

SQUID developments for high accuracy current measurements in the kHz frequency range

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INTRODUCTION

Superconducting quantum interference devices (SQUIDs) for high accuracy current measurements in the kHz frequency range are developed for utilization in the first spherical gravitational wave detector MiniGRAIL, a resonant mass detector based on a CuAl6% sphere with a diameter of 68 cm [1-3]. It is expected to show variations in diameter of the order of 10^{-20} m at its resonance frequency of about 3 kHz when it is excited by a gravitational wave. The operation temperature will be about 20 mK. For the readout of the MiniGRAIL, a capacitive transducer was developed which converts the displacement into a current.

Because of its low noise properties, direct current SQUIDs (dc-SQUIDs) were chosen for the readout of this current. For reasons of noise matching, the for high sensitivity optimized sensor SQUID has to be operated in a two-stage SQUID setup.

In this paper we focus on the design steps taken to develop this current measurement system.

SENSITIVITY OF SQUID-SENSORS

To compare the sensitivity of different SQUIDs, the energy resolution is commonly used. It represents the equivalent white noise energy per bandwidth referred to the SQUID inductance. For an uncoupled SQUID in the classical limit, it is given by [4]

$$\varepsilon_{uncoupled} \approx \frac{8 k_B T L_{sq}}{R_{sh}} \left(\frac{1 + \beta_L}{2 \beta_L} \right)^2 \quad (\text{for } \beta_L \approx 1), \quad (1)$$

where k_B is the Boltzmann constant, T is the operating temperature of the SQUID, R_{sh} is the shunt resistance of one Josephson junction and L_{sq} is the SQUID inductance. The screening-parameter β_L is calculated with $2I_0 L_{sq} / \Phi_0$, where $2I_0$ is the critical current of the SQUID and Φ_0 is the flux quantum.

The minimum reachable energy resolution, the quantum limit, is at about 1h. Here the sensitivity is limited by zero-point fluctuations [5], equation (1) does not hold anymore.

The white noise flux power spectral density (PSD) of an uncoupled SQUID is given by

$$S_\Phi = 2 \varepsilon L_{sq}. \quad (2)$$

Below also the sometimes more handy amplitude spectrum density (ASD), the square root of the PSD, is used.

Because the input current is the quantity to measure, it is useful to define an equivalent white noise PSD referred to the input current

$$S_{I,imp} = \frac{S_\Phi}{M_{sig}^2} = \frac{2 \varepsilon}{k^2 L_{sig}}, \quad (3)$$

where L_{sig} is the inductance of the signal coil and k is the coupling factor between this and the SQUID inductance L_{sq} .

Following (3), the first step to minimize the sensitivity is the usage of a high inductance signal coil (L_{sig}) with a good coupling to the SQUID loop (k), suggesting the usage of integrated coils.

Further one should focus on the minimization of the energy resolution ε . From (1) follows that β_L should be close to 1, which in practice also fixes L_{sq} for a given Josephson junction technology. Also the value of R_{sh} is strongly connected to the properties of the junctions, it has to be small enough to suppress the hysteresis. Finally the operation temperature should be minimized to reach the highest sensitivity.

Of course there are some practical limits in the reachable sensitivity. To reach a good coupling, an integrated signal coil has to be used, which adds problems with resonances. Special attention has to be paid to have no resonance frequencies close to the operation frequency of the intended working-point. Furthermore, the inserted parasitic capacitance across the washer can degrade the sensitivity, leading to an increased energy resolution [6] in the tightly coupled case

$$\varepsilon_{coupled} \approx \varepsilon_{uncoupled} \cdot \begin{cases} \sqrt{1 + 2 C_p / C_j} & (C_p / C_j \leq 2) \\ \sqrt{5} & (C_p / C_j > 2) \end{cases} \quad (4)$$

The additional factor in (4) was estimated by numerical simulation with an effective parasitic capacitances introduced by the input-coil C_p and the junction capacitance C_j .

Another physical obstacle in reaching the quantum limit is the hot-electron effect. At low temperatures, for conventional SQUIDs below 1 K, the conduction of thermal power from the electron system to the phonon system within the shunt resistors is degraded. So the dissipated electrical power leads to a, compared to the bath temperature, increased operating temperature of the SQUID. This leads to a minimum reachable temperature of the shunt resistors given by [7]

$$T_{min} = \sqrt[5]{\frac{P}{\Omega \Sigma} + T_{bath}^5}, \quad (5)$$

where P is the dissipated power in the resistor, Ω is the volume of the resistor, Σ is a material dependent electron-phonon coupling constant, typical in the order of $10^9 \text{ Wm}^{-3}\text{K}^{-5}$, and T_{bath} is the temperature of the bath. Thus the minimum reachable temperature can be reduced by increasing the volume Ω . This can be done by attaching a cooling fin to the resistors [7].

In conclusion, the ideal case would be a SQUID sensor that reaches the quantum limited sensitivity at the minimum operation temperature, which is limited by the hot-electron effect.

Besides the discussed white noise properties, SQUID sensors also have a $1/f$ noise dominated frequency region. This is typically below the, for MiniGRAIL interesting, frequency range.

TWO-STAGE SQUID SYSTEMS

Of course the gained signal has to be processed further and special attention has to be paid on noise-matching within the whole readout system. In practice, especially the room-temperature amplifier can affect the total sensitivity.

This can be eliminated with another SQUID used as a cryogenic pre-amplifier as shown in Fig. 1. Depending on the properties of the signal SQUID, the bias impedance $R_{b,1st}$ and the mutual inductance of the signal coil of the second stage, the flux is amplified by a factor $G_\Phi = \partial\Phi_{2nd}/\partial\Phi_{1st}$ from the first stage to the second stage.

The total sensitivity of the system at the interesting frequency range can be expressed by the flux noise PSD $S_{\Phi,tot}$ referred to the flux in the first stage SQUID

$$S_{\Phi,tot} = S_{\Phi,1st} + \frac{1}{G_\Phi^2} \left(S_{\Phi,2nd} + \frac{S_{V,amp}}{T_{V\Phi,2nd}^2} \right). \quad (6)$$

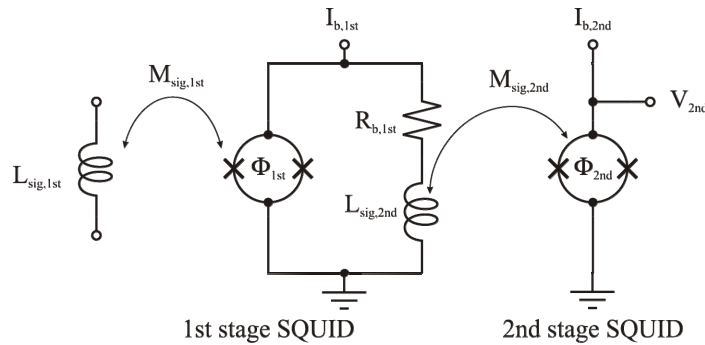


Fig. 1. Schematic of a two-stage SQUID system

Here, $S_{\phi,1st}$ and $S_{\phi,2nd}$ are the flux noise PSDs of the two SQUIDs, $S_{v,amp}$ is the input voltage noise PSD of the amplifier and $T_{V\phi,2nd}$ is the flux-to-voltage transfer of the second stage SQUID.

For a well implemented two-stage SQUID setup, $S_{\phi,tot}$ is close to $S_{\phi,1st}$. In practice the second stage SQUID does not have to be very sensitive, if G_{ϕ} is high enough. The emphasis should be placed on the amplification of the signal above the noise level of the room temperature amplifier, especially because the sensitivity of both SQUIDs is improving with lower temperature while the noise level of the amplifier stays the same. So a second stage with a high flux-to-voltage transfer $T_{V\phi,2nd}$ is preferred. In practice this could be realized by a series array of dc-SQUIDs, SQUIDs with additional positive feedback or double relaxation oscillation SQUIDs (DROSeS), which have the advantage of an increasing flux-to-voltage transfer $T_{V\phi,2nd}$ with decreasing temperature [8].

SUMMARY OF THE FIRST DESIGN- AND CHARACTERIZATION PROCESS

We have designed several SQUIDs [9] and they were fabricated at the HYPRES foundry [10]. Basically, there are three designs for the first stage: a dc-SQUID with a coupled flux-transformer and two dc-SQUIDs with two feedback and two signal washers, one in a series [11] and one in a parallel configuration [12]. For the second stage DROSeS were made.

The reason for designing dc-SQUIDs with multiple-washer configurations are back-action effects. Noise power fed back into MiniGRAIL, which represents a resonator with a very high quality factor, can lead to a loss in accuracy and an unstable operation [13]. The difference to a conventional single washer dc-SQUID is that the signal and the feedback coils are integrated on different washers, reducing a direct magnetic coupling to a minimum. This is believed to help to overcome the negative effects.

Unfortunately a lot of fabricated samples were damaged or showed a big spread in the values for critical current density and sheet resistance. The characterization showed that most of the samples did not fulfill the expected properties.

The working samples of the dc-SQUID with washers in a series-configuration as well as with flux-transformer showed strongly distorted characteristics, so no useful results could be obtained. The believed reason for this are resonance effects in the input coils.

In general we observed big differences in the measured mutual inductances, which could be explained by a more precise estimation of the slit-inductance using the numerical inductance calculation program ‘‘FastHenry’’ [14].

The results of the characterization of the dc-SQUID with parallel washers at 4.2 K are shown in Fig. 2. The minimum measured flux noise ASD of $2.5\mu\Phi_0/\sqrt{\text{Hz}}$ was a factor of 2.5 higher than the on (1) based design prediction.

Adapting the measured critical current and resistances and the re-calculated inductance, the with (1) and (2) calculated flux noise ASD is $1.5\mu\Phi_0/\sqrt{\text{Hz}}$. The effective parasitic capacitance introduced by the input coil was estimated to be about a factor of 7 higher than the capacitance of the Josephson junction. With (4) this suggests a flux noise ASD of $2.3\mu\Phi_0/\sqrt{\text{Hz}}$, which is close to the measured value.

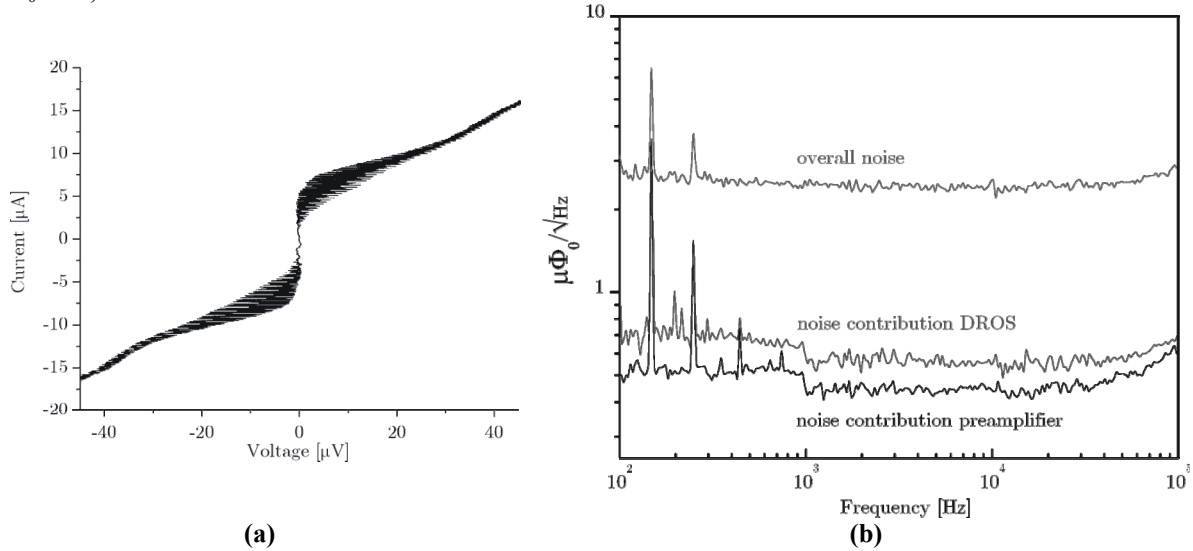


Fig. 2. Measurements on the dc-SQUID with parallel washers at a temperature of 4.2K. (a) Current-Voltage characteristic; (b) Flux noise measurement, conducted in a two-stage setup in open loop. The measured white flux noise ASD is $2.5\mu\Phi_0/\sqrt{\text{Hz}}$, corresponding to an input current noise ASD of $300\text{fA}/\sqrt{\text{Hz}}$ and an energy resolution of 550h. The noise levels of the second stage SQUID and the room-temperature amplifier were calculated and are shown in the same graph.

Thus, the degraded noise properties of the SQUID could be explained by a spread in fabrication, imprecise inductance calculation and the influence of the integrated signal coil.

Following the problems observed with the old samples, the main guidelines for the next design step were extended with the suppression of effects originating from coil resonances.

THE NEXT DESIGN-STEP

We joined a run with 110 A/cm² critical current density at the JeSEF foundry [15] with the possibility to add cooling-fins to the shunt-resistors. The samples are fabricated but not yet completely characterized.

The designed sensor SQUIDs are dc-SQUIDs with a multiple washer configuration and an integrated flux-transformer. They will be discussed more detailed below. The dc-SQUID with washers in a series configuration would lead to a screening parameter β_L much bigger than 1 and so we decided not to adapt this design.

For the second stage DROsEs and a series SQUID array were designed. To conduct experiments on the hot-electron effect, chips with test structures were designed.

New sensor SQUIDs

The sensor SQUIDs were optimized regarding the above mentioned steps for improving the sensitivity.

To reach the lowest possible effective operation temperature, cooling-fins were added to the sensor-SQUIDs. They are made in a PdAu layer of 100nm thickness.

Resonances were encountered by shifting the critical frequencies as far away as possible from the operation frequency. Besides the standard damping of resonances by external shunts on the input coil and washer-resistors [16], more experimental damping schemes like resistances connected between the coil and the washer [17] and intra-coil damping [18] were applied in variances of the standard designs.

The dc-SQUID with Flux Transformer

The advantage of using a flux transformer is that any input inductance can be matched to the, with the critical current of the SQUID varying, optimum SQUID inductance. A schematic and the layout are shown in Fig. 3.

The final design parameters are listed in Table 1. The listed inductances are the effective values from the input to the SQUID inductance, so the screening of the transformer loop is included.

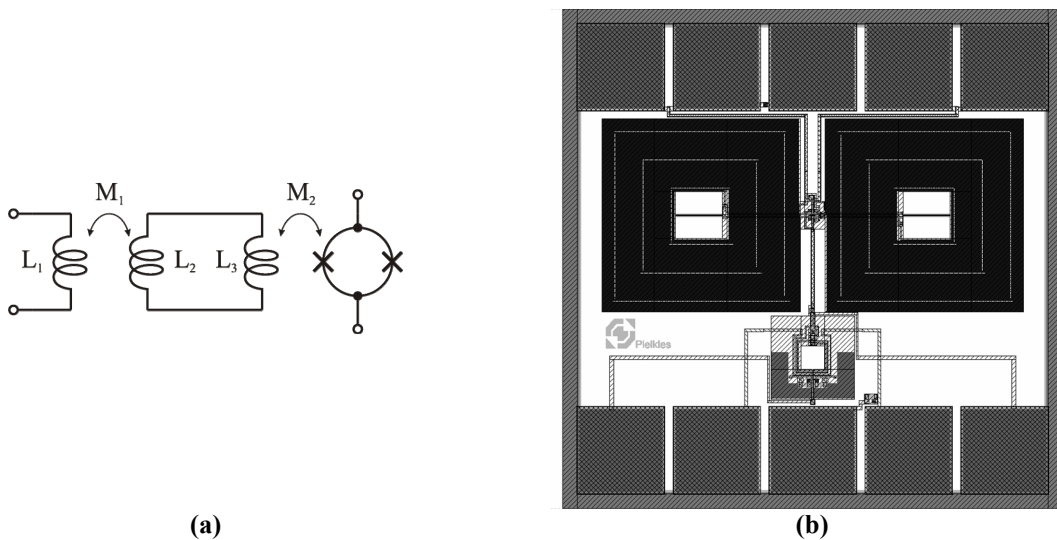


Fig. 3. The dc-SQUID with a multiple-washer configuration. (a) simplified schematic. (b) layout – in the top part the gradiometric pickup circuit (L_1 and L_2 in the schematic, which are both actually two gradiometric washers (L_2) and two gradiometric coils (L_1)) is visible, in the bottom part the single-hole dc-SQUID

Table 1. Design-values of the dc-SQUID with flux transformer

$2 I_0$	R_{sh}	L_{sq}	β_L	L_{sig}	M_{sig}
22.5 μ A	5.7 Ω	128 pH	1.4	1.6 μ H	10.4 nH

Using (1), (2) and (3) one can determine the sensitivity at a temperature of 4.2K to be $\varepsilon=73\hbar$, $\sqrt{S_\phi}\approx 0.68\mu\Phi_0/\sqrt{\text{Hz}}$ or $\sqrt{S_{I,inp}}\approx 140\text{fA}/\sqrt{\text{Hz}}$ respectively. The parasitic capacitance introduced by the pickup coil is ignored here, because it is not directly located at the SQUID. Anyway, negative effects cannot be ruled out, so the real sensitivity is expected to be worse.

The minimum reachable operation temperature strongly depends on the hot-electron effect. With (3) it is estimated for the stand-alone shunt resistor with a volume of $1.6\cdot 10^{-16}\text{ m}^3$ to be 270 mK. Here, the power P dissipated in one shunt resistance was guessed to be $\frac{1}{4}\text{ nW}$, the bath temperature was set to 20 mK and the material constant Σ was set to a guessed value of $1\cdot 10^9\text{ Wm}^{-3}\text{K}^{-5}$ – a median value of the until now measured constants for different metals [19]. Using the volume of the attached cooling fin $2.6\cdot 10^{-15}\text{ m}^3$ one gets a minimum temperature of 160 mK. It is believed that there exists an effective size of a cooling fin, not all the volume of the cooling fin will be used to cool down and conduct electrons [20]. So the real minimum operating temperature should be somewhere between those two values, our rough guess is 0.2K.

The sensitivity at 0.2 K is determined in the same way as above to be $\varepsilon=4\hbar$, $\sqrt{S_\phi}\approx 0.15\mu\Phi_0/\sqrt{\text{Hz}}$ or $\sqrt{S_{I,inp}}\approx 30\text{fA}/\sqrt{\text{Hz}}$ respectively.

To reduce possible effects of resonances in the input coil, it is possible to connect a damping circuit into the pickup loop. An additional version of this design with a damping resistor between the gradiometric pickup coils and the pickup-washers was made [17].

dc-SQUID with a Multiple-Washer Configuration

Because of its symmetric layout and the reduced coupling between the feedback- and the signal-coils, this design was adapted from [12]. The layout and the schematic are shown in Fig. 4. The differences to the original design are small integrated resistances R_b , for a symmetric splitting of the bias current, reducing a coupling to the SQUID inductance, as well as cooling fins.

The final design-parameters are listed in Table 2. The effective parasitic capacitance C_p was determined with a connection of the separate parasitic elements of each washer connected like the washers.

Using the coupled energy resolution from (4), (1) and (2), one can determine the sensitivity at a temperature of 4.2 K: $\varepsilon=140\hbar$, $\sqrt{S_\phi}\approx 1.3\mu\Phi_0/\sqrt{\text{Hz}}$ and $\sqrt{S_{I,inp}}\approx 220\text{fA}/\sqrt{\text{Hz}}$. Here, β_L is far off the condition $\beta_L\approx 1$, so the real sensitivity could differ.

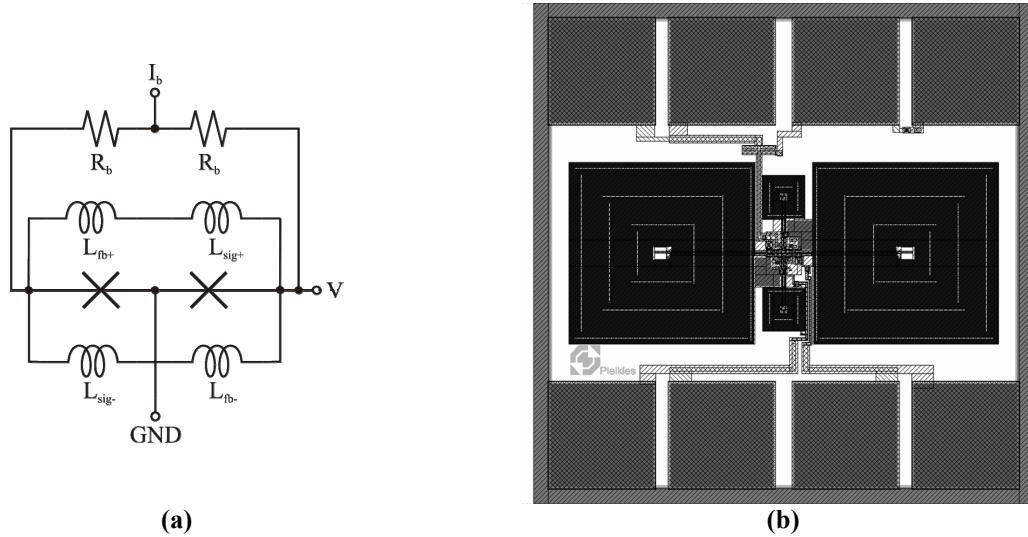


Fig. 4. The dc-SQUID with a multiple-washer-configuration; (a) simplified schematic of the washer configuration without the input coils, (b) layout, on the left and right side the gradiometric signal washers L_{sig} are visible, in the middle of the circuit on the upper and lower part the gradiometric feedback washers L_{fb} .

Table 2. Design-values of the dc-SQUID with multiple washers

$2I_0$	R_{sh}	L_{sq}	β_L	L_{sig}	M_{sig}	C_p/C_j
22.5 μA	5.6 Ω	234 pH	2.5	1.5 μH	12.2 nH	0.8

Following the calculation of the section above, we get a minimum operation temperature of 270 mK without the cooling fin and 180 mK with the attached cooling fin of $1.4 \cdot 10^{-15} \text{ m}^3$. The minimum reachable temperature is again roughly guessed to be 0.2 K.

The sensitivity at 0.2 K is determined in the same way as the one at 4.2K to be $\varepsilon = 7\hbar$, $\sqrt{S_\phi} \approx 0.28 \mu\Phi_0/\sqrt{\text{Hz}}$ or $\sqrt{S_{I_{inp}}} \approx 48 \text{ fA}/\sqrt{\text{Hz}}$ respectively.

Another version of the design with an intra-coil damping resistor on both feedback coils was made to reduce possible resonance effects [18].

OUTLOOK

The first characterization steps showed acceptable results, the critical current density and the sheet resistance are in good agreement to the design-values. The sensor SQUIDs are working and behave close to what we expected. The noise measurements are in progress at the moment. We will present the results at the workshop.

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