one of us (H.A.F.) [1], the phase problem was reviewed for circumstances under which the stochastic character of the image could be ignored. A statistical description becomes imperative for images obtained under low-intensity illumination as occurs e.g. in low-dose electron microscopy. Under these circumstances the images are heavily degraded by shot-noise. Often(e.g. [2,3])images are described as a signal plus signal-independent noise. Such a treatment does not apply to shot-noise, the quanta arriving in non-overlapping image cells are statistically independent Poisson-distributed random variables [4]. The variances of the data-counts also contain information about the signal. In this contribution we discuss the retrieval of the object wave function (w.f.) from shot-noise degraded images. Due to the stochastic imaging process, the reconstructed object w.f. is also stochastic. The purpose of this contribution is to establish the statistical characterization of the object w.f. We assume axial coherent quasi-monochromatic illumination. The analysis is presented for one lateral dimension only, the extension to two dimensions is obvious.

2. Statistical description of the image. The image plane is divided into N non-overlapping cells (pixels) with N the Shannon number (N=4β2d). The number of electrons arriving in the k-th cell is denoted by $\hat{n}_k$ where the hat "^" denotes that we deal with a random variable. $\hat{n}_k$ has a Poisson distribution with mean λ_k, where λ_k is proportional to the integral of the squared modulus of the image w.f. over the k-th cell. In our approach this noisy image is the input for the reconstruction procedure of the object w.f.. We do not smooth or filter by whatever method the data to be processed!

3. Reconstruction of the object wave function. The relation between the Fourier transform (F.T.) of the image and the w.f. $\psi_p(.)$ in the exit pupil (see [1] for details)is given by:

$$\hat{\imath}_k = \lambda_o N^{-1} \sum_{\ell=-\frac{1}{2}N+k}^{\frac{1}{2}N-1} \psi_p(\ell)\psi_p^*(\ell-k), \qquad k=0,1,\ldots \quad , \tag{1}$$

where
$$\hat{\imath}_k = \sum_{\ell=-\frac{1}{2}N}^{\frac{1}{2}N-1} \hat{n}_\ell \exp(2\pi i k\ell N^{-1}) \;. \tag{2}$$

$\psi_p(\ell)$ stands for $\psi_p(\ell/2d)$ and λ_o denotes the mean number of quanta incident per pixel and 2d is the size of the object. The phase problem is essentially solved when $\psi_p(.)$ is determined. As $\hat{\imath}_k$ is a stochastic quantity, the resulting $\psi_p(.)$ also acquires stochastic properties. From (2) we derive that $\hat{\imath}_k$, $k=-\frac{1}{2}N,\ldots,\frac{1}{2}N-1$ are correlated complex random variables; the autocorrelation function is:

$$R(k,k') = E(\hat{\imath}_k\hat{\imath}^*_{k'}) = E(\hat{\imath}_k)E(\hat{\imath}^*_{k'}) + \sum_{\ell=-\frac{1}{2}N}^{\frac{1}{2}N-1} \lambda_\ell \exp(2\pi i\ell(k-k')N^{-1}), \tag{3}$$

the symbol E(.) denotes the mathematical expectation value. The non-linear equation (1) is not simply solvable and to characterize the obtained solution statistically is even more difficult. In the next section we shall therefore treat the more tractable case of weak-object imaging.

4. Weak-object reconstruction. In this case the non-linear terms in the squared modulus of the image w.f. are neglected and λ_k depends linearly on the object w.f.. This allows us to enlarge the pixel area, the signal-to-noise ratio of the image data is increased. We have now $\frac{1}{2}N$ pixels. λ_k is written $\lambda_k=\lambda_o(1+s_k)$ where s_k is the contrast due to the object ($s_k<<1$). $\hat{s}_\ell$ is estimated from $\hat{n}_\ell$ by:

$$\hat{s}_\ell = \lambda_o^{-1}(\hat{n}_\ell-\lambda_o) \tag{4}$$

Calculating $\hat{\jmath}(.)$, the F.T. of the new random variables we obtain for the auto-correlation function:

$$E(\hat{\jmath}_k\hat{\jmath}^*_{k'}) = E(\hat{\jmath}_k)E(\hat{\jmath}^*_{k'}) + \sum_{\ell=-\frac{1}{4}N}^{\frac{1}{4}N-1} \mathrm{var}(\hat{s}_\ell)\exp(4\pi i\ell(k-k')N^{-1})$$

$$\simeq E(\hat{\jmath}_k)E(\hat{\jmath}^*_{k'})+\tfrac{1}{2}\lambda_o^{-1}\,\delta_{k,k'} \tag{5}$$

$$\text{with: } \hat{\jmath}_k = \sum_{\ell=-\frac{1}{4}N}^{\frac{1}{4}N-1} \hat{s}_\ell \exp(4\pi ik\ell N^{-1}) \tag{6}$$

In the derivation of (5) we used that the variance of $\hat{s}_\ell$ is equal to $\lambda_o^{-1}(1+s_\ell)\simeq\lambda_o^{-1}$. It can be shown that the probability distribution of $\hat{s}_\ell$ is in good approximation Gaussian with mean s_ℓ and variance equal to λ_o^{-1}. The real and imaginary part of $\hat{\jmath}_k$ are therefore also Gaussian distributed with mean the true deterministic values and with variances: $\lambda_o^{-1}\sum_\ell(1+s_\ell)\cos^2(4\pi k\ell N^{-1})$ and $\lambda_o^{-1}\sum_\ell(1+s_\ell)\times$ $\times\sin^2(4\pi k\ell N^{-1}$, respectively. The relation between $\hat{\jmath}_k$ and the object w.f. is given by: (see e.g. [5])

$$\hat{\jmath}_k = 2\, a_k\sin\phi(k)-2\, b_k\cos\phi(k), \tag{7}$$

$\phi(k)$ denotes the wave aberrations of the optical system and a_k and b_k are the F.T. of $\alpha(.)$ and $\beta(.)$, respectively. $\alpha(.)$ and $\beta(.)$ denote the phase and amplitude part of the object w.f.: $\psi_o(x_o)=1+i\alpha(x_o)-\beta(x_o)$. The aberration function $\phi(.)$ depends on the spherical aberration coefficient C_s and the defocusing δ_z:

$$\phi(k)=\phi(k/2d)=(2\pi/\lambda)[\tfrac{1}{4}C_s(k/2d)^4-\tfrac{1}{2}\delta_z(k/2d)^2]. \tag{8}$$

Taking two images with different defocus allows us to calculate a_k and b_k from two equations of the type (7). From (7) we see with (8) that Fourier components a_k for k in the neighborhood of zero do not contribute to the contrast in the image. Calculating these Fourier components would give rise to a very large noise-variance in the reconstructed object w.f.. We therefore exclude these components from the reconstruction procedure; a band-pass filtered object phase function $\bar{\alpha}(.)$ is reconstructed. The reconstruction of $\beta(.)$ follows directly from the two equations of the type (7):

$$\hat{\beta}(\ell/2\beta) = N^{-1} \sum_{k=-\frac{1}{2}N}^{\frac{1}{2}N-1} \hat{b}_k \exp(-4\pi i k\ell N^{-1}) = \beta(\ell/2\beta)+N(o,\sigma_\beta^2), \quad \forall\ell \tag{9}$$

where $N(o,\sigma_\beta^2)$ denotes a zero-mean Gaussian variable with variance σ_β^2. This variance is signal independent and is only a function of the parameters of the imaging system:

$$\sigma_\beta^2 = (8N\lambda_o)^{-1} \sum_{k=-\frac{1}{2}N}^{\frac{1}{2}N-1} \frac{\sin^2\phi_2(k)+\sin^2\phi_1(k)}{\sin^2(\phi_1(k)-\phi_2(k))} \tag{10}$$

where the suffixes "1" and "2" symbolise the two different defocusings. The phase reconstruction proceeds along similar lines:

$$\hat{\bar{\alpha}}(\ell/2\beta) = \bar{\alpha}(\ell/2\beta) + N(o,\sigma_{\bar{\alpha}}^2) \ , \quad \forall\ell \tag{11}$$

were

$$\sigma_{\bar{\alpha}}^2 = (8N\lambda_o)^{-1} \sum_k{}' \frac{\cos^2\phi_2(k)+\cos^2\phi_1(k)}{\sin^2(\phi_1(k)-\phi_2(k))} \tag{12}$$

were the accent (",") denotes that values of k in the immediate neighborhood of zero (specified more precisely in [4]) have been omitted.

5. Discussion. The disadvantage of the present approach is that the low spatial frequencies of the phase structure of the object are practically beyond retrieval. This obstacle is circumvented by illuminating the object from different directions. The results of this research will soon be reported.

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