Strong rarity value in view of hysteresis in a stochastic fishery game

Reinoud Joosten & Rogier Harmelink*

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Abstract

Strong rarity value is the phenomenon that an increase in scarcity of a species leads to a price increase which more than compensates increased search costs and decreased landings. Hysteresis here can be seen as a regime shift in which overfishing moves the system into a stable low resource-level state. Measures to restore the resource may take a long time for the system to move out of this state again.

We engineer a model in which agents wishing to maximize their limiting average rewards have two choices at every stage of the play: restraint or no-restraint ("overfishing"). Overfishing damages the resource and causes hysteresis to occur. The former induces scarcity which in turn creates scarcity value, and hysteresis makes it hard for the resource to recover.

Results show that Pareto efficient equilibrium outcomes for very patient agents may require substantial overexploitation of the resource which induces serious threats to the sustainability of the resource. Incentives of patient agents and very impatient ones (not modeled here) may point in the same direction, namely ruthless exploitation, which requires a careful long term management.

Keywords: stochastic games, endogenous stage payoffs, endogenous transition probabilities, rarity value, hysteresis, limiting average rewards, fishery games, jointly-convergent strategies

1 Introduction

Rarity value has been advanced as a phenomenon posing a substantial threat to the survival of rare species (cf., e.g., Courchamp et al. [2006], Hall et al. [2008], Berec et al. [2006]). Once a species of animals becomes rare, its value may increase, which may induce greater efforts to exploit it, leading to greater rarity, an even higher value etc. An anthropogenic Allee effect may then very well occur, i.e., human exploitation may push the population size or density below a threshold beyond which only negative growth rates are possible, obviously leading to extinction of the species.

The increase in value mentioned does not essentially stem from ‘moving upward on the inverse demand curve’ in the terminology of traditional industrial

*Both authors: IEBIS, School of Behavioral, Management and Social Sciences, University of Twente, POB 217, 7500 AE Enschede, The Netherlands. Corresponding author: r.a.m.g.joosten@utwente.nl
organization, i.e., increasing market clearing prices as supply of a commodity decreases. Rather, it is caused by the inverse demand curve shifting upwards (and then possibly tilting) as scarcity increases. Hence, for every quantity offered a higher market clearing price is found. So, in micro-economic terms a normal good obeying rather standard price-quantity relationships at standard levels of availability, turns into a status or Veblen good\(^1\) at (very) low levels of availability, and consumers are willing to pay the price for this.

In the same strand of literature, another phenomenon is noted namely hysteresis,\(^2\) a regime shift moving the dynamic system into a stable low resource-level state seemingly abruptly. If measures to restore the resource to previous levels of abundance are adopted, it may take a surprising amount of time for the system to move out of this stable low-availability state again (cf., e.g., Hutchings [2000], Lilly [2008], Kelly [2008]). The occurrence of such a (slow) recovery potential may be highly species-related and even situation-related. On the one hand, herring stocks in the North Sea seem to react rather quickly (cf., e.g., Dickey-Collas et al. [2010], Bjorndal & Lindroos [2004],) compared to cod stocks near Canada, under measures intended to restore the resource. On the other, there seems to be a difference in recovery between cod stocks close to mainland Canada and those located further away in the Atlantic Ocean due to a complex interplay of factors (cf., e.g., Mullowney & Rose [2014], Lilly [2008], Kelly [2008], Hutchings & Rangeley [2011], Mather [2013]).

We engineer a non-cooperative stochastic game\(^3\) with endogenous transition probabilities and endogenous state payoffs (ETP-ESP\(^4\)) to analyze the interplay of rarity value and hysteresis under the limiting average reward criterion, i.e., agents want to maximize their average rewards over an infinite time-horizon. In this game, the catches decrease and the transition probabilities change if over-exploitation of the resource occurs for prolonged periods of time. The model has two states, \textit{High}, in which the fish stock is at a high level inducing large catches, and \textit{Low}, in which landings are small. Transitions between states are random, crucial is that transition probabilities to \textit{High} decrease as the overexploitation continues, and they may even become zero. Then \textit{Low} becomes temporarily a temporarily absorbing state.\(^5\)

Strong rarity value is modeled by a unit profit function inspired by Courchamp et al. [2006], with the following features. If the fish stock decreases from maximum availability, unit profits decrease because unit costs increase quicker than the unit price does, for instance due to an increase in search efforts. For

\(^1\)See e.g., Leibenstein [1950], Veblen [1899]. For circumstantial evidence that fish (products) could qualify as status goods, consider the prices of kaviar. Furthermore, Van Dinther [2019] reports that endangered species are on the menu lists of Dutch Michelin guide restaurants. See also the development of prices of blue fin tuna at the Tsukiji market realized at the first auction of each year.

\(^2\)See e.g., Buite [2003], Lilly [2008]. Similar phenomena occur in shallow lakes (e.g., Scheffer [1998], Carpenter et al. [1999], Mäler et al. [2003], Wagenet [2003]), labor markets (e.g., Blanchard & Summen [1988]), climate change (e.g., Lenfant et al. [2012]), or tipping (Scheffer et al. [2001], Andersen et al. [2008]).

\(^3\)Engineer’ as in Aumann [2008]. Stochastic games are due to Shapley [1953], the infinite time version to Hoffman & Karp [1966], see also Amir [2006]; for game theoretic work on fisheries see, e.g., Haurie et al. [2012], Long [2010, 2018], Sumaila [1999], Bailey et al. [2010].

\(^4\)ETP-ESP games result from generalizations of the notion of FP-games (Joosten et al. [2003]), see also Joosten & Samuel [2018] for a classification. See also Section 6.

\(^5\)Temporarily absorbing: the agents can induce strictly positive transition probabilities from \textit{Low} to \textit{High} again by fishing sustainably for a sufficient amount of time.
certain ranges of the stock, unit profits drop to very low levels and may even drop below zero. However, necessary for strong rarity value is that below a certain value of the fish stock, unit profits continue to increase for decreasing levels of the fish stock. Hence, the rarer the resource becomes the higher the unit profits become. The latter however, is not sufficient for strong rarity value as Joosten [2016] shows. Strong rarity value requires that the increases in unit profits are so significant that they dominate the effects the lower landings may have due to the decreasing fish stock and hysteresis under overfishing.

For analytical purposes we split the model into a part which is kept fixed, and one in which two parameters vary. One parameter, the minimum fish stock, indicates how much damage the two agents do to the system by overfishing each and every period. For instance, agents with a small (large) catching capacity or harmless (harmful) catching technology bring about a rather high (low) minimum fish stock by overfishing. So, if the minimum fish stock is 0.05 for instance, persistent overfishing reduces to fish stock to 5% of its maximum stock level. The other parameter, the speed of evolution, indicates how fast the transition probabilities to High decrease to zero under overfishing. The faster this occurs the earlier the system displays hysteresis if overfishing occurs, i.e., play will remain in Low temporarily.

Our analysis reveals that overfishing causes a double whammy on landing sizes under the effects mentioned in the previous paragraphs. The lower the minimum fish stock and the greater the speed of evolution, whether taken in isolation or together, the more the average catches decrease due to overfishing (ceteris paribus). Therefore, Pareto efficient long term average landing sizes can only be achieved if the agents fish with a high degree of restraint, i.e., overfishing is rather infrequent and the sustainable action is used by both agents for a rather high proportion of the play. So, if unit profits were constant, a tragedy of the commons (Hardin [1968]) could be avoided by infinitely patient, rational agents.

Strong rarity value adds a layer of complexity which makes general conclusions for the whole range of the two parameters which we allow to change in our analysis, difficult, if not impossible. What may very well occur though, with respect to large ranges of the minimum fish stock and the speed of evolution, is that the rewards, i.e., the long term average profits, of the agents increase to huge values if they exploit the resource at their maximum capacity. Pareto efficient equilibrium outcomes require a fish stock level which is rather low and the only manner to accomplish this is by strategies which involve overfishing for a large proportion of the time.

Joosten [2016] coined the phrase the tragedy of the herd inspired by the above mentioned tragedy of the commons which, by the way, is a tragedy common to both the ‘herd’ and the ‘herdsmen’. Strong rarity value however, splits this common tragedy, the metaphorical herd suffers and the metaphorical herdsmen have immense potential benefits. The more the herd suffers, the more the herdsmen earn. This may be accompanied by the considerable risk that the resource is pushed to extinction by sheer greed.

Strict management of the resource seems required, after all, short term greed and long term greed point towards overfishing at maximum capacity. Now, for large ranges of the two parameters considered in our analysis, strong rarity value leaves room to compromise between economic and ecological goals as many equilibria yield sustainable rewards exceeding ‘perfect restraint’ rewards staying away from dangerously low fish stocks. This suggests that a basis for effective
policies for management and conservation exists. Holden [1994] attributes ineffectiveness of the common fishery policies of the European community, to their biological focus instead of an economic one (see also e.g., Brooks et al. [2008], BenDor et al. [2008], Sanchirico et al. [2007]). Dangers might be curbed if the resource is managed according to The Precautionary Approach of the International Council for the Exploration of the Sea (cf., e.g., ICES [2005]).

Next, we review the ETP-ESP Small Fish War of Joosten & Samuel [2017] and change it such that hysteresis may occur as in Joosten & Meijboom [2018]. For the time being, a linear relation between catch sizes and profits is assumed. In Section 3 we focus on strategies and rewards in general, introducing the work horse of our analysis, the so-called jointly-convergent strategies. Section 4 introduces rarity value to the model proposed in Section 2. Section 5 deals with equilibria for our infinitely patient agents, and we derive some Folk Theorem like results. We give an overview of related work in Section 6. Section 7 concludes.

2 A Small Fish War with hysteresis: landings

We briefly\(^6\) introduce a stochastic game with endogenous transition probabilities and endogenous stage payoffs, or ETP-ESP game for short (Joosten & Samuel [2017]), which forms an important building block of the total model to be engineered. We formulate the model without rarity value for the moment, as we eventually wish to compare the case in which rarity value and hysteresis are both present in the model, with the case in which only hysteresis occurs. Unit profits are fixed, hence payoffs depend linearly on catch sizes. In Section 4, we add rarity value to the model presented here.

The game is played by players A and B at discrete moments in time called stages. The game has two states \(H, L\), mnemonic for High and Low, and in both states both players have two actions, 1 and 2. We use the convention that for Player A (B) top (left) refers to the first action and bottom (right) to the second one. The first action for both players entails fishing with restraint and the second one fishing without restraint. Continued unrestrained fishing may lead to decreasing future catches; restrained fishing by both agents is sustainable.

The play continues forever at stages \(t = 1, 2, \ldots\). The stage payoffs and transition probabilities are endogenous, so, let us first attempt to capture this idea formally. The past play until stage \(t, t > 1\), is captured by two relative frequency matrices

\[
X^{s,t} = \begin{bmatrix}
  x_{1,1}^{s,t} & x_{1,2}^{s,t} \\
  x_{2,1}^{s,t} & x_{2,2}^{s,t}
\end{bmatrix}, \ s = H, L.
\]

Here, e.g., \(x_{H,1}^{t}\) is the relative frequency of action pair top-left in High having been chosen until stage \(t\), and \(x_{L,1}^{t}\) is the relative frequency of action pair bottom-left in Low occurring in past play. Therefore, logic dictates that \(x^t = (x_{H,1}^t, \ldots, x_{L,1}^t, x_{H,2}^t, \ldots, x_{L,2}^t) \in \Delta^7 = \{y \in \mathbb{R}^8 | y_i \geq 0 \text{ for all } i = 1, \ldots, 8 \text{ and } \sum_{j=1}^8 y_j = 1\}\). Vector \(x^t\) is called a relative frequency vector.

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\(^6\)For more elaborate definitions, modeling backgrounds and motivations, we gladly refer to Joosten & Meijboom [2018], Joosten & Samuel [2017].
The interaction during the play is represented by the following two matrices:

\[
\tilde{\omega}^s(\cdot) = \left[\begin{array}{cc}
(\theta^s_{1,1}(\cdot), p^s_{1,1}(\cdot)) & (\theta^s_{1,2}(\cdot), p^s_{1,2}(\cdot)) \\
(\theta^s_{2,1}(\cdot), p^s_{2,1}(\cdot)) & (\theta^s_{2,2}(\cdot), p^s_{2,2}(\cdot))
\end{array}\right], \ s = H, L.
\]

Here, all functions \( p^s_{i,j} : \Delta^7 \rightarrow [0,1] \), \( \theta^s_{i,j} : \Delta^7 \rightarrow \mathbb{R}_+ \cup \{0\} \) are assumed continuous for all \( i, j = 1, 2 \), and \( s = H, L \).

Matrix \( \omega^s \) incorporates everything necessary for the description of the play occurring in and from state \( s = H, L \) at a certain stage \( t \). So, suppose that the play is in state \( s \) at stage \( t \), then relative frequency vector \( x^t \) is known. Each entry of \( \omega^s \) contains an ordered pair \( (\theta^s_{i,j}(x^t), p^s_{i,j}(x^t)) \) denoting firstly the immediate payoffs to the players

\[
\theta^s_{i,j}(x^t) = \left(\theta^s_{i,j}^A(x^t), \theta^s_{i,j}^B(x^t)\right),
\]

and secondly the 2-dimensional probability vector

\[
p^s_{i,j}(x^t) = \left(p^s_{i,j}^H(x^t), p^s_{i,j}^L(x^t)\right) = \left(p^s_{i,j}^H(x^t), 1 - p^s_{i,j}^H(x^t)\right)
\]

that the system moves to \( H \) respectively \( L \) at stage \( t + 1 \), if the corresponding action pair \( (i, j) \) is chosen.

Since both the immediate, i.e., stage, payoffs to the players, and the probability vector of transitions of the play depend on the current relative frequency vector capturing the entire history, we call them endogenous stage payoffs and endogenous transition probability weights. We continue on the latter two subjects separately in the following two subsections.

### 2.1 Endogenous stage payoffs specified

In the sequel we assume \( \theta^H_{i,j}(x^t) \geq \theta^L_{i,j}(x^t) \) for all action combinations \((i, j)\) with \( i, j \in \{1, 2\} \) and all \( x^t \in \Delta^7 \). Hence, for both players the payoffs in \( \text{High} \) are higher than the payoffs in \( \text{Low} \) for every stage \( t \) and for every fixed action combination \((i, j)\).

Crucial is the **weighted current rate of overfishing** \( \rho : \Delta^7 \rightarrow [0,1] \), i.e., how often the agents have caught without restraint until then, weighted by the impact of the action choices on the resource. Given the relative frequency vector \( x \), the weighted current rate of overfishing is given by:

\[
\rho(x) = \xi_1 \left[x^H_{1,2} + x^H_{2,1} + \omega \cdot (x^L_{1,2} + x^L_{2,1})\right] + \xi_2 \left[x^H_{2,2} + \omega \cdot x^L_{2,2}\right]. \tag{1}
\]

Here, \( 0 < \xi_1 < \xi_2, \omega \geq 1 \) and \( \xi_2 \cdot \omega = 1 \). The condition \( \omega \geq 1 \) implies that unilateral fishing without restraint in \( \text{Low} \) has more impact than unilateral fishing without restraint in \( \text{High} \), moreover, mutual fishing without restraint in \( \text{Low} \) is more damaging to the resource than in \( \text{High} \). The condition \( 0 < \xi_1 < \xi_2 \) implies that mutual fishing without restraint in either state is more damaging to the resource than unilateral fishing without restraint (ceteris paribus). Finally, \( \xi_2 \cdot \omega = 1 \) is a normalization condition guaranteeing that if agents manage to reach a situation in which \( x^2_{2,2} = 1 \) then the rate of overfishing is 1.

The current **normalized fish stock** \( \mu : \Delta^7 \rightarrow [0,1] \) depends on the relative frequency vector \( x \) indirectly over \( \rho \) as follows

\[
\mu(x) = 1 + (1 - \mu) \left[\frac{n_2}{n_1 - n_2} \rho(x)^{n_1} - \frac{n_1}{n_1 - n_2} \rho(x)^{n_2}\right]. \tag{2}
\]
Here, \( m_k \in [0, 1] \), \( n_1 > n_2 > 1 \). Eq. (2) guarantees that the range of \( \rho(x) \) is \([m_k, 1]\), and that the curve is an inverted S-curve, never increasing in \( \rho(x) \). We refer to \( m_k \) as the minimal stock. This minimal fish stock is fixed, given the size or impact of the agents’ catching technologies on the resource. These parameters allow considerable freedom to approximate large families of inverted S-curves.

Now, we are ready to present the framework for the (endogenous) stage payoffs, i.e., \( \delta_{ij}^s : \Delta^7 \to \mathbb{R}_+ \cup \{0\} \) for all \( i, j = 1, 2 \), \( s = H, L \). The stage payoffs depend on the current level of the fish stock, and this depends on the current rate of overfishing which in turn depends on the current relative frequency vector.

Let in the sequel, \( x^{sus} \) denote the relative frequency vector such that in both states both agents always fish sustainably meaning with restrain, i.e., \([x^{sus}]_{i, 1}^{L} = 1\) and hence \( \mu(x^{sus}) = 1 \). Here, superscript \( sus \) is mnemonic for sustainable. Let \( \eta > 0 \), \( x \in \Delta^7 \), we impose the following

\[
\begin{align*}
\delta_{ij}^{H,A}(x^{sus}) &> \delta_{ij}^{L,A}(x^{sus}) \text{ and } \delta_{ij}^{H,B}(x^{sus}) > \delta_{ij}^{L,B}(x^{sus}) \\
\theta_{ij}^{H,A}(x) &> \theta_{ij}^{L,A}(x) \text{ and } \theta_{ij}^{H,B}(x) \geq \theta_{ij}^{L,B}(x) \\
\theta_{ij}^{A}(x) &> \mu(x)^\eta \cdot \theta_{ij}^{A}(x^{sus}) \text{ and } \theta_{ij}^{B}(x) = \mu(x)^\eta \cdot \theta_{ij}^{B}(x^{sus})
\end{align*}
\] (3)

The first line of restrictions in Eq. (3) implies that stage payoffs in \( High \) are never less than in \( Low \) ceteris paribus. The second line states that fishing with restraint should yield at most the catches of fishing without. The third line of the restrictions expresses that the catches increase monotonically in the level of the fish stock. The role of the parameter \( \eta \) is to allow some flexibility in the model, the standard would for instance be to have this parameter equal to one, indicating that it is harder to find fish if their density is lower. However, if for instance several technologies are in use to facilitate detection, such as sonar, helicopters, etc., and the fish happen to be a schooling species, one would like to put in a much higher value of this parameter, because then the effect of overfishing would be quite small initially, i.e., at high fish stock levels.

### 2.2 Endogenous transition probabilities specified

To formulate the endogenous transition probabilities crucial in engineering the occurrence of hysteresis, we use a simple mathematical relation.\(^7\) Let

\[
\pi^H = \left( \pi_{1,1}^{H,H}, \ldots, \pi_{2,2}^{H,H}, \pi_{1,1}^{L,H}, \ldots, \pi_{2,2}^{L,H} \right) \in (0, 1)^8
\] (4)

represent a given vector of transition probabilities to \( High \) if the resource is at maximal stock level, i.e., overfishing is negligible and the system recovers completely, and let \( p^H : \Delta^7 \to [0, 1]^8 \) be given as follows

\[
p^H(x) = [\pi^H - x \cdot P]^+ = \left( p_{1,1}^{H,H}(x), p_{1,2}^{H,H}(x), p_{2,1}^{H,H}(x), p_{2,2}^{H,H}(x), p_{1,1}^{L,H}(x), p_{1,2}^{L,H}(x), p_{2,1}^{L,H}(x), p_{2,2}^{L,H}(x) \right).
\] (5)

Here, \([z]^+\) implies \( [z]^+ = \max(0, z) \), \( k = 1, \ldots, 8 \). Note that \( p^H(x^{sus}) = \pi^H \); furthermore, \( p_6^H(x) = p_{1,2}^{L,H}(x) \) is the probability that the play continues in \( High \)

\(^7\)This can be generalized considerably, but results depend crucially on continuity (cf., e.g., Joosten et al. [2003], Joosten & Meijboom [2018]). More research is required to determine the modeling boundaries of this approach.
next stage, given that the play is currently in state Low and action pair \((1, 2)\) is chosen. \(P\) is an \(8 \times 8\)-matrix satisfying

\[ P_{4,2} > P_{3,2} > P_{2,2} = 0 \text{ and } P_{2,1} = P_{1,1} = P_{0,1} = P_{0,0} = 0 \]  \( \tag{6} \)

Eq. (6) captures that less restraint is more damaging to the resource, and that due to the implicit symmetry assumed, it does not matter which agent overfishes unilaterally.

**Example 1** In the remainder we use the following special cases (plural because our free parameters will be \(m, \Phi\) of Eqs. (1)-(6) for expository purposes:

\[
\rho(x) = \frac{3}{8} \left[ x_{1,2}^H + x_{2,1}^H + 2 \cdot (x_{1,2}^L + x_{2,1}^L) \right] + \frac{1}{2} \left[ x_{2,2}^H + 2 \cdot x_{2,2}^L \right]
\]

\[
\mu(x) = 1 + (1 - m) \left[ 2\rho(x)^3 - 3\rho(x)^2 \right]
\]

\[
\eta = 1
\]

partly describing the effects of overfishing on the resource, and furthermore

\[
\begin{bmatrix}
\theta^{H,A}_{1,1}(x^{sus}), \theta^{H,B}_{1,1}(x^{sus}) \\
\theta^{H,A}_{2,1}(x^{sus}), \theta^{H,B}_{2,1}(x^{sus}) \\
\theta^{H,A}_{2,2}(x^{sus}), \theta^{H,B}_{2,2}(x^{sus}) \\
\end{bmatrix}
= 
\begin{bmatrix}
16,16,14,28 \\
28,14,26,26 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\theta^{L,A}_{1,1}(x^{sus}), \theta^{L,B}_{1,1}(x^{sus}) \\
\theta^{L,A}_{2,1}(x^{sus}), \theta^{L,B}_{2,1}(x^{sus}) \\
\theta^{L,A}_{2,2}(x^{sus}), \theta^{L,B}_{2,2}(x^{sus}) \\
\end{bmatrix}
= 
\begin{bmatrix}
4,4,7,7 \\
7,7,14,12 \\
\end{bmatrix}
\]

describing what the agents get in an unspoiled system.

The effects of overfishing on the stochastic system are determined by

\[
\pi^H = (0.8 \ 0.7 \ 0.6 \ 0.5 \ 0.4 \ 0.4 \ 0.15)
\]

\[
P = \Phi,
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
.285 & .25 & .25 & .225 & .195 & .1575 \\
.285 & .25 & .25 & .225 & .195 & .1575 \\
.57 & .5 & .5 & .45 & .39 & .315 \\
0 & 0 & 0 & 0 & 0 & 0 \\
.285 & .25 & .25 & .225 & .195 & .1575 \\
.285 & .25 & .25 & .225 & .195 & .1575 \\
.57 & .5 & .5 & .45 & .39 & .315 \\
\end{bmatrix}
\]

Here, \(\Phi\) will be called the speed of evolution and we take \(\Phi \geq 0\). We intend to compare several cases of endogenous transition probabilities. For \(\Phi = 0\), the game reduces to a standard stochastic game.

For small values of \(\Phi > 0\), the feature of endogenous transitions occurs, and since all transition probabilities remain positive, the entire set of states is ergodic for every Markov chain induced. However, for \(\Phi = 1.35\), and \(x_{1,2}^H + x_{2,2}^L = 1\), i.e., both players fish without restraint, then

\[
p^H(x) = [\pi^H - xP]^+ = (0.0305 \ 0.025 \ 0.025 \ 0 \ 0 \ 0 \ 0 \ 0).
\]

Since the last four entries belong to state Low, it follows that if the play gets to Low, it will stay there, because all transition probabilities to High from Low have become equal to zero. So, Low has become temporarily absorbing. If this occurs, we may speak of a regime-shift as the system changes qualitatively.
3 Strategies and rewards: what can agents get?

The sets of all strategies for $A$ ($B$) is denoted by $\mathcal{X}^A$ ($\mathcal{X}^B$). The payoff to player $k$, $k = A, B$, at stage $t$, is stochastic and depends on the strategy-pair $(\pi, \sigma) \in \mathcal{X}^A \times \mathcal{X}^B$; the expected stage payoff is denoted by $R^k_{t} (\pi, \sigma)$. The players receive an infinite stream of stage payoffs during the play, and they wish to maximize their average rewards. For a given pair of strategies $(\pi, \sigma)$, player $k$’s average reward, $k = A, B$, is given by

$$
\gamma^k (\pi, \sigma) = \lim_{T \to \infty} \inf \frac{1}{T} \sum_{t=1}^{T} R^k_t (\pi, \sigma);
$$

and $\gamma (\pi, \sigma) \equiv (\gamma^A (\pi, \sigma), \gamma^B (\pi, \sigma))$. First, we focus on rewards from strategies which are pure and jointly convergent. Then, we extend our analysis to obtain larger sets of feasible rewards.

A strategy is pure, if at each stage a pure action is chosen, i.e., an action is chosen with probability 1. The set of pure strategies for player $k$ is $\mathcal{P}^k$, and $\mathcal{P} = \mathcal{P}^A \times \mathcal{P}^B$. Let for given $(\pi, \sigma) \in \mathcal{X}^A \times \mathcal{X}^B$

$$
x^t (\pi, \sigma) = (x^t_1 (\pi, \sigma), ..., x^t_8 (\pi, \sigma))
= (x^H_{1,1} (\pi, \sigma), ..., x^H_{2,2} (\pi, \sigma), x^L_{1,1} (\pi, \sigma), ..., x^L_{2,2} (\pi, \sigma))
$$
denote a relative frequency vector resulting from the agents using strategy pair $(\pi, \sigma)$. Then, the strategy pair $(\pi, \sigma) \in \mathcal{X}^A \times \mathcal{X}^B$ is jointly convergent if and only if $x \in \Delta^T$ exists such that for all $\varepsilon > 0, i = 1, ..., 8$:

$$
\limsup_{t \to \infty} \Pr_{\pi, \sigma} \left[ |x^t_i (\pi, \sigma) - x_i| \right] \geq \varepsilon = 0, \tag{7}
$$

where $\Pr_{\pi, \sigma}$ denotes the probability under strategy-pair $(\pi, \sigma)$. The terminology in Billingsley [1986, p.274] for this type of convergence is convergence with probability 1. $\mathcal{JC}$ denotes the set of jointly-convergent strategy pairs.

We now list a series of immediate consequences of Eq. (7), i.e., strategy pair $(\pi, \sigma) \in \mathcal{X}^A \times \mathcal{X}^B$ being jointly convergent:

$$
\lim_{t \to \infty} X^H_{t} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad \text{and} \quad \lim_{t \to \infty} X^L_{t} = \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}
$$

$$
\lim_{t \to \infty} \rho(x^t (\pi, \sigma)) = \rho(x)
\lim_{t \to \infty} \mu (\rho(x^t (\pi, \sigma))) = \mu (\rho(x))
\lim_{t \to \infty} \theta^*_{i,j} (x^t (\pi, \sigma)) = \theta^*_{i,j} (x)
\lim_{t \to \infty} p^H (x^t (\pi, \sigma)) = p^H (x). \tag{8}
$$

So, the expected long-term relative frequency matrices, the expected long-term rate of overfishing, the expected long-term fish stock, the expected long-term stage payoffs and the expected long-term transition probabilities to High converge to fixed numbers as well. As an immediate consequence, we have

$$
\lim_{t \to \infty} \sum_{i=1}^{4} (1 - p^H_i (x^t (\pi, \sigma))) \cdot x_i^t (\pi, \sigma) = \sum_{i=1}^{4} (1 - p^H_i (x)) \cdot x_i, \tag{9}
$$
\[
\lim_{t \to \infty} \sum_{i=1}^{8} p_{i}^{H}(x_{t}(\pi, \sigma)) \cdot x_{i}(\pi, \sigma) = \sum_{i=5}^{8} p_{i}^{H}(x) \cdot x_{i}.
\] (10)

Eq. (9) represents the long term transition frequency of the system moving from \textit{High} to \textit{Low}, and Eq. (10) is the same frequency from \textit{Low} to \textit{High}.

If Eqs. (9) and (10) are positive we have the following balance equation (cf., e.g., Kemeny & Snell [1985])

\[
4 \sum_{i=1}^{4} (1 - p_{i}^{H}(x)) \cdot x_{i} = \sum_{i=5}^{8} p_{i}^{H}(x) \cdot x_{i}
\] (11)

with at least one of the relative frequencies on either side positive. The system visits both states infinitely often as time goes to infinity albeit not necessarily equally often, however the relative frequency of transitioning from one state to the other must be equal to the relative frequency of transitioning from the latter to the former. Otherwise we have

\[
4 \sum_{i=1}^{4} (1 - p_{i}^{H}(x)) \cdot x_{i} = \sum_{i=5}^{8} p_{i}^{H}(x) \cdot x_{i} = 0
\] (12)

with all relative frequencies on the left hand side equal to zero, all probabilities on the right hand side equal to zero and all relative frequencies on the right hand side adding up to unity. Eq. (12) implies that the transition frequencies from \textit{Low} to \textit{High} are zero, i.e., the system enters \textit{Low} and it remains there in the long run due to the Law of Large Numbers.

The following is immediately implied by Eqs. (11) and (12) (cf., e.g., Kemeny & Snell [1985]).

\textbf{Lemma 2} Let \((\pi, \sigma) \in \mathcal{X}^A \times \mathcal{X}^B\) be jointly convergent and let \(x \in \Delta^T\) be the relative frequency vector such that Eq. (7) holds, then the stationary distribution of the Markov chain implied \(Q(x) = (Q^H(x), Q^L(x))\) is given by either

\[
Q(x) = (0, 1) \text{ if } \sum_{i=1}^{4} (1 - p_{i}^{H}(x)) \cdot x_{i} = \sum_{i=5}^{8} p_{i}^{H}(x) \cdot x_{i} = 0, \text{ or}
\]

\[
Q(x) = \left(\sum_{i=1}^{4} x_{i}, \sum_{i=5}^{8} x_{i}\right) \text{ if } \sum_{i=1}^{4} (1 - p_{i}^{H}(x)) \cdot x_{i} = \sum_{i=5}^{8} p_{i}^{H}(x) \cdot x_{i} > 0.
\]

The standard interpretation of this stationary distribution \((Q^H, Q^L)\) is that in the long run the play occurs with relative frequency \(Q^H\) of time, or stated differently a proportion \(Q^H\) of the stages, in \textit{High} and with the complementary probability \(Q^L\) in \textit{Low}. In the first line of the central implication of the lemma \textit{Low} has become a temporarily absorbing state. By logical and technical implications \(x_{i} = 0\) for \(i = 1, ..., 4\) whereas \(x_{i} \geq 0\) implies \(p_{i}^{H}(x) = 0\) for \(i = 5, ..., 8\).

It may be cumbersome to compute the stationary distribution of the Markov chain for a pair of jointly convergent strategies, but in some cases of special interest they are easily determined. If both agents use one action pair whenever the play occurs in that state we obtain the following convenient expressions.

\textbf{Corollary 3} Let \(i^* = 1, ..., 4, i^{**} = 5, ..., 8\) and let \(x\) satisfy \(x_{i^*} + x_{i^{**}} = 1\), then

\[
Q(x) = (x_{i^*}, x_{i^{**}}) = \left(\frac{p_{i^*}^{H}(x)}{1 - p_{i^*}^{H}(x) + p_{i^{**}}^{H}(x)}, \frac{p_{i^{**}}^{H}(x)}{1 - p_{i^*}^{H}(x) + p_{i^{**}}^{H}(x)}\right).
\]
Example 4 Particularly interesting are the relative frequency vectors arising from perfect restraint leading to the sustainable relative frequency vector $x^{sus}$ and its complete opposite. Let therefore, $x^{nr} = (0, 0, 0, x^{nr}_4, 0, 0, 0, x^{nr}_5)$ be the relative frequency vector resulting from the jointly-convergent pair of strategies $(\pi, \sigma)$ such that both agents always catch with no restraint. Then, we have

$$x^{sus} = \frac{1}{1 - \pi^{H,H}_{1,1} + \pi^{H}_{1,1}} \left( \pi^{L,H}_{1,1}, 0, 0, 0, 1 - \pi^{H,H}_{1,1}, 0, 0, 0 \right)$$

$$= \left( \frac{5}{7}, 0, 0, 0, \frac{2}{7}, 0, 0, 0 \right)$$

$$x^{nr} = \frac{1}{1 - [\pi^{H} - x^{nr}P]^+_x + [\pi^{H} - x^{nr}P]^+_x} \times \left( 0, 0, 0, [\pi^{H} - x^{nr}P]^+_x, 0, 0, 0, 1 - [\pi^{H} - x^{nr}P]^+_x \right).$$

So, $x^{sus}$ (associated with perfect restraint) is fixed regardless of $m$ or $\Phi$. Moreover, the latter expression reduces to $x^{nr} = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$ whenever Low becomes a temporarily absorbing state.

3.1 Jointly-convergent pure-strategy rewards

The set of jointly-convergent pure-strategy rewards $P^{JC}$ is then the set of pairs of rewards each of which can be obtained by using a pair of jointly-convergent pure strategies. Let for a given relative frequency vector $x \in \Delta^7$, the associated stage payoffs be given alternatively as follows

$$\theta(x) = (\theta_1(x), \theta_2(x), ..., \theta_8(x))$$

$$= (\theta^1(x), \theta^2(x), \theta^3(x), \theta^4(x), \theta^5(x), \theta^6(x), \theta^7(x), \theta^8(x)),$$

then we introduce the following pair

$$\langle \gamma^A, \gamma^B \rangle_x = \sum_{i=1}^{8} x_i \cdot \theta_i(x). \quad (13)$$

We interpret the above as follows. The vector $x$ is a relative frequency vector to which the system converges with probability 1 and the pair $\langle \gamma^A, \gamma^B \rangle_x$ as the resulting limiting average rewards. This requires a formal result of course, for which recall the following.

Proposition 5 (Joosten & Samuel [2017]) Let strategy-pair $(\pi, \sigma) \in JC$ and let $x \in \Delta^7$ for which (7) is satisfied, then the average payoffs are given by

$$\gamma(\pi, \sigma) = \langle \gamma^A, \gamma^B \rangle_x.$$  

To compute the rewards connected to a pair of jointly-convergent strategies is a matter of ‘book keeping’. We find the expected long-term stage payoff pairs for each state and then sum these expected long-term stage payoff pairs over all entries weighted by their long run relative frequencies.

Lemma 6 Under Eqs. (1)-(13) we have for $x^{sus}$ that the limiting average rewards are given by

$$\langle \gamma^A, \gamma^B \rangle_{x^{sus}} = \frac{\pi^{L,H}_{1,1}}{1 - \pi^{H,H}_{1,1} + \pi^{L,H}_{1,1}} \theta^H_{1,1}(x^{sus}) + \frac{1 - \pi^{H,H}_{1,1}}{1 - \pi^{H,H}_{1,1} + \pi^{L,H}_{1,1}} \theta^L_{1,1}(x^{sus}).$$
For $x^{nr}$ the limiting average rewards are

$$
(\gamma^A, \gamma^B)_{x^{nr}} = m \cdot \frac{[\pi^H - xP]^+_8}{1 - [\pi^H - xP]^+_4 + [\pi^H - xP]^+_8} \theta^L_{2,2}(x^{sus}) + m \cdot \frac{1 - [\pi^H - xP]^+_4}{1 - [\pi^H - xP]^+_4 + [\pi^H - xP]^+_8} \theta^L_{2,2}(x^{sus}).
$$

Moreover, if Low becomes temporarily absorbing we have the following:

$$
(\gamma^A, \gamma^B)_{x^{nr}} = m \cdot \theta^L_{2,2}(x^{sus}).
$$

Example 7 Lemma 6 implies for our continued example that

$$
(\gamma^A, \gamma^B)_{x^{sus}} = \frac{5}{7}(16, 16) + \frac{2}{7}(4, 4) = \left(\frac{88}{7}, \frac{88}{7}\right).
$$

If Low is temporarily absorbing we have an equally simple expression for the limiting average rewards associated with no restraint

$$
(\gamma^A, \gamma^B)_{x^{nr}} = m \cdot \left(\frac{13}{2}, \frac{13}{2}\right).
$$

Using Prop. 5, we generate (graphs of) the feasible limiting average rewards for the Small Fish War with rarity value. We have chosen quite a few parameters which we do not intend to examine further, we only intend to vary the parameters $m$, the minimum stock size, and $\Phi$, the speed of evolution.

For $m \leq 1$ and $\Phi > 0$, the game is a true ETP-ESP-game, but not necessarily one with a temporarily absorbing state by which we intend to capture hysteresis. For $\Phi \approx 1.2821$, Low becomes temporarily absorbing for $x^{nr}$, and only then we have hysteresis. We depict four cases to illustrate the effects of changes on average rewards (catches).

- $m = 1, \Phi = 0$ standard stochastic game (see Fig. 1)
- $m = 0.05, \Phi = 1.5$ ETP-ESP game (see Fig. 2)
- $m = 0.05, \Phi = 0$ ESP game (see Fig. 3)
- $m = 1, \Phi = 1.5$ ETP game (see Fig. 4)

4 Rarity value and hysteresis

The ETP-ESP Small Fish Wars introduced in the previous section implicitly model situations in which agents sell their catches at a competitive market while incurring fixed unit search costs, at least fixed with respect to the scarcity of the resource in their fishing environment. Alternatively, if neither prices on the market, nor search costs are fixed, then one can regard the model as pertaining to a situation in which unit prices go up approximately in the fashion as the unit search costs do for increasing scarcity.

In some cases, fishermen may extract smaller and smaller quantities, but may obtain higher and higher revenues. Plot 1 is inspired by a figure from Courchamp et al. [2006]. The unit costs of catching depend on the fish stock $\mu$ in our model.
Figure 1: $P_{JC}^{m}$ for $m = 1$ and $\Phi = 0$, a standard stochastic game. The yellow (resp. red) dot indicates the perfect (resp. no) restraint rewards.

Figure 2: $P_{JC}^{m}$ for $m = 0.05$ and $\Phi = 1.5$. In this ETP-ESP game mutual over-fishing reduces the fish stock to 5% of its maximum level and for some relative frequency vectors hysteresis occurs. Perfect restraint rewards (the yellow dot) are Pareto efficient, no restraint is Pareto inferior in this set.
Figure 3: $P_{JC}$ for $m = 0.05$ and $\Phi = 0$, the resulting game is an ESP game. Differences with respect to Fig. 1 are due to endogenous stage payoffs. Perfect restraint (the yellow dot) is not Pareto efficient, no restraint (the red dot) is Pareto inferior.

Figure 4: $P_{JC}$ for $m = 1$ and $\Phi = 1.5$. Differences with respect to Fig. 1 are due to endogenous transitions and hysteresis. Perfect restraint (yellow dot) is Pareto efficient, no restraint (red dot) is not Pareto inferior any more.
(on the horizontal axis), and these increase as the species becomes rarer, i.e., $\mu$ becomes smaller. This can be motivated for instance by increased efforts and energy expenses per unit, giving rise to a unit cost function $c : (0, 1] \to \mathbb{R}_+$ satisfying $\frac{\partial c}{\partial \mu} < 0$ for all $\mu \in (0, 1]$.

Unit prices may be assumed rather constant near maximal availability of the fish stock, but we assume that they increase whenever the availability of the fish stock becomes low, certainly at a rate considerably higher than the cost increase in the same situation. This gives rise to a unit price function $\pi : (0, 1] \to \mathbb{R}_+$ satisfying $\frac{\partial \pi}{\partial \mu} < 0$ for all $\mu \in (0, 1]$.

The unit profits $\Pi : (0, 1] \to \mathbb{R}$ must be engineered from those two functions such that such that

$$(\mu - \mu^*) \frac{\partial \Pi}{\partial \mu} < 0 \quad \text{for some } \mu^* \in (0, 1) \text{ and all } \mu \neq \mu^*$$

$$\Pi(m) \gg 0$$

Observe that the unit profit function is merely related to the fish stock, it does not vary with the quantity caught. Given the fish stock, the unit price is fixed, as are the unit costs, hence also the unit profits.

![Graph](image.png)

Figure 5: The unit costs (solid), unit prices (top dashed) and unit profits (bottom dashed) as functions of the level of the fish stock.

We model strong rarity value by adding the unit profit function multiplicatively to the stage payoffs of the model with constant prices as presented. This allows us to proceed quite efficiently as we can keep much of the framework presented in the previous sections the same, i.e., in the set of Eqs. (1)-(13) one change needs to be made. In Eq. (3) the last line should be replaced by

$$\hat{\theta}_{i,j}^{c_A}(x) = \mu(x)^{n} \cdot \Pi(\mu) \cdot \hat{\theta}_{i,j}^{c_A}(x^{aux}) \quad \text{and} \quad \hat{\theta}_{i,j}^{c_B}(x) = \mu(x)^{n} \cdot \Pi(\mu) \cdot \hat{\theta}_{i,j}^{c_B}(x^{aux}).$$

The immediate consequences formulated in Eqs. (9) as the third line, remain the same with the understanding that the above substitutes the previous formulation. All other parts of the Eqs. (1)-(12) are logically unrelated to addition of

14
the unit profit function. Eq. (13) is still correct with the same understanding that the above substitutes the original formulation in the last line of Eq. (3).

**Example 8** The formula for the unit profit curve to be used in the sequel is based on the following unit cost and unit price functions

\[
\begin{align*}
  c(\mu) &= \frac{1}{3.75} \left( 12 + \frac{1 - 12 \cdot \mu^{2.5}}{\mu^{1.5}} \right) \\
  \pi(\mu) &= \frac{1}{3.75} \left( 4 + \frac{0.75}{\mu^{2}} \right)
\end{align*}
\]

It can be confirmed that the above restrictions regarding the unit costs and revenues are fulfilled. The resulting unit profit curve is therefore

\[
\Pi(\mu) = \frac{1}{3.75} \left( \left( 4 + \frac{0.75}{\mu^{2}} \right) - \left( 12 + \frac{1 - 12 \cdot \mu^{2.5}}{\mu^{1.5}} \right) \right). \tag{14}
\]

\( \Pi(1) = 1 \) by design. As \( \lim_{\mu \to 0} \Pi(\mu) \) does not exist, we take an additional condition here, namely that \( \mu > 0 \). Then, the unit profit function is continuous on \([m, 1]\) and we may proceed as before.

So, unit profits decrease as fish stocks decrease from maximal level as the unit price remains almost constant, but unit search costs increase steadily. If the fish stock continues to fall below \( \mu^* \approx 0.675 \), unit profits become negative, i.e., the agents would incur losses by catching fish. However, at \( \mu^* \approx 0.3665 \) the unit profits increase again if the fish stock decreases. If the fish stock would fall below \( \mu^* \approx 0.228 \), then the unit price driven by scarcity exceeds unit costs again. ■

Figure 6 illustrates the effects of adding Eq. (14) to the model used for expository purposes so far with \( m = 0.05 \) and \( \Phi = 1.5 \). So, comparing Figures 2 and 6 yields some insights. Firstly, since a range of fish stocks with negative unit profits exists, we find negative average rewards as well. However, for fish stocks with \( \mu < 0.228 \) unit profits increase steadily as fish stocks decline. The Pareto-dominant rewards are the ‘no-restraint’ rewards, i.e., the agents have the highest total and individual rewards if they exploit the resource ruthlessly. Note that the no-restraint rewards were located in the South-West in Fig. 2, but in Fig. 6 they are to be found in the North-East. The sustainable outcome in which both agents show ‘perfect restraint’ provides relatively high rewards of \( (\gamma^A, \gamma^B)_{x^{\text{sus}}} = (\frac{83}{2}, \frac{83}{2}) = (12.571, 12.571) \), but many rewards to the North-East of this point in Fig. 2 Pareto dominate this outcome, i.e., give both players strictly more than 12.571. Note that

\[
(\gamma^A, \gamma^B)_{x^{\text{sus}}} = m \cdot \Pi(m) \cdot \left( \frac{13}{2}, \frac{13}{2} \right) = 0.05 \cdot 54.175 \cdot \left( \frac{13}{2}, \frac{13}{2} \right) = (17.607, 17.607).
\]

Hence, the relative improvement of no-restraint over perfect restraint is

\[
\frac{(\gamma^A, \gamma^B)_{x^{\text{sus}}} \cdot (1, 1) - (\gamma^A, \gamma^B)_{x^{\text{sus}}} \cdot (1, 1)}{(\gamma^A, \gamma^B)_{x^{\text{sus}}} \cdot (1, 1)} = 0.40057.
\]

This shows that under strong rarity value a 40% increase of long term average rewards over perfect restraint rewards may be obtained by both agents choosing
no restraint at each stage of the play. This constitutes a complete turn around of affairs as absence of rarity value induces a relative improvement of

\[
\frac{(\gamma^A, \gamma^B)_{\text{max}} \cdot (1, 1) - (\gamma^A, \gamma^B)_{\text{max}} \cdot (1, 1)}{(\gamma^A, \gamma^B)_{\text{max}} \cdot (1, 1)} = -0.97415,
\]

which means that a decrease of more than 97% is to be expected.

Figure 6: $P^Z_C$ for $m = 0.05$, $\Phi = 1.5$ and the unit profit function given by Eq. (14). Negative rewards occur on the lower left hand side. On the upper right hand side the rewards associated with the agents showing less and less restraint. The Pareto efficient symmetric outcome is no restraint (the red dot) which yields 40% more than the perfect restraint (the yellow dot).

5 Threats and equilibria

The strategy pair $(\pi^*, \sigma^*)$ is an equilibriuim, if no player can improve his limiting average rewards by unilateral deviation. An equilibrium is called subgame perfect if for each possible state and possible history (even unreached states and histories) the subsequent play corresponds to an equilibrium, i.e., no player can improve his limiting average rewards by deviating unilaterally from then on. In the construction of equilibria for repeated or stochastic games, ‘threats’ play an important role (cf. e.g., Hart [1985], Thuijsman & Vrieze [1998]). A threat specifies the conditions under which one player will punish the other, as well as the subsequent measures. For more details see e.g., Joosten et al. [2003].

We call $v = (v^A, v^B)$ the threat point, where

\[
v^A = \min_{\sigma \in \mathcal{X}^B} \max_{\pi \in \mathcal{X}^A} \gamma^A(\pi, \sigma) \quad \text{and} \quad v^B = \min_{\pi \in \mathcal{X}^A} \max_{\sigma \in \mathcal{X}^B} \gamma^B(\pi, \sigma).
\]

So, $v^A$ is the highest amount player A can get if his opponent B tries to minimize his average payoffs. Under a pair of individually rational (feasible) rewards each player receives at least the threat-point reward. We can now present the major result of Joosten [2007].
Proposition 9 Each pair of rewards in the convex hull of all jointly-convergent pure-strategy rewards giving each player strictly more than the threat-point reward, can be supported by a subgame-perfect equilibrium.

The proposition is illustrated in Figure 7 for the continued example. This Folk Theorem type result hinges on the possibility of punishing unilateral deviations, as in e.g., Hämäläinen et al. [1985]. So, we need history-dependent strategies. There is no contradiction between strategy pairs being both jointly-convergent and history-dependent, or for that matter cooperative, e.g., Tolwinski [1982], Tolwinski et al. [1986], Krawczyk & Tolwinski [1993], or incentive strategies, or combinations, e.g., Ehtamo & Hämäläinen [1986,1989,1993,1996].

Nor is there one regarding subgame-perfectness: if the equilibrium path in the terminology of Hart [1985] induces convergence with probability 1, the off-equilibrium part may be arbitrarily sophisticated.

Example 10 It is very hard to compute the threat point. For wide ranges of the parameters our algorithm, based on Harnelink & Joosten [2019], finds an exact threat point. In others, we can only find what must be considered approximations. Specifically for the most interesting case treated here namely \( m = 0.05 \) and \( \Phi = 1.5 \), we do not find that our algorithm yields a threat point with the precision that we desire. However, we can state that all positive rewards to both agents can be supported by a subgame perfect (Nash) equilibrium. This is based on an incentive strategy (in our reading of Ehtamo & Hämäläinen [1986,1989,1993,1996]) for the punisher yielding negative profits. This means

\[
(v^A, v^B) < (0, 0).
\]

Then, by Prop. 9 all rewards in the set formed by the convex hull of all positive jointly-convergent pure-strategy rewards can be supported by a subgame perfect equilibrium (see also the Appendix).

![Figure 7](image)

Figure 7: The area enclosed by the black lines represents the set of subgame perfect Nash equilibrium rewards implied by Prop. 9, for \( m = 0.05 \) and \( \Phi = 1.5 \). Both perfect restraint (yellow) and no restraint (red) can be supported by equilibria, no restraint is the unique Pareto efficient subgame perfect equilibrium.
6 Related previous work and technicalities

The first FD-games, i.e., games with frequency dependent stage payoffs, were designed by Brenner & Witt [2003] in order to show that plausible learning mechanisms in a pollution game, may induce the worst possible. Crucial in their initial idea was the adaptation to a game theoretical framework of frequency dependent utilities originally proposed in the framework of distributed choice (cf., e.g., Herrnstein [1997] for an extensive overview).

Joosten et al. [2003] classified and analyzed the class of FD-games extensively by deriving a series of Folk Theorems, i.e., identifying large sets of awards obtained by Naeh (or subgame perfect) equilibria involving threats. Instrumental in the analysis were so-called jointly convergent strategies, i.e., combinations of strategies inducing long term play under which the expected relative frequency of each possible action combination converges to a constant with probability 1 in the terminology of Billingsley [1986].

The Small Fish War of Joosten [2007] brought two new features. In Brenner & Witt [2003] and Joosten et al. [2003] the history was summarized in the two relative frequencies with which the agents used their actions in the past, but in the new set up the relative frequencies of all action combinations matter, yielding a more general model. Secondly, a multiplicative approach was used to model the endogenous stage payoffs, which were previously modeled linearly.

In such a Small Fish War, two agents possess the fishing rights to a body of water, and they have two options, to fish with or without restraint. Restraint in practice may take various forms, e.g., on catching seasons, quantities caught, or technologies allowed. Essential is that unrestrained fishing yields a higher intermediate catch, but continued unrestrained fishing may lead to decreasing future landings; restrained fishing by both agents is sustainable. The agents want to maximize their average catches over an infinite time-horizon. There, a ‘tragedy of the commons’ (Hardin [1968]) is avoided, as Pareto-efficient outcomes can be sustained by subgame perfect equilibria inducing high fish stocks.

Joosten [2016] incorporated rarity value (Courchamp et al. [2006]) by adding a price-scarcity mechanism to the Small Fish War. To deal with subtleties disregarded by Courchamp et al. [2006], Joosten [2016] considered total profits, i.e., unit profits times quantities, averaged over an infinite period by very patient agents with (possibly) bounded catching capacities in an interactive decision making framework with both short and long term strategic externalities. Analysis showed that only strong rarity value induces the environmental and economic effects sketched by Courchamp et al. [2006], i.e., Pareto-efficiency requires very low resource stocks. It is without contradiction that very patient agents achieve the necessary scarcity levels to exploit strong rarity value.

Joosten & Meijboom [2018] examined the effects of hysteresis by engineering a stochastic game based on the original Small Fish War in which a certain state with low levels of availability of the resource, becomes temporarily absorbing due to overfishing, i.e., the transition probabilities leading the play out of this state all become zero temporarily. Only if the agents fish with restraint for sufficiently many periods after this occurs, the transition probabilities may become nonzero again and as a result, the play may move out of this state.

To obtain results, efficiency of algorithms to find large sets of feasible limiting average rewards needed to be improved. Joosten & Samuel [2017] designed efficient algorithms to obtain sets of feasible rewards, based on Joosten & Samuel...
[2018]. Table 1 may be found in the latter contribution and for each of the three types mentioned rather efficient algorithms have been formulated. Here, the term stochastic games has a broader interpretation than usual, hence traditional ‘stochastic games’ appears here as a subclass.

<table>
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<tr>
<th>Table 1: A computation-inspired ordering among stochastic games.</th>
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<tr>
<td>Type I</td>
</tr>
<tr>
<td>((p_0))</td>
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<tr>
<td>CSP ((\theta_0))</td>
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<tr>
<td>CSP ((\theta(x)))</td>
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An even greater obstacle to progress was the computational complexity of finding threat points in ETP games. In the contributions mentioned above these threat points were determined analytically requiring some ingenuity and painstaking efforts. To illustrate the difficulty somewhat, finding a threat point in a stochastic game boils down to finding solutions to a set of stochastic zero-sum games. Existence of a value under the limiting average reward criterion is implied by Mertens & Neymann [1981], but optimal strategies may belong to a class of strategies of considerable complexity as demonstrated in the Big Match of Blackwell & Ferguson [1968]. Several methods to obtain results in this framework are available if it is known beforehand that the value can be obtained by simpler strategies such as stationary strategies. However, ETP games do not belong to the subclasses of stochastic games to which such methods can be applied, and the previously developed modes of analysis regarding ESP and ETP-games revolved on jointly-convergent strategies which are independent from the usually distinguished classes of strategies facilitating computations.

Recently, Harmelink & Joosten [2019] came up with intelligent brute force algorithms in order to find threat points in arbitrary ETP-ESP games with one ergodic set of states. It was based on an adaptation of solving a generalized Markov Decision Problem for a player if his opponent uses a fixed stationary strategy (cf., e.g., Hordijk et al. [1983], Blackwell [1962], Filar & Vrieze [1996]). The solution against such a fixed strategy was determined by looking for optimal pure stationary strategies. The global solution had to be determined by brute force to a large sequence of such fixed strategies. This is a slightly risky method because of the complexity of this type of games, but the brute force procedures

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4AIT games are games with action independent transitions, this subclass includes but is by no means restricted to repeated games. CAIT games are games with current-action independent transition games, so transition probabilities only depend on past behavior.

keep track of both maxmin and minmax, and if they are found to converge they provide a threat point. The algorithms were applied in the present model, too. The good news is that for large ranges of the parameters, the algorithm works as expected, despite the fact that in the present context, one state can become temporarily absorbing. However, a parameter range exists for which the algorithm does not provide the right kind of answers. We conjecture this is due to the unit profit curve used lacking monotonicity in $\mu$. Further research should shed more light on the issues concerned.

7 Conclusion and discussion

The main purpose of this paper was to model the combination of hysteresis and rarity value in a Small Fish War (Joosten [2007, 2016], Joosten & Meijboom [2018]) and analyze consequences. We engineered an appropriate model within the class of stochastic games by adding to the ETP-ESP games with two states High and Low first treated in Joosten & Samuel [2017], a unit profit function dependent on the present fish stock. For decreasing availability of the resource the unit profits decrease steadily, because search costs are relatively more affected than unit prices by the decrease of the fish stock initially. Below a certain threshold however, unit prices increase more than unit costs whenever the fish stock decreases, and they may eventually lead to very high (total) profits due to strong rarity value.

With respect to the landings in our model, we have clear and intuitive findings presented largely in Section 3, namely overfishing deals a double blow to the quantities caught, as it decreases the fish stock on which catches depend directly. Moreover, continued overfishing causes the system to spend more and more time in Low. It may even happen that hysteresis occurs, the system can not return to High anymore, which means that Low becomes temporarily absorbing. For simplicity, we analyze changes in just two parameters for our model, $m_{H}$ the minimum fish stock the agents can achieve, and $\Phi$, the speed of evolution of the transition probabilities. For wide ranges of $m_{H}$ and $\Phi$, perfect restraint is Pareto efficient (see for instance Fig. 2 and Fig. 3). In other cases, Pareto efficient outcomes in quantities caught, involve a high degree of restraint by the agents. So, with respect to catches (which is equivalent to keeping constant prices as argued) we can confidently state: high sustainable landings can only be achieved by keeping the fish stocks at rather high levels.

Rarity value, modeled in Section 4 by adding a unit profit function depending on the fish stock to the ‘catches submodule’, yields a complex interaction with the resource system and as a consequence this changes the characteristics of the set of feasible rewards and the behavioral requirements to obtain high long term average rewards, fundamentally and dramatically. The roles of $m_{H}$ and $\Phi$ are crucial, yet complex. For instance, for high values of $m_{H}$, we do not even have strong rarity value, as the agents cannot diminish the fish stock sufficiently for the phenomenon to kick in, and there is a parallel to fishing without price changes: fishing with restraint yields rather efficient rewards. If $m_{H}$ becomes smaller, no restraint rewards gradually move upwards, eventually even beyond the perfect restraint rewards and no restraint yields the Pareto efficient outcome. Suddenly, the economic system and the resource system have ‘conflicting interests’ even for very patient agents. High(est) sustainable equilibrium
rewards under strong rarity value can only be accomplished by reaching (the) low(est) possible sustainable fish stock.

Qualitatively, the current model and the one examined in Joosten [2016] bear striking resemblance in outcomes despite notable differences in their technical setups. The reason is probably that rarity value largely overrides the relatively subtle effects of the innovations added to the earlier modeling framework.

Section 5 demonstrates that equilibrium behavior under strong rarity value need not necessarily imply ruthlessness. On the one hand, many equilibria exist which induce stocks above the minimum fish stock. On the other, many equilibria induce rewards above the ‘perfect restraint’ rewards. In the intersection of these two sets of equilibria, there is room for policies that compromise between ecology and economy, overcoming the one-sidedness of management policies for natural resources (e.g., Holden [1994], Brooks et al. [2008]), thus improving chances of success cf., e.g., BenDor et al. [2008], Sanchirico et al. [2007].

We might be accused of stacking the cards in favor of sustainability (according to common wisdom). We then plead guilty as charged, as we proposed two very patient agents,\(^\text{10}\) perfect information, perfect monitoring, absence of other types of stochasticity than the ones treated here, which form aspects known to contribute to solving social dilemmas (cf., Komorita & Parks [1996]) and inducing Pareto-efficient equilibria in repeated and stochastic games. For strong rarity value sustainability of Pareto-optimal behavior may depend crucially on these abstractions made, and hence must be regarded as vulnerable in practice.

In the strong rarity value case, Pareto-efficient (equilibrium) behavior may imply flirting with disaster, as the slightest mistake in actions or estimations of the parameters involved, or any change in environmental, ecological or climatic conditions might bring about an anthropogenic Allee effect (AAE) which should always be assumed to lurk in the background. Joosten [2016] also treats the AAE and its consequences are rather straightforward: this phenomenon simply eliminates part of the set of feasible rewards which under strong rarity value yield considerably higher rewards than perfect restraint, but quite seldomly the entire set of rewards Pareto-dominating the perfect restraint outcome. In Joosten [2016] perfect information regarding the Allee threshold was assumed, a requirement which might be unrealistic in reality.

To make matters worse, Berec et al. [2007] claim that Allee effects and other low-density phenomena such as hysteresis may work in combinations. Lilly [2008] attributes the slow recovery of Atlantic cod to a predator pit, namely increased predation of cod by seals. Molloney & Rose [2014] hypothesize that deteriorating food sources (shellfish) for the cod may play a role in the persistence of low stocks of cod. Externalities from other fisheries may push the fish stock targeted over the Allee threshold or into hysteresis irrespective of the behavior of the agents in the fishery under examination. For instance, Döring et al. [2005] give an overview of negative externalities on own and other fisheries by different types of gear used in the Baltic Sea. Mather [2013] adds an interesting note to this namely that the catching moratorium on cod induced an entire occupational branch to switch from catching and processing cod to shellfish. An interesting hypothesis is thus imminent: societal transformations

\(^{10}\)Our agents are countries, regions, villages or cooperatives of fishermen. It is debatable whether even the latter care sufficiently for the future to induce sustainability (see e.g., Ostrom [1990], Ostrom et al. [1994] for optimistic views), but individual fishermen’s preferences seem too myopic (cf., e.g., Hillis & Wheelan [1994]).
to cope with measures to help recovery of a renewable resource, may potentially deteriorate conditions crucial to this aim. Kim et al. [2008] build a framework to assess damages by marine sand mining on fisheries in Korea, demonstrating that phenomena quite unrelated to any fishery may affect fish stocks targeted.

There may be some wisdom in the Precautionary Approach of the International Council for the Exploration of the Sea (ICES) establishing limits on stock levels in order to manage fisheries in a ‘safe’ way (see ICES [2005]). Two limits are relevant to our concerns: the biomass limit and the precautionary biomass limit.\textsuperscript{11} The former is the stock level below which the probability of total breakdown is very high and reproductive capacity is reduced. Two variants seem to be possible, one is similar to the scenario pictured with respect to the Allee effect, the second one is similar to the poaching pit (cf., e.g., Bulte [2003]), in which a ban on fishing may not even be sufficient to guarantee recovery and the species may remain vulnerable to extinction (see also Hall et al. [2008]). The latter limit is a level such that if the stocks should fall below it, short-term measures to reduce fishing should suffice for recovery.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure8.png}
\caption{$P^{J_C}$ for $\mu = 0.05$, $\Phi = 1.5$ and unit profit given by Eq. (14), under strict adherence to a precautionary biomass limit of 0.06 (ICES [2005]). The Pareto efficient symmetric outcome (the orange triangle) yields 12\% more than perfect restraint (the yellow dot). Note that the red dot is unattainable.}
\end{figure}

Rarity value is unrelated to increasing marginal returns. Maroto & Moran [2008] show in a standard model using net present value optimization that increasing marginal returns and weak dependence of marginal costs on stock, induce collapse of schooling fisheries of species with high reproduction rates even if managed by a very patient single agent (‘owner’). Their arguments work in two steps. First, they bring to the fore empirical work showing that for several schooling species limited dependence of marginal costs on stock levels holds, cf., e.g., MacCall [1976], Csirke [1989]. Hence, in the case of schooling fisheries marginal returns independent from fish stocks as in e.g., Bjørndal [1988], might be more justified than boundlessly increasing marginal harvesting costs for decreasing stock levels as analyzed by Gordon [1954]. Then, they cite several

\textsuperscript{11}See e.g., Döring & Egelkraut [2008] and Maroto & Moran [2008] for studies using the same limits guiding management strategies in fisheries.
papers giving a rationale for increasing marginal returns on catching efforts due to positive externalities and technological advances, e.g., Bjørndal & Conrad [1987], Bjørndal et al. [1993], Hannesson [1975]. Maroto & Moran [2008] proceed however, with constant unit prices of harvest and constant variable cost independent from stock levels. We suspect that rarity value increases the effects pictured by Maroto & Moran [2008].

The case of the bluefin tuna brings, in all likelihood, together strong rarity value, increasing marginal returns (tuna is a schooling species), low reproduction rates, and a multitude of myopic agents (instead of few very patient owners). Moreover, bluefin tuna is a crucial part in sushi or sashimi which are gaining popularity in the Far East (cf., e.g., Veldkamp [2007]). The way in which sushi or sashimi packages are priced may induce psychological effects similar to the ones driving the tulip crisis in The Netherlands in the seventeenth century (cf., e.g., Dash [1999]), or the collateralized debt obligation crisis more recently.

8 Appendix

Proof of the claim in Example 11: We show that player A possesses a strategy guaranteeing that unit profits are negative, no matter what his opponent does. So, \( v^B \) is negative as player A can always punish his opponent by using this strategy whenever B deviates from a certain equilibrium path (see e.g., Hart [1986]). Let \( \mu^* \) be the fish stock minimizing the unit profits. Define

\[
\pi^t = \begin{cases} 
1 & \text{if } \mu^t \leq \mu^*, \\
2 & \text{if } \mu^t > \mu^*.
\end{cases}
\]

Now, if B cannot prevent the play from inducing \( \mu^t \to \mu^* \), then unit profits are indeed smaller than zero. So, this case is not interesting. Suppose B can prevent the play from inducing \( \mu^t \to \mu^* \), then one of two cases occurs

\[
\pi^t = \begin{cases} 
1 & \text{and } \mu^t < \mu^*, \\
2 & \text{and } \mu^t > \mu^*.
\end{cases}
\]

Take \( \pi^t = 1 \) and \( \mu^t < \mu^* \), then the only possibility for player B to reach or to maintain positive unit profits is to play in a manner that \( \mu^t \) decreases if \( \mu^t > \mu^L \), or to prevent the same to reach values above this threshold (see Plot 1). This means that B must use his second action at a significant number of stages, if not all. So, suppose player B uses his most powerful manner to achieve this by playing his second action at all stages. Then, in the long run the transition probability to remain in (switch to) High if the play is in High (Low) converge to (recall \( \Phi = 1.5 \))

\[
p_2^H = 0.7 - 0.25(1.5) = 0.325, \\
\]

\[
p_0^H = 0.4 - 0.1575(1.5) = 0.16375.
\]

Hence, the long run stationary distribution resulting from that is

\[
Q = \left( \frac{0.16375}{1 - 0.325 + 0.16375} \frac{1 - 0.325}{1 - 0.325 + 0.16375} \right) = (0.19523, 0.80477).
\]

Which implies in turn that the resulting rate of overfishing is

\[
\rho = \frac{3}{8} (0.19523 + 2 \cdot (0.80477)) = 0.67679,
\]
leading to long run fish stocks of (recall \( m = 0.05 \))

\[
\mu = 1 + (0.95) (2(0.67679)^3 - 3(0.67679)^2) = 0.28357.
\]

For the latter level of the resource we have

\[
\Pi(\mu) = \frac{1}{3.75} \left( \left( 4 + \frac{0.75}{(0.28357)^2} \right) - \left( 12 + \frac{1 - 12 \cdot (0.28357)^{2.5}}{(0.28357)^{1.5}} \right) \right)
= -0.50467 < 0.
\]

The other case \( \mu^t > \mu^* \) is rather similar. If player B wants to influence the long term fish stock upwards (to the right in Plot 1), then the only way to do it is to play his first action for significant stages. The most forceful way to do this is to always play action 1. However, the associated transition probabilities are equal to the case above due to the symmetry of this example. Hence, the long term distribution moves towards \( \mu = 0.28357 \). If B wants to influence the long term fish stock downwards, he can do this by playing his second action for significant instances of the play. The system will move quickly in the direction of \( \mu^* \), player A uses the other action once the system reaches fish stocks below \( \mu^* \) and the system converges to \( \mu = 0.28357 \). We already demonstrated that for this level of the resource unit profits are negative.

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