Dynamic speed control and lane management in the general link transmission model

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1 Introduction

Dynamic network loading models for simulating traffic on networks are applied both for transport planning purposes as well as traffic management purposes. For large scale networks, macroscopic link models consistent with first order kinematic wave theory are particularly popular, especially in conjunction with first order node models. The link transmission model (LTM) is an efficient algorithm with relatively small numerical errors which is used increasingly in dynamic traffic assignment procedures. The original algorithm in [1] adopt a triangular fundamental diagram (FD). Recent extensions consider more general concave FDs ([2], [3], [4]).

For transport planning purposes, it is common to keep the FD fixed during the simulation. However, for traffic management purposes an FD may change when properties of a road segment vary over time. In particular, dynamic speed control may change the maximum speed imposed on a road segment, and dynamic lane management may open or close certain lanes. Implementation of changes in the FD in a first or order second order cell-based model can be achieved by instantaneously changing initial conditions, see e.g. [5]. While this is relatively straightforward, it assumes that all drivers on (part of the) link immediately react to the change, which may temporarily result in infeasible traffic states.[1] In LTM only boundary conditions are used and hence accounting for changes in the FD is more challenging. Variable speed limits were considered through an extension of LTM in [6] assuming a triangular FD. A more general extension to variable fundamental diagrams for LTM is presented in [7]. However, this method adds significant complexity to LTM and still suffers from possible infeasible

[1] For example, instantaneously reducing a 2-lane road to a 1-lane road may not be feasible (i.e., results in a density larger than the jam density) when vehicles are not physically able to merge onto the same lane due to existing traffic conditions.

traffic states due to the adoption of initial conditions that assume an instantaneous change of the FD across the entire link.

In this paper we therefore propose a new method that obviates the need for the assumption that the FD changes instantaneously across the entire link. Instead, we advocate an approach where the FD progressively changes in accordance with the information on the variable message sign (VMS), which travels with the speed of drivers. In other words, in case the speed limit changes from 120 km/h to 90 km/h then this will only affect drivers upstream the VMS while drivers downstream the VMS are not yet informed of the reduction in maximum speed. This effectively results in a situation where multiple FDs are active across a single link akin to multiclass traffic. The result is a behaviourly justifiable method revolving around the way information propagates (resulting in time-varying FDs) alongside the regular propagation of traffic flow. These two propagation mechanisms fit nicely into the recently proposed event-based formulations of LTM ([8] and [9]) which allow for such a separation.

2 Constrained fundamental diagram

Let the following physical parameters be given for each link: length L [km], capacity $q^{\max}$ [veh/h], jam density k^{jam} [veh/km], critical density k^{crit} [veh/km], maximum wave speed $\gamma^{\max}$ [km/h], and minimum wave speed $\gamma^{\min}$ [km/h]. For each link we assume that we consider the following fundamental relationship between flow q [veh/h] and density k [veh/km],

$$q = \Phi(k), \tag{1}$$

where $\Phi:[0,k^{\max}] \rightarrow [0,q^{\max}]$ is a continuous concave function with $\Phi(k^{\text{crit}}) = q^{\max}$, $\Phi(0) = \Phi(k^{\max}) = 0$, $q^{\max} = \Phi(k^{\text{crit}})$, $d\Phi(0)/dk = \gamma^{\max}$, and $d\Phi(k^{\max})/dk = \gamma^{\min}$. Let $\Phi_{\text{I}}(k)$ and $\Phi_{\text{II}}(k)$ denote the hypocritical branch (where flows are increasing with density) and hypercritical branch (where flows are decreasing with density) of the FD, i.e.

$$\Phi(k) = \begin{cases} \Phi_{\text{I}}(k), & 0 \le k \le k^{\text{crit}}; \\ \Phi_{\text{II}}(k), & k^{\text{crit}} \le k \le k^{\max}. \end{cases} \tag{2}$$

Eqn. (2) refers to the *physical* FD in the absence of any driving constraints. Let $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3)$ denote the vector of parameters that constrain the vehicle speed, flow, and density, respectively, where $0 \le \theta_1, \theta_2, \theta_3 \le 1.$ Expanding the idea presented in [10] we denote the *constrained* FD by $\tilde{\Phi}(k \mid \boldsymbol{\theta}),$ which can be formulated as

$$\tilde{\Phi}(k \mid \boldsymbol{\theta}) = \min\left\{\theta_1 \gamma^{\max} k, \theta_2 \Phi\left(\frac{k}{\theta_3}\right)\right\}. \tag{3}$$

If $\boldsymbol{\theta} = (1,1,1)$ then $\tilde{\Phi}(k \mid \boldsymbol{\theta}) = \Phi(k)$ and no constraints are imposed. Using $\boldsymbol{\theta}$ several dynamic traffic management measures can be simulated. If a maximum speed of $\sigma^{\max}$ is imposed on the link, then $\theta_1 = \min\left\{\sigma^{\max}/\gamma^{\max}, 1\right\},$ while if one lane of a two-lane road segment is closed then $\theta_2 = \theta_3 = \frac{1}{2}$ (assuming that the the lane closure affects flows and densities proportionally).

Figure 1 illustrates different shapes of the constrained FD. The physical FD (with a quadratic hypocritical branch and a linear hypercritical branch for illustration purposes) is the same in all cases, while the constrained FD shown at the bottom varies depending on imposed speed, flow, and density constraints.

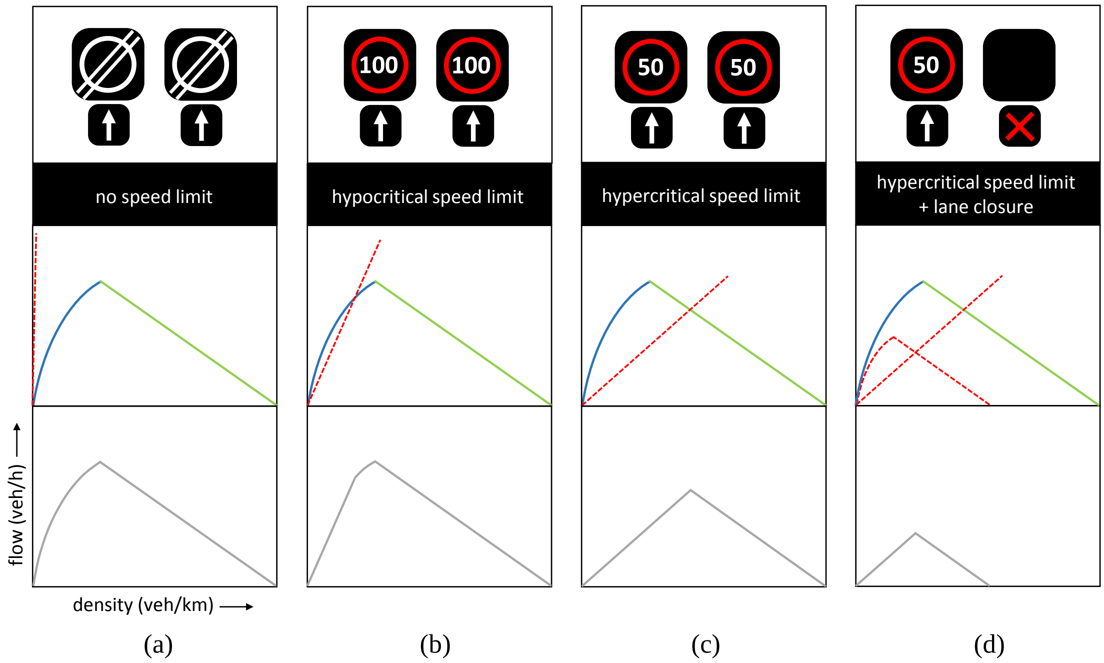

Figure 1 Constrained fundamental diagrams in case of (a) no speed limit, (b) hypocritical speed limit, (c) hypercritical speed limit, (d) hypercritical speed limit and lane closure.

3 Transitions between fundamental diagrams

We are interested in determining link outflow rates when there is a change in the FD starting at the upstream link boundary. Note that information about the FD (dynamic speed limits, dynamic lane management) only propagates downstream (with the driver of the vehicle), never upstream.

In the case of a fixed FD, an instantaneous flow rate increase at the link entrance may result in an acceleration fan in which traffic states at the downstream link boundary follow the shape of the FD [4]. Similarlarly, when we allow the FD to change shape, we consider changes in traffic states due to the transition from one (constrained) FD to the other. Such transitions between FDs occur conditional on the vehicle speed with which this information propagates.

Consider traffic state D in Figure 2(a). When there is a decrease in the maximum speed and the number of lanes available, then the traffic state changes to A (with the same flow rate but at a lower speed). This results in a temporary traffic state E with zero flow exiting the link, see Figure 2(b). Now consider traffic state A. When there is an increase in the maximum speed and the number of lanes available, then the traffic state eventually changes to D (with the same flow rate but at a high speed). This change is not instantanous but happens gradually via traffic states B and C and all traffic states in between. Practically, this can be simplified by only considering a subset of traffic states (e.g., B and C).

These (counter) clockwise transitions along the fundamental diagrams can be implemented as additional events in the algorithm proposed in [9]. In this solution method, the more traditional ‘flow rate change’ events propagate with the wave speed, while ‘route choice’ events propagate with the vehicle speed. We now propose to add ‘fundamental diagram change’ events that also propagate with the vehicle speed, albeit with a very different impact. Case studies on networks will be presented in the full paper.

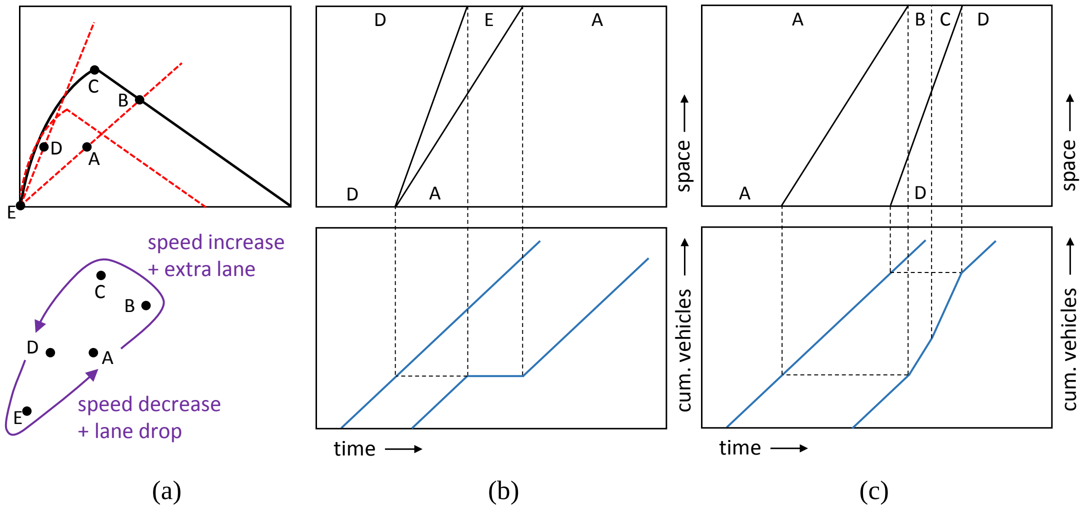

Figure 2 Constrained fundamental diagrams in case of (a) no speed limit, (b) hypocritical speed limit, (c) hypercritical speed limit, (d) hypercritical speed limit and lane closure.

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