Averaging methods for static traffic assignment with junction modelling

Erwin M. Bezembinder
Area Development Research Group, Department of Technology
Windesheim University of Applied Sciences, The Netherlands
Email: e.bezembinder@windesheim.nl

Luc J.J. Wismans
Centre for Transport Studies, Faculty of Engineering Technology
University of Twente, Enschede, The Netherlands

Eric C. Van Berkum
Centre for Transport Studies, Faculty of Engineering Technology
University of Twente, Enschede, The Netherlands

1. Introduction

Traditional traffic assignment models in transportation planning allocate origin-destination trips to routes according to Wardrop’s Principle (Wardrop, 1952) of equal minimum cost per used route. In addition, they assume that travel time on each link is a separable function of the flow on the link itself. This simplistic assumption of separable cost functions allows a convenient mathematical treatment using convex optimisation formulations that guarantees solution uniqueness, and assists in proving algorithms convergence. However, the assumption of separable costs is not always satisfied, especially at junctions, where various traffic movements interact and influence each other’s travel time in a non-separable and potentially highly asymmetric manner. As a consequence, conventional solution approaches, such as the Frank-Wolfe algorithm (Frank and Wolfe, 1956) cannot be used. Throughout the years, various solution approaches for the static traffic assignment problem with junction modelling have been suggested. The most straightforward solution is based on the Method of Successive Averages (MSA), whose major distinction from the Frank-Wolfe algorithm is the determination of the step size; where the Frank-Wolfe algorithm calculates the step size such that the objective function is minimized, MSA uses a predetermined step size per iteration. A major drawback of MSA however, is the slow convergence speed (Sheffi, 1985). This is primarily caused by the fact that in the classic MSA, the step size ($\phi$) is determined by $1/k$, where $k$ is the iteration number. The flow of each iteration is thus weighted equally.

Alternative averaging methods have been developed, primarily aiming to give higher weights to solutions that are closer to the optimal solutions, thus improving convergence speed. Due to the use of averaging methods as a subroutine of Network Design Problems, research concerning the convergence speed of these approaches has been revived in recent years. Gallo et. la. (2015) recently tested various averaging methods to solve the local optimisation of signal settings on a real-scale network in Italy, and concluded that 70% of computing time
can be gained with alternative averaging approaches. Liu et.al. (2009) earlier compared similar approaches with two additional methods called the method of successive weighted averages (MSWA) and the self-regulating averaging method (SRAM). The latter two provided the best convergence speed for solving the stochastic user equilibrium problem. In this research, we examine whether the MSWA and SRAM also provide superior convergence properties for solving the traffic assignment problem with junctions. In addition, we compare the convergence properties for networks with different junction types, being signalized junctions, priority junctions and roundabouts. Test are conducted on the Sioux Falls network.

2. Solution algorithms

We examine seven different solution approaches, which are summarized in Table 1. The first approach (MSA) is the classic MSA algorithm with $\varphi = 1/k$. The second approach (MSA-POL) is based on work of Polyak (1990), were $k$ is replaced by $k^{2/3}$. In the third approach (MSA-BGB) $k$ is replaced by a constant factor. Bar-Gera and Boyce (2006) suggested this value to be 5. The fourth approach (MSA-RM) is called the Refresh Memory approach (Cascetta et.al., 2006), which resets the step size to an earlier value at increasing intervals, thus forcing an ‘acceleration step’. The fifth approach (MSA-NAZ) is based on work of Nagurney and Zang (1996). The sixth approach (MSWA) is based on the method of successive weighted averages as suggested by Liu et.al. (2009). This approach is based on a generalized formulation with a parameter $d$. When $d=0$, the approach equals the classic MSA. Liu et.al. tested various values for $d$ (between 0 and 10) and concluded that higher accuracy solutions required higher values for $d$. We test the values 1, 2, 5 and 10. The seventh method (SRAM) is called the self-regulated averaging method, also suggested by Liu et.al. (2009), for which the step size depends on the distance between the link flow differences ($x - y$) of intermediate solutions. Solutions closer to the optimal solution get a larger step size, whereas solutions lying farther away from the optimal solution get a smaller step size. Liu et.al. suggested the values 1/0.01 and 1/1.90 and for the two different situations.

<table>
<thead>
<tr>
<th>Approach</th>
<th>Explanation</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSA</td>
<td>$\varphi = 1/k$</td>
<td></td>
</tr>
<tr>
<td>MSA-POL</td>
<td>$\varphi = 1/k^{2/3}$</td>
<td>Polyak (1990)</td>
</tr>
<tr>
<td>MSA-BGB</td>
<td>$\varphi = 1/5$</td>
<td>Bar-Gera and Boyce (2006)</td>
</tr>
<tr>
<td>MSA-RM</td>
<td>$\varphi = 1/[1,2, \ldots, 10, 2,3, \ldots, 20, 4,5, \ldots, 40, \ldots]$</td>
<td>Cascetta et.al. (2006)</td>
</tr>
<tr>
<td>MSA-NAZ</td>
<td>$\varphi = 1/[1,2,3,3,3,4,4,4,4,5,5,5,5,5,\ldots]$</td>
<td>Nagurney and Zang (1996)</td>
</tr>
<tr>
<td>MSWA</td>
<td>$\varphi = k^d/(1^d + 2^d + 3^d + \ldots + k^d)$</td>
<td>Liu et.al. (2009)</td>
</tr>
</tbody>
</table>
| SRAM     | $\varphi = 1/\beta_k$, where $\beta_k = \begin{cases} \beta_{k-1} + 1.90, & \text{if } \|x^k - y^k\| \geq \|x^{k-1} - y^{k-1}\| \\
\beta_{k-1} + 0.01, & \text{if } \|x^k - y^k\| < \|x^{k-1} - y^{k-1}\| \end{cases}$ | Liu et.al. (2009) |

Table 1 - Solution algorithms.

The junction delays, which are recalculated every iteration, are determined by using the Highway Capacity Manual 2010 (HCM 2010) (TRB, 2010) methodologies. The ‘optimal’ signal settings (cycle time, green times) are determined using the Quick Estimation Method as described in the HCM 2010.
3. Results

Numerical tests were conducted on the Sioux Falls network, which was extended with 20 junctions. Five network variants were tested, respectively containing no junctions (NET), signalized junctions (NET-SIG), priority junctions (NET-PR), single lane roundabouts (NET-RA1) and two-lane roundabouts (NET-RA2). Traffic demand was assumed fixed. For all averaging methods the number of iterations was determined based on a stopping criterion of a Relative Gap (RG) less than 0.001, as suggested by Rose et.al. (1988). Results show a considerable reduction of the number of iterations for the alternative solution approaches in comparison with the classic MSA. Table 2 shows the results for the four best alternative solution approaches.

<table>
<thead>
<tr>
<th></th>
<th>NET</th>
<th>NET-SIG</th>
<th>NET-PR</th>
<th>NET-RA1</th>
<th>NET-RA2</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSA</td>
<td>669</td>
<td>664</td>
<td>740</td>
<td>884</td>
<td>931</td>
</tr>
<tr>
<td>MSA-RM</td>
<td>222</td>
<td>127</td>
<td>130</td>
<td>243</td>
<td>229</td>
</tr>
<tr>
<td></td>
<td>(67%)</td>
<td>(80%)</td>
<td>(82%)</td>
<td>(73%)</td>
<td>(75%)</td>
</tr>
<tr>
<td>MSA-POL</td>
<td>316</td>
<td>173</td>
<td>142</td>
<td>398</td>
<td>486</td>
</tr>
<tr>
<td></td>
<td>(53%)</td>
<td>(73%)</td>
<td>(81%)</td>
<td>(55%)</td>
<td>(48%)</td>
</tr>
<tr>
<td>MSWA (d=2)</td>
<td>152</td>
<td>121</td>
<td>100</td>
<td>140</td>
<td>143</td>
</tr>
<tr>
<td></td>
<td>(81%)</td>
<td>(81%)</td>
<td>(86%)</td>
<td>(84%)</td>
<td>(85%)</td>
</tr>
<tr>
<td>SRAM</td>
<td>307</td>
<td>486</td>
<td>482</td>
<td>550</td>
<td>482</td>
</tr>
<tr>
<td></td>
<td>(54%)</td>
<td>(25%)</td>
<td>(35%)</td>
<td>(38%)</td>
<td>(48%)</td>
</tr>
</tbody>
</table>

Table 2 – Number of iterations for RG=0.001 (and relative difference to MSA) for all network variants and five solution approaches.

The table shows that the alternative approaches can reduce the number of iteration, and thus the computing time with 81-86%. The benefit of the alternative approaches is higher for the networks with junctions. Indeed, convergence is better in the network with signalized junctions in comparison with the network without junctions. This is probably because the automated (local) optimisation of the signal settings gives extra options to spread the traffic flows over the highly saturated Sioux Falls network. The network variants with roundabouts have the slowest convergence, which could be explained by the fact that the flow on each turn is influenced by almost all other turns. MSWA has the best convergence speed for all network variants, as also can be seen in Figure 1, which shows the RG for NET-PR.

Figure 1 - Convergence of solution algorithms for NET-PR.
A value of $d=2$ gave the best convergence. The SRAM algorithm gave less favourable results as suggested by Liu et.al. (2009), which could be caused by the fact that the parameters used for weighting the different situations are not optimised for networks with junctions. Tests with altered traffic demand (times 0.6, 0.8, 1.2 and 1.6), junction weights (0.25, 0.50, 0.75 and 1.25) and other values for the RG (0.01 and 0.0001) predominantly gave the same results.

4. Conclusions

In this research we tested seven MSA-based averaging algorithms in order to increase the convergence speed. Results show that computing time in a network with junctions can be reduced with 81-86% in comparison with the classic MSA algorithm. MSWA and MSA-POL show the best results, while SRAM performs less favourable than suggested in earlier studies. Networks with signalized junctions converge the fastest, while networks with roundabouts converge slowest.

References


Gallo, M., L. D’Acicrino and B. Montella. MSA algorithms for solving the combined assignment-control problem. In: Rudas, I.J. (Eds.), Recent Researches in Mechanical and Transportation Systems (pp. 90-96), WSEAS Press. 2015.


