# Pricing Mechanism Based on Losses Using Grid Topology

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***Abstract*—The currently used pricing mechanisms for the network costs in the domestic sector do not give incentives to lower the stress on the grid. In this paper, we present a novel pricing mechanism for the grid costs based on the losses caused by the transport of energy in the low voltage (LV) grid. The mechanism is based on the Shapley value. In a first step, we derive the Shapley value based on the given locations of the households in the grid. As the locations influence the prices heavily, we present a second mechanism by averaging the Shapley value over all permutations of households. This leads to a contribution to the costs as if the household is on an average location. We show that this value can be calculated efficiently by an explicit expression if the households are connected to a single cable. Furthermore, we present an approximation for this value which may be useful for extensions to LV grids of a different topology.**

## I. Introduction

The currently used pricing mechanisms for electricity in the domestic sector do not give customers incentives to e.g. consume electricity at times when it is better for the grid or to spread their consumption evenly over the day. However, also more advanced mechanisms like time-of-use pricing, peak pricing or real-time pricing have their disadvantages as they tend to only shift peaks and may even increase the synchronisation of loads, e.g. for charging electric vehicles [1], [2]. Furthermore, time-of-use pricing or peak pricing do not reflect the actual costs incurred by the consumption or production of energy [3], [4].

Several new pricing mechanisms have already been developed in recent years. In [5] the authors have looked at demand response as a game with a monotone price function, in particular with a quadratic price function. Similar to this, in [6] a game was defined for determining prices whereby no specific properties were assumed for the price function, implying that pricing mechanisms proposed in our work could be integrated into this game. In [2] an optimization framework was developed for real-time pricing environments, where some adjustments to the prices to compensate for load synchronisation were added. Locational marginal prices have also received some attention (see [7], as well as some pilot and commercials projects [8]). Here prices depend on both the location as well as the extra cost incurred from raising the consumption. In [9] the authors take the total load in the grid as a variable for a quadratic price function. Using the Aumann-Shapley value to determine transportation prices for medium voltage networks has been explored in [10].

In this paper, we present a novel pricing mechanism based on the losses caused by the transport of energy in the low voltage (LV) grid. The mechanism is based on the Shapley value. Normally, the calculation of this value is computationally expensive, however, in our setting an efficient structure can be used to reduce this complexity [11]. As the resulting energy prices are highly dependent on the location in the network, these prices possibly do not lead to a fair mechanism. Therefore, we propose two variants based on permutations of the locations within the network. This way, a pricing mechanism is obtained that fully distributes all costs based on the individual consumption, assuming that everyone is at an "average location" in the grid.

This paper is organised as follows. In Section II we explain the model we use. After that, we introduce the new pricing mechanisms using a small example. Then in Section IV, we extend this to instances with $n$ households. We demonstrate our mechanisms on a few small cases in Section V and finish this paper with some conclusions and directions for future work.

## II. Model

This work is part of the Grid Flex Heeten project, in which innovative price mechanisms are developed and tested in a setting with local electricity production and storage. The used incentives have the goal to get a better match of local supply and demand. Within the project in the town of Heeten, all 47 households behind a single transformer are involved [12], [13]. Our approach is based on a model of a neighbourhood, taking into account the houses, the underground cables, the transformer, and their geographical layout. This model can be represented as a graph, where houses, cable joints, and the transformer are the nodes, and the cables are the edges. This forms the base for analysing the network. To present the concept, we focus on a single feeder without any branches and do not take into account any losses in the cable connecting a household to the feeder, as these are negligible and make the modelling unnecessarily difficult. Instead, we assume the power of a house is directly inserted at the joint, implying that the considered graph is a chain.

The starting point for the costs are the losses, which scale quadratically with the power. We denote the set of households by $H = \{h_1, \dots, h_n\}$, and the set of locations for the households in the network by vertices $L = \{1, \dots, n\}$. We

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assume the locations are numbered consecutively from furthest to closest to the feeder. We introduce a mapping $p : L \to H$ that gives for each location which household is at the given location. The transformer $T$ in the grid is connected to location $n$ via cable segment $(n, T)$. Furthermore, we assume each cable segment $(i, i+1)$ has a function $F_i^p(f_i^p)$ that specifies the costs of a power flow $f_i^p$ on the cable segment (i.e. losses). As losses are assumed to be quadratic in the power flow, we model $F_i^p$ as follows:

$$F_i^p(f_i^p) = e_i(f_i^p)^2, \quad (1)$$

where $e_i$ is a parameter depending on length, age, condition, etc. Note that $f_i^p$ depends on the location of the households, so on the function $p$. To denote the costs of the whole network, we define $F^p := \sum_i F_i^p(f_i^p)$. To determine the power flow on an edge, we need to know the power of the households. Let $X_{h_j}$ denote this power for household $h_j$. Then adding up the power of all households using cable segment $(i, i+1)$ gives us $f_i^p$. As we sometimes need to look at the power flow of a subset $S \subseteq H$ of households in our analysis, we denote by $f_i^p(S)$ the power flow on cable segment $(i, i+1)$ induced by $S$. This flow is given by

$$f_i^p(S) = \sum_{j=1, p(j) \in S}^{i} X_{p(j)}. \quad (2)$$

Note that $f_i^p = f_i^p(H)$. Similarly, we denote the overall costs of $S$ in the grid by $F^p(S) = \sum_i F_i^p(f_i^p(S))$.

The presented model is based on the following assumptions. Firstly, we address the network costs and the electricity costs in one price mechanism. Both of these scale quadratically [4], so we use the same structure for both. Constant and linear terms of the network and electricity cost (e.g. overhead costs) can be added to the developed price mechanism as linear and constant terms without having to do the analysis again. Secondly, we only consider total power, meaning we do not explicitly take into account reactive power. This could be included by introducing imaginary numbers, however, this would complicate the analysis and distract attention from the main issue considered in this paper. Lastly, we assume a single-phase cable in this paper, though an extension to three-phase cables can be done by splitting a three-phase cable into three separate single-phase cables.

## III. Small Example

To get some intuition for the considered problem, we start with a small example of three houses and one transformer as shown in Fig. 1. To ease the explanation, we assume that all energy flows are in the same direction (so either all from or towards the transformer). Note that this has no further effect on the analysis to come.

In the example we have $p(1) = h_1$, $p(2) = h_2$ and $p(3) = h_3$. Using Equation (1) on this model leads to the following costs for the cable segments:

$$F_1^p(f_1^p) = e_1(f_1^p)^2 = e_1 X_{h_1}^2$$
$$F_2^p(f_2^p) = e_2(f_2^p)^2 = e_2(X_{h_1} + X_{h_2})^2$$
$$F_3^p(f_3^p) = e_3(f_3^p)^2 = e_3(X_{h_1} + X_{h_2} + X_{h_3})^2.$$

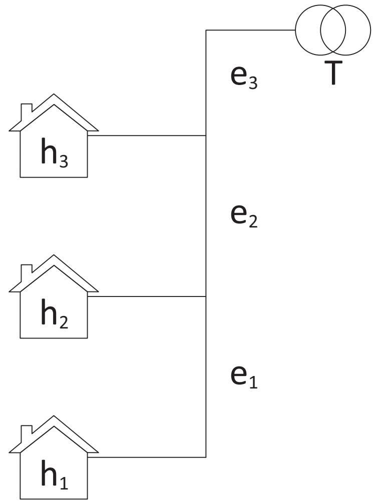


Fig. 1. Toy Example.

Taking the costs for all cable segments together gives us

$$F^p = e_1 X_{h_1}^2 + e_2(X_{h_1} + X_{h_2})^2 + e_3(X_{h_1} + X_{h_2} + X_{h_3})^2. \quad (3)$$

If we look at the individual costs each household creates (so assuming the other households are not there), we would get $(e_1 + e_2 + e_3)X_{h_1}^2$, $(e_2 + e_3)X_{h_2}^2$ and $e_3 X_{h_3}^2$ for households $h_1$, $h_2$ and $h_3$ respectively. Note that these do not add up to the actual total costs as calculated in (3). This means that we cannot use these individual costs but need to find a different way to divide the total costs among the households.

### A. Shapley Value

One way of dividing the costs among a group of participants used in literature is the Shapley value [14]. The Shapley value divides the costs based on how much a player adds to the total costs of each possible coalition in a game. In our case a coalition corresponds to a set of households. For such a coalition we only take into account the power of those households when calculating the costs. Based on this, the Shapley value of household $h_i$, denoted by $\phi_{h_i}$, is given as:

$$\phi_{h_i} = \sum_{S \subseteq H \setminus \{h_i\}} \frac{|S|!(|H| - |S| - 1)!}{|H|!} (F^p(S \cup \{h_i\}) - F^p(S)).$$

The Shapley value has some nice properties:

- Efficiency: $\sum_{h_i \in H} \phi_{h_i} = F^p$
- Symmetry: if $F^p(S \cup \{h_i\}) = F^p(S \cup \{h_j\})$ for all $S \subseteq H \setminus \{h_i, h_j\}$ then $\phi_{h_i} = \phi_{h_j}$
- Linearity: $(\phi + \Phi)_{h_i} = \phi_{h_i} + \Phi_{h_i}$, where $\Phi$ is equal to $\phi$ except $F^p$ is exchanged for any other cost function.
- Zero player: if $F^p(S \cup \{h_i\}) = F^p(S)$ for all $S \subseteq H \setminus \{h_i\}$, then $\phi_{h_i} = 0$.

Furthermore, it has been shown that the Shapley value is the only possible division that has these four properties [14].

If we apply this formula on the small example introduced in Fig. 1, the costs of household $h_1$ become:

$$
\begin{aligned}
\phi_{h_1} = & \frac{0!\ 2!}{3!}(F^p(\{h_1\}) - F^p(\emptyset)) \\
& + \frac{1!\ 1!}{3!}(F^p(\{h_1, h_3\}) - F^p(\{h_3\})) \\
& + \frac{1!\ 1!}{3!}(F^p(\{h_1, h_2\}) - F^p(\{h_2\})) \\
& + \frac{2!\ 0!}{3!}(F^p(\{h_1, h_2, h_3\}) - F^p(\{h_2, h_3\})) \\
= & \tfrac{1}{3}(e_1 + e_2 + e_3)X_{h_1}^2 \\
& + \tfrac{1}{6}(e_1 X_{h_1}^2 + e_2 X_{h_1}^2 e_3 (X_{h_1} + X_{h_3})^2 - e_3 X_{h_3}^2) \\
& + \tfrac{1}{6}(e_1 X_{h_1}^2 + (e_2 + e_3)(X_{h_1} + X_{h_3})^2 \\
& - (e_2 + e_3)X_{h_2}^2) + \tfrac{1}{3}(e_1 X_{h_1}^2 \\
& + e_2(X_{h_1} + X_{h_2})^2 + e_3(X_{h_1} + X_{h_2} + X_{h_3})^2 \\
& - e_2 X_{h_2}^2 - e_3(X_{h_2} + X_{h_3})^2) \\
= & e_1 X_{h_1}^2 + e_2 X_{h_1}(X_{h_1} + X_{h_2}) \\
& + e_3 X_{h_1}(X_{h_1} + X_{h_2} + X_{h_3}).
\end{aligned}
$$

Similar to that of household $h_1$, we get the following costs for households $h_2$ and $h_3$:

$$
\begin{aligned}
\phi_{h_2} &= e_2 X_{h_2}(X_{h_1} + X_{h_2}) + e_3 X_{h_2}(X_{h_1} + X_{h_2} + X_{h_3}), \\
\phi_{h_3} &= e_3 X_{h_3}(X_{h_1} + X_{h_2} + X_{h_3}).
\end{aligned}
$$

The total allocated costs $\phi$ in this network are now:

$$\phi := \phi_{h_1} + \phi_{h_2} + \phi_{h_3},$$

which equals the total costs $F^p$ as calculated in (3).

When comparing $\phi_{h_1}$ with $\phi_{h_3}$, it can be seen that with identical power usage, household $h_1$ has to pay much more than household $h_3$. This is due to the bigger distance to the transformer and only looking from the perspective of produced losses. This makes sense, as household $h_1$ also is responsible for more losses. In this sense the Shapley value preserves a sense of fairness, however, it is strongly location dependent.

Implementing such a locationally biased pricing mechanism is most likely not going to be accepted, as people do not see their location in the grid as a righteous criterion justifying these large price differences. As such, the mechanism would be judged as not treating everyone (formally) equal [15].

To be able to integrate the Shapley value in a pricing scheme, we need to get locational independence to some extent. This is what we are doing in the following sections.

### B. Average Location

In the above example, household $h_i$ was considered to be on location $i$, so $p(i) = h_i$. However, if we want locational independence, we need to get rid of this specific location. Note that if we switch the locations of all households, the contribution to the losses of each household changes.

For this, we consider each permutation $p$ of the households and calculate what their contribution to the losses would be according to the Shapley value. If we then take the average over all possible permutation $p \in \mathbb{P}$ (in the example there are 3! possible permutations), we get the contributions of a household if it would be on an "average location" in the grid. If we denote the average contribution of household $h_i$ over all possible permutations by $\mu_{h_i}$, for household $h_1$ we get

$$
\begin{aligned}
\mu_{h_1} = & X_{h_1}(\tfrac{1}{3}e_1 X_{h_1} + \tfrac{1}{3}e_2(2X_{h_1} + X_{h_2} + X_{h_3}) \\
& + e_3(X_{h_1} + X_{h_2} + X_{h_3})).
\end{aligned}
$$

Similarly, for $h_2$ and $h_3$ we get:

$$
\begin{aligned}
\mu_{h_2} = & X_{h_2}(\tfrac{1}{3}e_1 X_{h_2} + \tfrac{1}{3}e_2(X_{h_1} + 2X_{h_2} + X_{h_3}) \\
& + e_3(X_{h_1} + X_{h_2} + X_{h_3})), \\
\mu_{h_3} = & X_{h_3}(\tfrac{1}{3}e_1 X_{h_3} + \tfrac{1}{3}e_2(X_{h_1} + X_{h_2} + 2X_{h_3}) \\
& + e_3(X_{h_1} + X_{h_2} + X_{h_3})).
\end{aligned}
$$

Adding these up gives us the total average costs:

$$
\begin{aligned}
\mu := & \mu_{h_1} + \mu_{h_2} + \mu_{h_3} \\
= & \tfrac{1}{3}e_1(X_{h_1}^2 + X_{h_2}^2 + X_{h_3}^2) + \tfrac{2}{3}e_2(X_{h_1}^2 + X_{h_2}^2 \\
& + X_{h_3}^2 + X_{h_1}X_{h_2} + X_{h_1}X_{h_3} + X_{h_2}X_{h_3}) \\
& + e_3(X_{h_1} + X_{h_2} + X_{h_3})^2.
\end{aligned}
$$

Note that, as to be expected, these total average costs are not equal to the total actual costs $F^p$ in (3). So in order to divide the actual cost over the households, we have to use the values $\mu_{h_i}$ as a proportion of the actual costs attributed to each household in the average case:

$$\tilde{\mu}_{h_i} = \frac{\mu_{h_i}}{\mu}\phi. \tag{4}$$

Comparing this with the properties of the Shapley value, we gave up symmetry to achieve locational independence. Note, that with identical loads, the costs for each households are the same. In addition, the price of electricity is equal for all households at each point in time. As such, all locational dependency is averaged out in this pricing. Even though that is what we aimed to achieve, offering the same marginal prices might be too averaged, so we present an alternative.

### C. Approximate Average Location

In the previous subsection, we used the average location defined by all possible permutations of all households to determine the costs allocated to a household. However, as the number of permutations grows exponentially with the number of houses, this calculation gets quite inefficient. Even though the formulas presented in the previous section grow quadratically for chain networks, we want to have a more efficient alternative when extending this approach to non-chain networks. Therefore, we propose the following alternative as an approximation of the average location. For the approximate average contributions of household $h_i$, we consider the permutations $p_i^j$ where household $h_i$ is exchanged with household $h_j$, for $j = 1, \ldots, n$. More formally, permutations $p_i^j$ are defined by:

$$
p_i^j(k) = \begin{cases} h_j, & \text{if } k = i, \\ h_i, & \text{if } k = j, \\ h_k, & \text{otherwise.} \end{cases} \tag{5}
$$

Now, to determine the approximate average contribution of household $h_i$ to the losses, we do not take the average of the Shapley value over all $n!$ permutations, but only the $n$ permutations $p_i^1, \ldots, p_i^n$. This cost is denoted by $\rho_{h_i}$. Here we can see the computational advantage of this calculation (scaling down from $n!$ to $n$). If we work this out for $h_1$ and $n = 3$, we get:

$$\begin{aligned}\rho_{h_1} = X_{h_1}(&\tfrac{1}{3}e_1 X_{h_1} + \tfrac{2}{3}e_2(X_{h_1} + X_{h_2}) \\ &+ e_3(X_{h_1} + X_{h_2} + X_{h_3})).\end{aligned}$$

Similarly for $h_2$ and $h_3$ we get:

$$\begin{aligned}\rho_{h_2} &= X_{h_2}(\tfrac{1}{3}e_1 X_{h_2} + \tfrac{2}{3}e_2(X_{h_1} + X_{h_2}) \\ &\quad + e_3(X_{h_1} + X_{h_2} + X_{h_3})), \\ \rho_{h_3} &= X_{h_3}(\tfrac{1}{3}e_1 X_{h_3} + \tfrac{1}{3}e_2(X_{h_1} + X_{h_2} + 2X_{h_3}) \\ &\quad + e_3(X_{h_1} + X_{h_2} + X_{h_3})).\end{aligned}$$

Note that for the case $n = 3$, coincidentally $\rho_{h_3}$ is equal to $\mu_{h_3}$. Furthermore, if the loads are identical for any two households, the costs of these households are identical. For the total costs we get:

$$\begin{aligned}\rho &:= \rho_{h_1} + \rho_{h_2} + \rho_{h_3} \\ &= \tfrac{1}{3}e_1(X_{h_1}^2 + X_{h_2}^2 + X_{h_3}^2) + \tfrac{1}{3}e_2(2X_{h_1}^2 + 2X_{h_2}^2 \\ &\quad + 2X_{h_3}^2 + 4X_{h_1}X_{h_2} + X_{h_1}X_{h_3} + X_{h_2}X_{h_3}) \\ &\quad + e_3(X_{h_1} + X_{h_2} + X_{h_3})^2.\end{aligned}$$

Again, the total allocated costs are not the total actual costs as calculated in (3). Thus, to determine the actual cost, we have to scale the costs appropriately:

$$\tilde{\rho}_{h_i} = \frac{\rho_{h_i}}{\rho}\phi. \tag{6}$$

If we have identical loads for all households, so $X_{h_i} = X$ for all $h_i \in H$, again we get identical costs for all households:

$$\rho_{h_i} = (\tfrac{1}{3}e_1 + \tfrac{4}{3}e_2 + 3e_3)X^2.$$

Also with the approximate average contributions, we do not have that this cost function is symmetric.

## IV. General Case

In the previous section, we have derived three different price mechanisms in the case of three households. In this section we derive the result for the general case of $n$ households, $h_1, \ldots, h_n$.

### A. Shapley Value

With a simple generalization of the reasoning in Section III-A, we get the following explicit expression for the Shapley value of household $h_i$ if we base the costs on the incurred losses:

$$\phi_{h_i} = X_{h_i}\left(\sum_{j=i}^{n} e_j \left(\sum_{k=1}^{j} X_{h_k}\right)\right).$$

This means that for each cable segment $(j, j+1)$ between household $h_i$ and the transformer (i.e. $j \geq i$), we take the summed load of the households using this segment (households $h_1$ to $h_j$) and multiply this by the load of household $h_i$. The total cost then becomes

$$\phi = \sum_{j=1}^{n} e_j \left(\sum_{k=1}^{j} X_{h_k}\right)^2.$$

### B. Average Location

To derive an expression for the total average location cost, we need to get some intuition of how it is constructed. First of all, note that as we average over all possible permutations $p \in \mathbb{P}$, it does not matter which household we take to calculate the costs, meaning we end up with the same structural expression for each house. Secondly, as we have $n$ households, we have a total of $n!$ possible permutations. In the previous sections, we have seen that the expression $\mu_{h_i}$ for the average contribution of household $h_i$ always looks like

$$\mu_{h_i} = X_{h_i}\left(\sum_{j=1}^{n} e_j y_{i,j}\right),$$

where $y_{i,j}$ is the term for the contribution of cable segment $(j, j+1)$ to the average costs of household $h_i$. It remains to find these terms $y_{i,j}$ for each cable segment. Note that only the permutations where household $h_i$ is on a location $k$ with $k \leq j$ contribute to segment $(j, j+1)$. Thus, only permutations with $p(k) = h_i$ where $k \leq j$ have to be considered. For the first segment $(1, 2)$, this means that only the permutations where $p(1) = h_i$ contribute. This happens $(n-1)!$ times out of the $n!$ permutations and each time the load is given by $h_i$. So, we get

$$y_{i,1} = \frac{(n-1)!}{n!}X_{h_i} = \tfrac{1}{n}X_{h_i}.$$

For the second segment, we only have to take into account the permutation where $p(1) = h_i$ or $p(2) = h_i$. Each of these options happens $(n-1)!$ out of the $n!$ permutations. If $p(1) = h_i$, any of the $n-1$ other households $h_k$ with $k \neq i$, are evenly likely to be on location 2. Furthermore, each combination of $h_i$ and $h_k$ on the first two positions occurs in $(n-2)!$ of the $n!$ permutations. A similar argument holds for $p(2) = h_i$. This means the contribution of the second segment is

$$\begin{aligned}y_{i,2} &= 2\left(\frac{(n-2)!}{n!}\sum_{k=1,k\neq i}^{n}(X_{h_i} + X_{h_k})\right) \\ &= 2\left(\frac{(n-1)!}{n!}X_{h_i} + \frac{(n-2)!}{n!}\left(\sum_{j=1,j\neq i}^{n} X_{h_j}\right)\right) \\ &= \frac{2}{n}X_{h_i} + \frac{2}{n(n-1)}\sum_{j=1,j\neq i}^{n} X_{h_j}.\end{aligned}$$

To derive an expression for $y_{i,j}$, note that we only need to take into account permutations where $h_i$ is on location 1 to $j$. Each option happens $(n-1)!$ times. If $p(j) = h_i$, any of the $n-1$ other households $h_k$ with $k \neq i$, are evenly likely to be on location 1 to $j-1$. Furthermore, each $h_k$ on the first $j-1$ positions occurs in $(j-1)\cdot(n-2)!$ of the $n!$ permutations. Similar arguments hold for $p(1)=h_i$ up to $p(j-1)=h_i$. This means the contribution to the $j$-th segment is

$$y_{i,j} = j\left(\frac{(n-1)!}{n!}X_{h_i} + \frac{(j-1)\cdot(n-2)!}{n!}\left(\sum_{k=1,k\neq i}^{n} X_{h_k}\right)\right) = \frac{j}{n}X_{h_i} + \frac{j(j-1)}{n(n-1)}\sum_{k=1,k\neq i}^{n} X_{h_k}.$$

Combining the contributions of all segments, we get

$$\mu_{h_i} = X_{h_i}\sum_{j=1}^{n} e_j\left(\frac{j}{n}X_{h_i} + \frac{j(j-1)}{n(n-1)}\sum_{k=1,k\neq i}^{n} X_{h_k}\right).$$

The total costs in this case are then

$$\mu = \sum_{j=1}^{n} e_j\frac{j}{n}\left(\frac{j-1}{n-1}\left(\sum_{i=1}^{n} X_{h_i}\right)^2 + \frac{n-j}{n-1}\sum_{i=1}^{n} X_{h_i}^2\right).$$

### C. Approximate Average Location

To derive the expression for the approximate average location cost, we need to get some intuition on how it is constructed. Opposed to the average location cost, we do not take the average over all possible permutations, but for household $h_i$ only over the $n$ permutation $p_i^1,\ldots,p_i^n$, as defined in (5). This implies that the costs of each household do not need to be identical (though with identical loads, the costs will be identical again). As in Section IV-B, we use the same structure in the cost contribution function meaning that for each cable segment $(j,j+1)$, we need to find the term $y_{i,j}$ that specifies the contribution of this segment for household $i$. Note that only the permutations with $p(k)=h_i$ with $k\leq j$ lead to a contribution of household $h_i$ to segment $(j,j+1)$. For the first segment $(1,2)$, this means that the only contributions come from the permutations where $p(1)=h_i$. This happens exactly once, so the contribution for $h_i$ is

$$y_{i,1} = \tfrac{1}{n}X_{h_i}.$$

For the second segment, we only have to take into account the permutation where $h_i$ is on location 1 or 2. Here we have to consider two cases. If $i\in\{1,2\}$, only the permutation with $p(i)=h_i$ for all $i$ and the permutation exchanging the first two positions have to be considered. In both cases the contribution is $X_{h_1}+X_{h_2}$. If $i\geq 3$, only the permutations $p_i^1$ and $p_i^2$ lead to a contribution of $h_i$ to the second segment. The total contribution of these permutations is $X_{h_1}+X_{h_2}+2X_{h_i}$. This means that the contribution to the second segment for $h_i$ is

$$y_{i,2} = \begin{cases} \frac{2}{n}(X_{h_1}+X_{h_2}), & \text{if } i\in\{1,2\},\\ \frac{1}{n}(X_{h_1}+X_{h_2}+2X_{h_i}), & \text{if } i\in\{3,\ldots,n\}.\end{cases}$$

TABLE I
POWERS $X_{h_i}$ AND RESULTING TOTAL COSTS FOR THE THREE CASES.

| | $X_{h_1}$ | $X_{h_2}$ | $X_{h_3}$ | Total costs |
|---|---|---|---|---|
| Case 1 | 3 | 3 | 3 | 126 |
| Case 2 | 3 | -9 | 0 | 81 |
| Case 3 | 3 | 6 | 9 | 414 |

To derive the expression for $y_{i,j}$, note that we need to take into account only permutations $p_i^k$ with $k\leq j$, where $h_i$ is on location 1 to j. Each of these options happens exactly once. If $i\leq j$, then all permutations lead to a contribution of $\sum_{k=1}^{j} X_{h_k}$ to segment $(j,j+1)$. If $i>j$, the permutations $p_i^1,\ldots,p_i^j$ lead to a contribution to segment $(j,j+1)$, but now for $p_i^\ell$ with $\ell\leq j$, the contribution is $X_{h_i}+\sum_{k=1,k\neq\ell}^{j} X_{h_k}$. This means that the contribution to the $j$-th segment is

$$y_{i,j} = \begin{cases} \frac{j}{n}\sum_{k=1}^{j} X_{h_k}, & \text{if } i\in\{1,\ldots,j\},\\ \frac{j}{n}X_{h_i} + \frac{j-1}{n}\sum_{k=1}^{j} X_{h_k}, & \text{if } i\in\{j+1,\ldots,n\}.\end{cases}$$

Combining the contributions of all segments, we get

$$\rho_{h_i} = X_{h_i}\left(\sum_{j=1}^{i-1} e_j\left(\tfrac{j}{n}X_{h_i} + \tfrac{j-1}{n}\sum_{k=1}^{j} X_{h_k}\right) + \sum_{j=i}^{n} e_j\left(\tfrac{j}{n}\sum_{k=1}^{j} X_{h_k}\right)\right).$$

We can see this expression depends on the original location of household $h_i$, as the summation splits depending on this $i$. However, if $X_{h_i}=X$ for all $h_i\in H$, this dependency disappears:

$$\rho_{h_i} = X^2\left(\sum_{j=1}^{n} e_j\tfrac{j^2}{n}\right).$$

Summing up the $\rho_{h_i}$ for all $h_i\in H$ gives us the total costs:

$$\rho = \sum_{j=1}^{n-1}\tfrac{e_j}{n}\left(j\left(\sum_{i=1}^{j} X_{h_i}\right)^2 + j\sum_{i=1}^{n} X_{h_i}^2 + (j-1)\left(\sum_{i=1}^{j} X_{h_i}\right)\left(\sum_{k=j+1}^{n} X_{h_k}\right)\right) + e_n\left(\sum_{i=1}^{n} X_{h_i}\right)^2.$$

## V. NUMERICAL COMPARISON

To get an idea of the differences between the newly introduced price mechanisms and some of the well-known mechanisms such as linear and quadratic pricing, we give a numerical comparison of these prices. For the small example presented in Fig. 1, we use three different cases corresponding

TABLE II
UNSCALED AND SCALED COSTS FOR EACH HOUSEHOLD IN THE CASES PRESENTED IN TABLE I FOR DIFFERENT PRICE MECHANISMS.

| | Case 1 | | | | | | Case 2 | | | | | | Case 3 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Unscaled costs | | | Scaled costs | | | Unscaled costs | | | Scaled costs | | | Unscaled costs | | | Scaled costs | | |
| | $h_1$ | $h_2$ | $h_3$ | $h_1$ | $h_2$ | $h_3$ | $h_1$ | $h_2$ | $h_3$ | $h_1$ | $h_2$ | $h_3$ | $h_1$ | $h_2$ | $h_3$ | $h_1$ | $h_2$ | $h_3$ |
| Linear | 3 | 3 | 3 | 42 | 42 | 42 | 3 | -9 | 0 | -40.5 | 121.5 | 0 | 3 | 6 | 9 | 69.0 | 138.0 | 207.0 |
| Quadradic | 9 | 9 | 9 | 42 | 42 | 42 | 9 | 81 | 0 | 8.1 | 72.9 | 0 | 9 | 36 | 82 | 29.3 | 117.4 | 267.3 |
| Shapley | 54 | 45 | 27 | 54 | 45 | 27 | -27 | 108 | 0 | -27.0 | 108.0 | 0 | 90 | 162 | 162 | 90.0 | 162.0 | 162.0 |
| Average | 42 | 42 | 42 | 42 | 42 | 42 | -18 | 99 | 0 | -18.0 | 99.0 | 0 | 78 | 168 | 270 | 62.6 | 134.8 | 216.6 |
| Approx. Av. | 42 | 42 | 42 | 42 | 42 | 42 | -27 | 117 | 0 | -24.3 | 105.3 | 0 | 75 | 156 | 243 | 65.5 | 136.3 | 212.2 |

to a possible morning, afternoon and evening situation respectively. The details of these cases are presented in Table I, where the corresponding loads $X_{h_i}$ and the total costs based on (3) are given. For the cable parameters, we assume $e_1 = e_2 = e_3 = 1$.

The costs for the different pricing mechanisms in the three cases for households $h_1$, $h_2$ and $h_3$ are presented in Table II. We give the unscaled costs for each pricing mechanism in each case and then divide the total costs over households according to their ratios (similar to (4) and (6)). For linear and quadratic pricing, we use $X_{h_i}$ and $X_{h_i}^2$ respectively for the unscaled costs. As shown for Case 1, with identical usage, the costs are identical for all households (except for the Shapley value). This at least shows a form of fairness in the (approximate) average mechanism. In Case 2, household $h_1$ gets paid for its consumption, while household $h_2$ pays for its production in the newly presented price mechanisms. It is interesting that household $h_3$ has to pay more than its Shapley value in Cases 1 and 3, as it causes relatively little losses, though would cause more in an average location. Here it can clearly be seen that the Shapley value penalizes households further from the transformer, and favours households close to it. For all three cases, the differences between the average and approximate average location price mechanism are relatively small. The largest deviation occurs in Case 2, where the difference between the cost according to the average location for household $h_1$ and $h_2$ is the largest as well.

Overall, we can state that with the average and approximate average price mechanism, the polluter pays and the ones helping to solve the congestion get rewarded. Also, we see that the costs are hardly locationally biased (especially compared to the Shapley value in Case 3).

## VI. CONCLUSIONS AND FUTURE WORK

In this paper, we have presented two novel price mechanisms based on the losses caused by the transport of energy in the LV grid. Both mechanisms are based on the Shapley value. We have shown that these prices can be determined efficiently using an application specific structure that arises.

We conclude this paper with some possible extensions to the presented price mechanisms. As we only focused on networks that are represented by a chain, it is of interest to also consider networks represented by arbitrary trees. Although this would destroy part of the structure used in this paper, an efficient approximation should still be possible. Another interesting question is how households respond to the presented prices in an optimization context, especially considering automatically steerable equipment. Lastly, it is of interest to investigate the influence of the parameters $e_j$ on the resulting prices.

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## REFERENCES


[1] K. McKenna and A. Keane, "Discrete elastic residential load response under variable pricing schemes," in *Innovative Smart Grid Technologies Conference Europe (ISGT-Europe), 2014 IEEE PES*. IEEE, 2014.

[2] A. H. Mohsenian-Rad and A. Leon-Garcia, "Optimal residential load control with price prediction in real-time electricity pricing environments," *IEEE Trans. Smart Grid*, vol. 1, no. 2, pp. 120–133, 2010.

[3] A. J. Wood and B. F. Wollenberg, *Power generation, operation, and control*. John Wiley & Sons, 2012.

[4] Y. Z. Li and A. K. David, "Wheeling rates of reactive power flow under marginal cost pricing," *IEEE transactions on power systems*, vol. 9, no. 3, pp. 1263–1269, 1994.

[5] A. H. Mohsenian-Rad, V. W. S. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE transactions on Smart Grid*, vol. 1, no. 3, pp. 320–331, 2010.

[6] A. De Paola, D. Angeli, and G. Strbac, "Price-based schemes for distributed coordination of flexible demand in the electricity market," *IEEE Transactions on Smart Grid*, vol. 8, no. 6, pp. 3104–3116, 2017.

[7] X. Gong, A. De Paola, D. Angeli, and G. Strbac, "A distributed price-based strategy for flexible demand coordination in multi-area systems," in *8th IEEE PES Innovative Smart Grid Technologies Conference Europe 2018*, 2018.

[8] C. Edmunds, S. Galloway, and S. Gill, "Distributed electricity markets and distribution locational marginal prices: A review," in *Universities Power Engineering Conference (UPEC), 2017 52nd International*. IEEE, 2017.

[9] M. Pilz, J.-C. Nebel, and L. Al-Fagih, "A practical approach to energy scheduling: A game worth playing?" in *8th IEEE PES Innovative Smart Grid Technologies Conference Europe 2018*, 2018.

[10] M. Junqueira, L. C. da Costa, L. A. Barroso, G. C. Oliveira, L. M. Thomé, and M. V. Pereira, "An aumann-shapley approach to allocate transmission service cost among network users in electricity markets," *IEEE Transactions on Power Systems*, vol. 22, no. 4, pp. 1532–1546, 2007.

[11] M. E. T. Gerards and J. L. Hurink, "A mechanism design for a sustainable integration of renewable energy," *Energy-Open*, p. 20, 2017.

[12] V. M. J. J. Reijnders, M. E. T. Gerards, J. L. Hurink, and G. J. M. Smit, "Testing grid-based electricity prices and batteries in a field test," in *CIRED Workshop 2018: Microgrids and Local Energy Communities*, 2018, p. 0500.

[13] "GridFlex Heeten," https://gridflex.nl/, accessed: 2019-03-22.

[14] L. S. Shapley, "A value for n-person games," *Contributions to the Theory of Games*, vol. 2, no. 28, pp. 307–317, 1953.

[15] S. Neuteleers, M. Mulder, and F. Hindriks, "Assessing fairness of dynamic grid tariffs," *Energy Policy*, vol. 108, pp. 111–120, 2017.