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Squared-down passivity–based state synchronization of homogeneous continuous-time multiagent systems via static protocol in the presence of time-varying topology

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Summary

This paper studies state synchronization of homogeneous multiagent systems (MAS) via a *static protocol* with partial-state coupling in the presence of a time-varying communication topology, which includes general time-varying graphs as well as switching graphs. If the agents are squared-down passive or squared-down passifiable (via static output feedback or static input feedforward), then a static protocol can be designed for balanced, time-varying graphs. Moreover, this static protocol works for arbitrary switching directed graphs if the agents are squared-down minimum phase with relative degree one. The static protocol is designed for each agent such that state synchronization is achieved without requiring exact knowledge about the time-varying network.

KEYWORDS

multiagent systems, squared-down passivity and passifiability, state synchronization, static protocol, time-varying graph

1 | INTRODUCTION

The synchronization problem for multiagent systems (MASs) has attracted substantial attention in recent decades because of its potential applications in cooperative control of autonomous vehicles, distributed sensor networks, swarming, flocking, and others. The objective of synchronization is to secure asymptotic agreement on a common state or output trajectory through decentralized control protocols (see for instance the books of Ren and Cao[1] and Wu[2] or the survey paper by Olfati-Saber et al[3]).

So far, there are many results focused on state synchronization based on diffusive partial-state coupling via a dynamic protocol or a static protocol.[4-10] For state synchronization via partial-state coupling with a *static protocol*, agents are usually required to be passive or passifiable via static output feedback. For example, the work of Xia and Scardovi[11] considers linear agents that are either passive or passifiable via static output feedback. In that case, a certain set of graphs is identified for which state synchronization can be achieved. In the work of Dzhunusov and Fradkov,[12] agents are strictly G-passifiable via static output feedback, while the work of Feng and Hu[13] deals with linear agents that are either passive or passifiable via static state/output feedback agents in presence of communication delay. Then, the work of Liu et al[14] considers the synchronization results for a MAS with input saturation by using the G-passivity. Further, Liu et al[15] developed a squared-down passivity to establish a state synchronization result for MAS with nonsquare passive or passifiable agents. Nonlinear input-affine passive agents are considered in previous studies,[16-20] while general nonlinear passive agents are studied in others as well.[21-24] Passivity based on a complex dynamic network (or a neural network) was developed by Wang et al.[25,26] Moreover, a communication topology with switching is also considered.[20-22]

Meanwhile, the extension from fixed networks to time-varying networks is also done for communication networks. At present, time-varying networks are generally done in the framework of switching, using the concepts of dwell time and average dwell time. The network switches within an infinite set of graphs with some, a priori given, properties and a minimum dwell time (examples are available for full-state coupling,[27-32] partial-state coupling,[33-37] and heterogeneous MAS[38-40]). On the other hand, a general time-varying network including time-varying weights is developed by Stoorvogel et al,[41] which can be varying continuously or in the form of switches. In this case, we do not impose a minimum dwell time on the switches. Moreover, we do not need a minimum amount of time between switches. Some related researches have also been developed recently. Zhang et al[42,43] gave the state synchronization and leader-follower state synchronization results via full-state coupling. Liu et al[44] developed a synchronization result for MAS with input delay. A state-dependent graph was used by Jing and Wang.[45] However, there are no result available using partial-state coupling with a *static protocol* for a MAS with a general time-varying network.

The main objective of this paper is to derive conditions for squared-down passivity–based state synchronization of homogeneous MASs with two classes of time-varying network graphs, ie, general time-varying graph and switching graph with dwell time. The contributions include twofold:

- We design *static protocols* for agents which are either squared-down passive or squared-down passifiable (via either static output feedback or static input feedforward) for time-varying networks that are balanced and contain a spanning tree.
- We also design a *static protocol* for agents that are squared-down minimum phase with relative degree one with a switching graph. In this case, the restriction regarding a balanced network can be removed.

In both cases, we do not need explicit knowledge regarding the graph but only some rough information on the Laplacian matrix.

1.1 | Notations and definitions

Given a matrix $A \in \mathbb{R}^{m\times n}$, A^{T} and A^* denote the transpose and conjugate transpose of A, respectively, and $\|A\|$ denotes the induced two-norm of A. A square matrix A is said to be Hurwitz stable if all its eigenvalues are in the open left-half complex plane. $A \otimes B$ depicts the Kronecker product between A and B. I_n denotes the n-dimensional identity matrix, and 0_n denotes $n \times n$ zero matrix; we will use I or 0 if the dimension is clear from the context.

A *weighted directed graph* $\mathcal{G}$ is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, \dots, N\}$ is a node set, $\mathcal{E}$ is a set of *edges* indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighting matrix where $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1, \dots, i_k\}$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \dots, k-1$. The graph consisting of these N nodes is *undirected* if $a_{ij} = a_{ji}$.

A *directed tree* is a directed subgraph of $\mathcal{G}$, consisting of a subset of the nodes and edges, such that every node has exactly one parent, except a single root node with no parents. In that case, there exists a directed path from the root to every other agent in the subgraph. A *directed spanning tree* is a directed tree that contains all the nodes of $\mathcal{G}$. In that case, the root node with no parents is called a *root agent*.

A directed graph may contain many directed spanning trees; thus, there may be several choices for the root agent. The set of all possible root agents for a graph $\mathcal{G}$ is denoted by $\Pi_{\mathcal{G}}$. For a weighted graph $\mathcal{G}$, a matrix $L = [\ell_{ij}]$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j, \end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$.

2 | REVIEW ON SQUARED-DOWN PASSIVITY AND PASSIFIABILITY

We developed the squared-down passivity and passifiability in our previous study.[15] In this section, we will review these definitions.

Consider a general square (dimensions of input and output are the same) system Σ:

$$\Sigma: \begin{cases} \dot{x} = Ax + Bu, \\ y = Cx + Ru, \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m$, and $y \in \mathbb{R}^m$, while $(B^{\mathrm{T}} R^{\mathrm{T}})^{\mathrm{T}}$ and $(C \;\; R)$ have full-column and full-row ranks, respectively. We first define passive system:

Definition 1. The system (1) is called *passive* if the system is square, and for initial condition $x(0) = 0$, for any input u, and for any $T \geq 0$, we have

$$\int_0^{\mathrm{T}} y^{\mathrm{T}}(t) u(t) \,\mathrm{d}t \geq 0.$$

The positive real lemma (see, for example, the work of Anderson[46]) gives an easy characterization when systems are passive.

Lemma 1. *Assume that* (A, B) *is controllable and* (A, C) *is observable. The system (1) is passive if and only if there exists a matrix* $P > 0$ *such that*

$$\begin{pmatrix} PA + A^{\mathrm{T}}P & PB - C^{\mathrm{T}} \\ B^{\mathrm{T}}P - C & -R - R^{\mathrm{T}} \end{pmatrix} \leq 0. \tag{2}$$

If the system (1) is strictly proper, ie, $R = 0$, then (2) is reduced to

$$\begin{array}{c} PA + A^{\mathrm{T}}P \leq 0, \\ PB = C^{\mathrm{T}}. \end{array} \tag{3}$$

Next, we consider a strictly proper linear system

$$\Sigma: \begin{cases} \dot{x} = Ax + Bu, \\ y = Cx, \end{cases} \tag{4}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^m$, and $y \in \mathbb{R}^p$. We define the concepts of *squared-down passive* and *squared-down passifiable via static output feedback* for a system (4) based on the idea of *squaring down* in the work of Saberi and Sannuti.[47]

A system (4) is called *squared-down passive* with a precompensator G_1 and a postcompensator G_2 if the interconnection in Figure 1 with input $\hat{u}$ and output $\hat{y}$ is passive. Assuming G_1 and G_2 are such that (A, BG_1) is controllable and (A, G_2C) is observable where BG_1 has a full-column rank and G_2C has a full-row rank, then this is equivalent to the existence of a positive definite matrix P, such that

$$\begin{array}{c} PA + A^{\mathrm{T}}P \leq 0, \\ PBG_1 = C^{\mathrm{T}}G_2^{\mathrm{T}}. \end{array} \tag{5}$$

Remark 1. Note that when $G_1 = I$, squared-down passivity is reduced to G-passivity as used in the work of Fradkov.[48] For a square system, ie, $G_1 = G_2 = I$, squared-down passivity becomes conventional passivity.

Similarly, a system (4) is called *squared-down passifiable via static output feedback* with a precompensator $G_1 \in \mathbb{R}^{m \times q}$, a postcompensator $G_2 \in \mathbb{R}^{q \times p}$, and an output feedback gain $H \in \mathbb{R}^{q \times q}$ when

$$\hat{u} = -H\hat{y} + v \tag{6}$$

makes the system (4) squared-down passive with respect to the new input v and output $\hat{y}$, as shown in Figure 2.

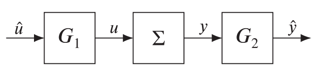

FIGURE 1 A squared-down passive system

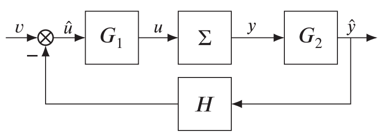

FIGURE 2 A squared-down passive system via static output feedback

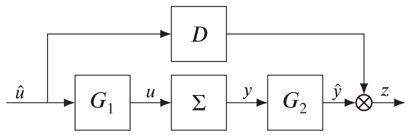

FIGURE 3 A squared-down passive system via static input feedforward

Next, a system (4) is called *squared-down passifiable via static input feedforward* with a precompensator $G_1 \in \mathbb{R}^{m\times q}$, a postcompensator $G_2 \in \mathbb{R}^{q\times p}$, and an input feedforward gain $D \in \mathbb{R}^{q\times q}$ when

$$z = D\hat{u} + \hat{y} \tag{7}$$

makes the system (4) squared-down passive with respect to the input $\hat{u}$ and the new output z, as shown in Figure 3.

Remark 2. When $G_1 = G_2 = I$ and $D = \alpha I$, squared-down passifiable via static input feedforward is reduced to static input feedforward passivity as used in the work of Proskurnikov and Mazo.[24]

Finally, we will define a class of agents, which are *squared-down minimum phase with relative degree one*. A system (4) is called *squared-down minimum phase with relative degree one* based on a precompensator $G_1 \in \mathbb{R}^{m\times q}$ and a postcompensator $G_2 \in \mathbb{R}^{q\times p}$ if the square system (A, BG_1, G_2C) is a minimum phase with relative degree one, ie, $\det(G_2CBG_1) \neq 0$. Note that, in this case, for the system (A, BG_1, G_2C), with input $\hat{u}$, where $u = G_1\hat{u}$, and output $\hat{y} = G_2y$, there exist nonsingular state transformation matrices T_x and T_u with

$$\tilde{x} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = T_x x, \qquad \tilde{u} = T_u\hat{u}$$

such that the dynamics of $\tilde{x}$ is represented by

$$\begin{aligned} \dot{\tilde{x}}_1 &= A_{11}\tilde{x}_1 + A_{12}\tilde{x}_2, \\ \dot{\tilde{x}}_2 &= A_{21}\tilde{x}_1 + A_{22}\tilde{x}_2 + \tilde{u}, \\ \hat{y} &= \tilde{x}_2, \end{aligned} \tag{8}$$

where $\tilde{x}_1 \in \mathbb{R}^{n-m}$ and $\tilde{x}_2 \in \mathbb{R}^m$. Moreover, A_{11} is Hurwitz stable.

3 | PROBLEM FORMULATION

Consider a MAS consisting of N identical linear dynamic agents:

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i, \\ y_i = Cx_i \end{cases} \tag{9}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$ and $y_i \in \mathbb{R}^p$ are the state, input, and output of agent $i = 1, \dots, N$, respectively.

The network provides agent i with the following information:

$$\zeta_i = \sum_{j=1}^{N} a_{ij}(t)(y_i - y_j) \tag{10}$$

where $a_{ij}(t) \geq 0$ and $a_{ii}(t) = 0$, are right-continuous functions of time t, indicating time-varying communication among agents. This time-varying communication topology of the network can be described by a weighted, time-varying graph $\mathcal{G}(t)$ with nodes corresponding to the agents in the network and the weight of edges at time t given by the coefficient $a_{ij}(t)$. Specifically, $a_{ij}(t) > 0$ indicates that at time t there is an edge with weight $a_{ij}(t)$ in the graph from agent j to agent i. The Laplacian matrix associated with $\mathcal{G}(t)$ is defined as $L(t) = [\ell_{ij}(t)]$. In terms of the coefficients of $L(t)$, ζ_i can be rewritten as

$$\zeta_i = \sum_{j=1}^{N} \ell_{ij}(t)y_j. \tag{11}$$

Next, we define a set of general time-varying graphs based on some rough information of the graph.

Definition 2. For any given real numbers β, γ and a positive integer N, the set $\mathbb{G}^{b,N}_{t,\beta,\gamma}$ is the set of all time-varying graphs $\mathcal{G}_t(t)$ such that, for each $t \in \mathbb{R}$, the network has a directed spanning tree and is balanced:

$$L(t) + L^{\mathrm{T}}(t) \geq \beta I$$

while $\|L(t)\|$ is smaller than γ.

For any given real number β and positive integer N, the set $\mathbb{G}^{b,N}_{t,\beta}$ is the set of all time-varying graphs $\mathcal{G}_t$ for which there exists a γ such that $\mathcal{G}_t \in \mathbb{G}^{b,N}_{t,\beta,\gamma}$.

For any given real number γ and positive integer N, the set $\mathbb{G}^{b,N}_{t,\gamma}$ is the set of all time-varying graphs $\mathcal{G}_t$ for which there exists a β such that $\mathcal{G}_t \in \mathbb{G}^{b,N}_{t,\beta,\gamma}$. By $\mathbb{G}^{u,N}_{t,\gamma}$, we denote the subset of $\mathbb{G}^{b,N}_{t,\gamma}$ of undirected graphs.

Finally, for any positive integer N, the set $\mathbb{G}^{b,N}_{t}$ is the set of all time-varying graphs $\mathcal{G}(t)$ for which there exist β and γ such that $\mathcal{G} \in \mathbb{G}^{b,N}_{t,\beta,\gamma}$.

Remark 3. Note that the distinction between the set $\mathbb{G}^{b,N}_{t,\beta}$ and $\mathbb{G}^{b,N}_{t,\beta,\gamma}$ is that if we only know that the graph is in the set $\mathbb{G}^{b,N}_{t,\beta}$ then an upper bound for the Laplacian exists but the bound is not known and therefore the parameter γ cannot be used in the protocol design.

Remark 4. According to the definition in the work of Stanoev and Smilkov,[49] the network is called *weakly connected* if it contains a path (disregarding the directions) from every node to every other node in the network and *strongly connected* if it contains a directed path from every node to every other node in the network. We immediately find that a network with a spanning tree is always weakly connected. Since our time-varying graphs $\mathcal{G}(t)$ are also balanced, we find that the network is strongly connected as well, using Theorem 1 in the work of Stanoev and Smilkov.[49]

Next, we define a set of time-varying, switching graphs.

Definition 3. For any given real numbers β, γ and a positive integer N, the set $\mathbb{G}^{N}_{\beta,\gamma}$ is the set of all graphs $\mathcal{G}$ for which the network has a directed spanning tree and the associated Laplacian is such that $\|L\| \leq \gamma$ and all nonzero eigenvalues are larger than β.

The set $\mathbb{G}^{\tau,N}_{\beta,\gamma}$ is defined as the set of all time-varying graphs $\mathcal{G}_t$ for which

$$\mathcal{G}_t(t) = \mathcal{G}_{\sigma(t)} \in \mathbb{G}^{N}_{\beta,\gamma}$$

for all $t \in \mathbb{R}$, where $\sigma : \mathbb{R} \to \mathcal{L}_N$ is a piecewise-constant, right-continuous function with minimal dwell time τ, ie, if σ has discontinuities in t_1 and t_2, then $|t_2 - t_1| > \tau$.

Remark 5. Note that the minimal dwell time is assumed to avoid chattering problems. However, our design is able to obtain a suitable protocol for arbitrarily small $\tau > 0$.

We therefore either have a balanced time-varying graph or a general directed switching graph. In both cases, it then follows from Lemma 2.5 in the work of Ren and Cao[1] that the Laplacian matrix $L(t)$ at time t has a simple eigenvalue at the origin, with the corresponding right eigenvector $\mathbf{1}$ (a vector with each element equal to 1) and all the other eigenvalues are in the open right-half complex plane. Let $\lambda_{t,1}, \ldots, \lambda_{t,N}$ denote the eigenvalues of $L_t(t)$ such that $\lambda_{t,1} = 0$ and $\mathrm{Re}(\lambda_{t,i}) > 0$, $i = 2, \ldots, N$. Meanwhile, the Laplacian matrix $L(t)$ has a spanning diverging tree where all nodes can receive information from the root. In this paper, we will employ the spanning diverging tree to establish our results. As in the work of Ren and Cao,[1] we will use "spanning tree" rather than "spanning diverging tree" for simplicity.

Note that ζ_i can be represented in term of the Laplacian matrix as in (11). Our goal in this paper is to achieve state synchronization among agents in MAS, that is

$$\lim_{t\to\infty} x_i(t) - x_j(t) = 0, \tag{12}$$

for all $i,j \in \{1, \ldots, N\}$.

We formulate our problem as follows.

Problem 1. Let $\gamma > \beta > 0$ be given. Consider a MAS described by (9) and (10). Assume the associated directed time-varying graph $\mathcal{G}_t$ is in the set $\mathbf{G}$. The *state synchronization* problem for a time-varying MAS is to find a distributed parameterized static protocol of the following form:

$$u_i = F\zeta_i \tag{13}$$

for each agent such that state synchronization is achieved for any time-varying graph $\mathcal{G}_t \in \mathbf{G}$.

4 | SYNCHRONIZATION FOR MAS WITH TIME-VARYING GRAPH

In this section, we will consider the static protocol design for MAS with squared-down passive agents, squared-down passifiable via static input feedforward agents, and squared-down minimum phase with relative degree one agents. We consider time-varying graphs $\mathcal{G}_t$.

4.1 | Squared-down passive agents

For a MAS with agent that are squared-down passive, we consider a static protocol of the following form:

$$u_i = -\rho G_1 G_2 \zeta_i, \tag{14}$$

where $\rho > 0$ is a design parameter, while G_1 and G_2 are given in (5).

Theorem 1. *Consider a MAS described by (9) and (10). Assume (A, B, C) is squared-down passive with respect to G_1 and G_2 such that (A, BG_1) is controllable and (A, G_2C) is observable, while BG_1 and G_2C have full-column and full-row ranks, respectively.*

In that case, the state synchronization problem stated in Problem 1 is solvable for the set of graphs $\mathbf{G} = \mathbb{G}_t^{b,N}$. In particular, for any $\rho > 0$, the static protocol (14) solves the state synchronization problem for any graph $\mathcal{G}_t \in \mathbb{G}_t^{b,N}$.

To prove the above theorem, we need the following technical lemma.

Lemma 2. *Define $T_1 \in \mathbb{R}^{(N-1)\times N}$ and $T_2 \in \mathbb{R}^{N\times(N-1)}$,*

$$T_1 = \begin{pmatrix} I & -\mathbf{1} \end{pmatrix}, \qquad T_2 = \begin{pmatrix} I \\ 0 \end{pmatrix} \tag{15}$$

while

$$V = \left(T_1 T_1^{\mathrm{T}}\right)^{-1/2}. \tag{16}$$

Consider:

$$\bar{L}(t) = V T_1 L(t) T_2 V^{-1}. \tag{17}$$

In that case, $L(t)$ being balanced implies that $\bar{L}(t)$ is balanced.

Moreover, assume that the Laplacian $L(t)$ is associated to a graph with a directed spanning tree. In that case, the eigenvalues of $\bar{L}(t) + \bar{L}^{\mathrm{T}}(t)$ are equal to the nonzero eigenvalues of $L(t) + L^T(t)$.

Proof. We first note that $L(t)\mathbf{1} = 0$ implies that $L(t)T_2T_1 = L(t)$. Moreover, since $L(t)$ is balanced, we find that $L^{\mathrm{T}}(t)\mathbf{1} = 0$ as well. This in turn yields that $L^{\mathrm{T}}(t)T_2T_1 = L^{\mathrm{T}}(t)$.

Let $x_1, \dots, x_N$ be the orthogonal eigenvectors of $L(t) + L^{\mathrm{T}}(t)$ with associated nonnegative eigenvalues $\lambda_1, \dots, \lambda_N$. Without loss of generality, we choose $x_1 = \mathbf{1}$ and $\lambda_1 = 0$. In that case, we find that

$$\left[\bar{L}(t) + \bar{L}(t)^{\mathrm{T}}\right] V T_1 x_i = V T_1 \left[L(t) + L^{\mathrm{T}}(t)\right] T_2 T_1 x_i = \lambda_i V T_1 x_i$$

Note that $VT_1x_1 = 0$, while $VT_1x_i \neq 0$ for $i = 2, \dots, N$ (it can actually be easily verified that VT_1x_i and VT_1x_j are orthogonal for $i \neq j$). Therefore, the eigenvalues of $\bar{L}(t) + \bar{L}^{\mathrm{T}}(t)$ are equal to the nonzero eigenvalues of $L(t) + L^{\mathrm{T}}(t)$. Moreover, $\bar{L}(t) + \bar{L}^{\mathrm{T}}(t) > 0$ since its eigenvalues $\lambda_2, \dots, \lambda_N$ are all nonnegative. □

Proof of Theorem 1. First, we need to transform the model of system (9). By using (14), we have

$$\dot{x}_i = Ax_i - \rho \sum_{j=1}^{N} \ell_{ij}(t) BG_1G_2Cx_j. \tag{18}$$

We define $\bar{x}_i = x_i - x_N$, and we obtain

$$\dot{\bar{x}}_i = A\bar{x}_i - \rho \sum_{j=1}^{N-1} \hat{\ell}_{ij}(t) BG_1G_2C\bar{x}_j, \tag{19}$$

where $\hat{\ell}_{ij}(t) = \ell_{ij}(t) - \ell_{Nj}(t)$. Define

$$\bar{x} = (V \otimes I) \begin{pmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \vdots \\ \bar{x}_{N-1} \end{pmatrix}.$$

Then, the overall dynamics of the network can be written as

$$\dot{\bar{x}} = (I \otimes A)\bar{x} - \rho(\bar{L}(t) \otimes BG_1G_2C)\bar{x}, \tag{20}$$

where $\bar{L}(t)$ is defined by (17).

Since the agents are squared-down passive, we find there exists a $P > 0$ satisfying (5). We obtain that

$$\begin{aligned} \frac{d}{dt}\bar{x}^{\mathrm{T}}(I \otimes P)\bar{x} &= \bar{x}^{\mathrm{T}}\left[I \otimes (A^{\mathrm{T}}P + PA)\right]\bar{x} - \rho\bar{x}^{\mathrm{T}}\left[(\bar{L}(t) + \bar{L}^{\mathrm{T}}(t)) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x} \\ &\leq -\rho\bar{x}^{\mathrm{T}}\left[(\bar{L}(t) + \bar{L}^{\mathrm{T}}(t)) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x} \end{aligned}$$

with $\rho > 0$. We find that

$$\int_0^\infty \bar{x}^{\mathrm{T}}\left[I \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x}\,\mathrm{d}t < \infty.$$

In other words, the output of the stabilizable and detectable system $(I \otimes A, I \otimes BG_1, I \otimes G_2C)$ is in $\mathcal{L}_2$. Since $\bar{L}(t)$ is also bounded, this also implies that the input is in $\mathcal{L}_2$. However, we then immediately find that the state $\bar{x}(t)$ converges to zero and therefore synchronization is achieved. □

4.2 | Squared-down passifiable via static output feedback agents

A system (4) is squared-down passifiable via static output feedback (6) if there exist matrices G_1 and G_2 and H and a positive-definite matrix P such that

$$\begin{aligned} P(A - BG_1HG_2C) + (A - BG_1HG_2C)^{\mathrm{T}}P \leq 0, \\ PBG_1 = C^{\mathrm{T}}G_2^{\mathrm{T}}. \end{aligned} \tag{21}$$

This condition is necessary for a system to be squared-down passifiable via static output feedback if (A, BG_1) is controllable and (A, G_2C) is observable, while BG_1 and G_2C have full-column and full-row ranks, respectively.

Here, we will use the same protocol as in the squared-down passive case:

$$u_i = -\rho G_1G_2\zeta_i.$$

However, in this case, the parameter $\rho > 0$ cannot be arbitrary but needs to be sufficiently large.

The main result in this section can be stated as follows.

Theorem 2. *Consider a MAS described by (9) and (10). Assume (A, B, C) is squared-down passifiable via static output feedback with G_1, G_2, and H such that (A, BG_1) is controllable and (A, G_2C) is observable, while BG_1 and G_2C have full-column and full-row ranks, respectively. Let any $\beta > 0$ be given.*

The state synchronization problem stated in Problem 1 is solvable for the set of graphs $\mathbf{G} = \mathbb{G}_{t,\beta}^{b,N}$. In particular, for any $\rho > \rho_*$, the static protocol (14) solves the state synchronization problem for any graph $\mathcal{G}_t \in \mathbb{G}_{t,\beta}^{b,N}$.

Proof of Theorem 2. As noted in the beginning of this section there exists a positive-definite matrix P such that (21) is satisfied. Obviously, there exists a real number $b > 0$ such that

$$H + H^{\mathrm{T}} \leq bI. \tag{22}$$

Similar to the proof of Theorem 1, we obtain that

$$\begin{aligned}\frac{d}{dt}\bar{x}^{\mathrm{T}}(I \otimes P)\bar{x} &= \bar{x}^{\mathrm{T}}\left[I \otimes (A^{\mathrm{T}}P + PA)\right]\bar{x}\\ &\quad - \rho\bar{x}^{\mathrm{T}}\left[(\bar{L}(t) + \bar{L}^{\mathrm{T}}(t)) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x}\\ &\leq \bar{x}^{\mathrm{T}}\left[I \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}(H + H^{\mathrm{T}})G_2C\right]\bar{x}\\ &\quad - \rho\bar{x}^{\mathrm{T}}\left[(\bar{L}(t) + \bar{L}^{\mathrm{T}}(t)) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x}\\ &\leq (b - \rho\beta)\bar{x}^{\mathrm{T}}\left[I \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x}.\end{aligned}$$

Choosing $\rho_* = \frac{b}{\beta}$ and $\rho > \rho_*$ then immediately ensures that synchronization is achieved using the same argument as in the proof of Theorem 1. □

4.3 | Squared-down passifiable via static input feedforward agents

A system (4) is squared-down passifiable via static input feedforward (7) if there exist matrices G_1, G_2, and D and a positive definite matrix P such that

$$G(P) = \begin{pmatrix} PA + A^{\mathrm{T}}P & PBG_1 - C^{\mathrm{T}}G_2^{\mathrm{T}} \\ G_1^{\mathrm{T}}B^{\mathrm{T}}P - G_2C & -D - D^{\mathrm{T}} \end{pmatrix} \leq 0. \tag{23}$$

This condition is necessary for a system to be squared-down passifiable via static input feedforward if (A, BG_1) is controllable and (A, G_2C) is observable, while BG_1 and G_2C have full-column and full-row ranks, respectively. Note this class of systems is always neutrally stable, which follows directly from (23).

Here, we still use the same protocol as in the squared-down passive case:

$$u_i = -\rho G_1 G_2 \zeta_i. \tag{24}$$

However, in this case, the parameter $\rho > 0$ cannot be arbitrary but needs to be sufficiently small.

We have the following result for undirected graphs.

Theorem 3. *Consider a MAS described by agents (9) and (10) where (A, B, C) is squared-down passifiable via static input feedforward with G_1, G_2, and D such that (A, BG_1) is controllable and (A, G_2C) is observable, while BG_1 and G_2C have full-column and full-row ranks, respectively. Let any $\gamma > 0$ be given.*

The state synchronization problem stated in Problem 1 is solvable for the set of graphs $\mathbf{G} = \mathbb{G}_{t,\gamma}^{u,N}$. In particular, there exists ρ_* such that for any $\rho \in (0, \rho_*]$, the static protocol (24) solves the state synchronization problem for any graph $\mathcal{G}_t \in \mathbb{G}_{t,\gamma}^{u,N}$.

To prove the above theorem, we need the following technical lemma.

Lemma 3. *Consider a Laplacian matrix $L(t)$ associated to an undirected graph in $\mathbb{G}_{t,\gamma}^{u,N}$. In that case, we have*

$$\bar{L}^2(t) \leq \gamma \bar{L}(t)$$

where $\bar{L}(t)$ is defined by (17).

Proof. Note that, since the graph is undirected and hence $L(t)$ is symmetric, it is also easily verified that $\bar{L}(t)$ is symmetric. The result is then trivial. □

Proof of Theorem 3. Similar to the proof of Theorem 1 but using (23), we obtain that

$$\begin{aligned}
\frac{d}{dt}\bar{x}^{\mathrm{T}}(I \otimes P)\bar{x} &= \bar{x}^{\mathrm{T}}\left[I \otimes (A^{\mathrm{T}}P + PA)\right]\bar{x} \\
&\quad - 2\rho\bar{x}^{\mathrm{T}}\left[\bar{L}(t) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x} \\
&\leq \begin{pmatrix} \bar{x} \\ -\rho(\bar{L}(t) \otimes G_2C)\bar{x} \end{pmatrix}^{\mathrm{T}} \bar{G}(P) \begin{pmatrix} \bar{x} \\ -\rho(\bar{L}(t) \otimes G_2C)\bar{x} \end{pmatrix} \\
&\quad - 2\rho\bar{x}^{\mathrm{T}}\left[\bar{L}(t) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x} \\
&\quad + \rho^2\bar{x}^{\mathrm{T}}\left[\bar{L}^2(t) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}(D + D^{\mathrm{T}})G_2C\right]\bar{x} \\
&\leq (\rho b\gamma - 2)\rho\bar{x}^{\mathrm{T}}\left[\bar{L}(t) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x}
\end{aligned}$$

where

$$D + D^{\mathrm{T}} \leq bI$$

and

$$\bar{G}(P) = \begin{pmatrix} I \otimes (PA + A^{\mathrm{T}}P) & I \otimes \left(PBG_1 - C^{\mathrm{T}}G_2^{\mathrm{T}}\right) \\ I \otimes \left(G_1^{\mathrm{T}}B^{\mathrm{T}}P - G_2C\right) & -I \otimes (D + D^{\mathrm{T}}) \end{pmatrix} \leq 0.$$

Choosing $\rho_* = \frac{2}{b\gamma}$ and $\rho < \rho_*$ then immediately ensures that synchronization is achieved using the same argument as in the proof of Theorem 1. □

For balanced graphs, we need some additional prior knowledge as clarified in the following theorem.

Theorem 4. *Consider a MAS described by agents (9) and (10), where (A, B, C) is squared-down passifiable via static input feedforward with G_1, G_2, and D such that (A, BG_1) is controllable and (A, G_2C) is observable, while BG_1 and G_2C have full-column and full-row ranks, respectively. Let any $\beta, \gamma > 0$ be given.*

The state synchronization problem stated in Problem 1 is solvable for the set of graphs $\mathbf{G} = \mathbb{G}_{t,\beta,\gamma}^{b,N}$. In particular, there exists ρ_* such that for any $\rho \in (0, \rho_*]$, the static protocol (24) solves the state synchronization problem for any graph $\mathcal{G}_t \in \mathbb{G}_{t,\beta,\gamma}^{b,N}$.

Proof of Theorem 4. The only difference with the proof of Theorem 3 are the first inequalities:

$$\begin{aligned}
\frac{d}{dt}\bar{x}^{\mathrm{T}}(I \otimes P)\bar{x} &= \bar{x}^{\mathrm{T}}\left[I \otimes (A^{\mathrm{T}}P + PA)\right]\bar{x} \\
&\quad - \rho\bar{x}^{\mathrm{T}}\left[(\bar{L}(t) + \bar{L}^{\mathrm{T}}(t)) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x} \\
&\leq \begin{pmatrix} \bar{x} \\ -\rho(\bar{L}(t) \otimes G_2C)\bar{x} \end{pmatrix}^{\mathrm{T}} \bar{G}(P) \begin{pmatrix} \bar{x} \\ -\rho(\bar{L}(t) \otimes G_2C)\bar{x} \end{pmatrix} \\
&\quad - \rho\bar{x}^{\mathrm{T}}\left[(\bar{L}(t) + \bar{L}^{\mathrm{T}}(t)) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x} \\
&\quad + \rho^2\bar{x}^{\mathrm{T}}\left[(\bar{L}^{\mathrm{T}}(t)\bar{L}(t)) \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}(D + D^{\mathrm{T}})G_2C\right]\bar{x} \\
&\leq (\rho b\gamma^2 - \beta)\rho\bar{x}^{\mathrm{T}}\left[I \otimes C^{\mathrm{T}}G_2^{\mathrm{T}}G_2C\right]\bar{x}
\end{aligned}$$

where

$$D + D^{\mathrm{T}} \leq bI.$$

Choosing $\rho_* = \frac{\beta}{b\gamma^2}$ and $\rho < \rho_*$ then immediately ensures that synchronization is achieved. □

4.4 | Squared-down minimum phase with relative degree one agents

Consider a MAS whose agents are squared-down minimum phase with relative degree one while BG_1 and G_2C have full-column and full-row ranks, respectively. We design the following static protocol:

$$u_i = -\rho G_1 T_u^{-1} G_2 \zeta_i, \tag{25}$$

where $\rho > 0$ is a design parameter. Here, T_u is the input transformation matrix, which together with the state transformation matrix T_x, brings the system in the form (8).

The main result in this subsection can be stated as follows.

Theorem 5. *Consider a MAS described by (9) and (10) whose agents are squared-down minimum phase with relative degree one while BG_1 and G_2C have full-column and full-row ranks, respectively. Let any $\beta > 0$ be given.*

The state synchronization problem stated in Problem 1 is solvable for the set of graphs $\mathbf{G} = \mathbb{G}_{t,\beta}^{b,N}$. In particular, there exists $\rho_* > 0$ such that, for any $\rho > \rho_*$, the static protocol (25) solves the state synchronization problem for any graph $\mathcal{G}_t \in \mathbb{G}_{t,\beta}^{b,N}$.

Proof of Theorem 5. Since all agents are squared-down minimum phase with relative degree one, we can use input transformation matrix T_u, and state transformation matrix T_x brings the system in the form (8). We obtain

$$\begin{aligned} \dot{\tilde{x}}_{1i} &= A_{11}\tilde{x}_{1i} + A_{12}\tilde{x}_{2i}, \\ \dot{\tilde{x}}_{2i} &= A_{21}\tilde{x}_{1i} + A_{22}\tilde{x}_{2i} + \tilde{u}_i, \\ \hat{y}_i &= \tilde{x}_{2i}, \end{aligned} \tag{26}$$

and a protocol

$$\tilde{u}_i = -\rho\sum_{j=1}^{N}\ell_{ij}(t)\hat{y}_j. \tag{27}$$

Here, we used that

$$G_2CT_x^{-1} = \begin{pmatrix} 0 & I \end{pmatrix}.$$

The closed-loop system of (26) and (27) is written as

$$\begin{aligned} \dot{\tilde{x}}_{1i} &= A_{11}\tilde{x}_{1i} + A_{12}\tilde{x}_{2i}, \\ \dot{\tilde{x}}_{2i} &= A_{21}\tilde{x}_{1i} + A_{22}\tilde{x}_{2i} - \rho\sum_{j=1}^{N}\ell_{ij}(t)\tilde{x}_{2j}. \end{aligned} \tag{28}$$

Since A_{11} is Hurwitz stable, there exists a $P_1 > 0$ such that

$$P_1A_{11} + A_{11}^{\mathrm{T}}P_1 = -I.$$

Meanwhile, we choose $b > 0$ such that

$$A_{22} + A_{22}^{\mathrm{T}} - bI \leq 0. \tag{29}$$

Let $\bar{x}_{1i} = \tilde{x}_{1i} - \tilde{x}_{1N}$ and $\bar{x}_{2i} = \tilde{x}_{2i} - \tilde{x}_{2N}$. We have

$$\begin{aligned} \dot{\bar{x}}_{1i} &= A_{11}\bar{x}_{1i} + A_{12}\bar{x}_{2i}, \\ \dot{\bar{x}}_{2i} &= A_{21}\bar{x}_{1i} + A_{22}\bar{x}_{2i} - \rho\sum_{j=1}^{N-1}\hat{\ell}_{ij}(t)\bar{x}_{2j}. \end{aligned} \tag{30}$$

Define

$$\bar{x}_1 = (V \otimes I)\begin{pmatrix} \bar{x}_{11} \\ \bar{x}_{12} \\ \vdots \\ \bar{x}_{1(N-1)} \end{pmatrix}, \quad \bar{x}_2 = (V \otimes I)\begin{pmatrix} \bar{x}_{21} \\ \bar{x}_{22} \\ \vdots \\ \bar{x}_{2(N-1)} \end{pmatrix},$$

where V is defined in Lemma 2. The overall dynamics of the network can then be written as

$$\begin{cases} \dot{\bar{x}}_1 = (I \otimes A_{11})\bar{x}_1 + (I \otimes A_{12})\bar{x}_2 \\ \dot{\bar{x}}_2 = (I \otimes A_{21})\bar{x}_1 + (I \otimes A_{22})\bar{x}_2 - \rho(\bar{L}(t) \otimes I)\bar{x}_2. \end{cases} \tag{31}$$

Next, we choose the following candidate Lyapunov function:

$$V_e = \bar{x}_1^{\mathrm{T}}(I \otimes P_1)\bar{x}_1 + \bar{x}_2^{\mathrm{T}}\bar{x}_2 \tag{32}$$

with $P_1 > 0$. Then, we calculate the time derivatives of V along the trajectories of system (31) as follows:

$$\begin{aligned}\dot{V}_e \leq & -\bar{x}_1^{\mathrm{T}}\bar{x}_1 + 2\bar{x}_1^{\mathrm{T}}(I \otimes P_1A_{12})\bar{x}_2 + 2\bar{x}_2^{\mathrm{T}}(I \otimes A_{21})\bar{x}_1 \\ & + \bar{x}_2^{\mathrm{T}} I \otimes (A_{22} + A_{22}^{\mathrm{T}})\bar{x}_2 - \rho\bar{x}_2^{\mathrm{T}}\left[(\bar{L}(t) + \bar{L}^{\mathrm{T}}(t)) \otimes I\right]\bar{x}_2 \\ \leq & -\bar{x}_1^{\mathrm{T}}\bar{x}_1 + 2\bar{x}_1^{\mathrm{T}}(I \otimes P_1A_{12})\bar{x}_2 + 2\bar{x}_2^{\mathrm{T}}(I \otimes A_{21})\bar{x}_1 \\ & + b\bar{x}_2^{\mathrm{T}}\bar{x}_2 - \rho\beta\bar{x}_2^{\mathrm{T}} I \otimes I\bar{x}_2 \\ \leq & \begin{pmatrix}\bar{x}_1 \\ \bar{x}_2\end{pmatrix}^{\mathrm{T}} (I \otimes \Theta) \begin{pmatrix}\bar{x}_1 \\ \bar{x}_2\end{pmatrix}\end{aligned}$$

where

$$\Theta = \begin{pmatrix} -I & P_1A_{12} + A_{21}^{\mathrm{T}} \\ A_{12}^{\mathrm{T}}P_1 + A_{21} & bI - \rho\beta I \end{pmatrix}. \tag{33}$$

Let

$$\rho_* = \frac{1}{\beta}\left(\|P_1A_{12} + A_{21}^{\mathrm{T}}\|^2 + b\right).$$

Clearly $\Theta \leq 0$ for $\rho > \rho_*$. Therefore, for $\rho > \rho_*$, we have

$$\dot{V}_e < 0.$$

It is then easily verified that we have achieved synchronization. □

5 | STATE SYNCHRONIZATION FOR MAS WITH SWITCHING GRAPH

In the previous section, we have seen that static protocols can be designed for a general time-varying graph. In this section, we will consider switching graphs with a minimum dwell time. For the switching graph, we consider agents that are squared-down minimum phase with relative degree one. The main advantage is that, for this class of systems, we do not need the assumption that the graph is balanced. Assume our graph is in the set $\mathbb{G}_{\beta,\gamma}^{\tau,N}$ with associated Laplacian matrix $L(t)$. Then, the Schur form of

$$\bar{L}(t) = T_1L(t)T_2$$

is given by $\bar{Q}_t^{-1}\bar{L}(t)\bar{Q}_t = \bar{U}_t$, where $\bar{U}_t$ is an upper triangular matrix. Moreover, the diagonal elements of $\bar{U}_t$ are the nonzero eigenvalues of the Laplacian matrix $\bar{L}(t)$ at time t while $\bar{Q}_t$ is unitary. Given the upper bound γ, there exists $\bar{\gamma}$ such that $\|\bar{U}_t\| < \bar{\gamma}$. The real part of the eigenvalues of $\bar{U}_t$ are all larger than or equal to β.

Next, we can design a protocol for a MAS that is squared-down minimum phase with relative degree one. The static protocol is the same as in Subsection 4.4:

$$u_i = -\rho G_1T_u^{-1}G_2\zeta_i,$$

where $\rho > 0$ is a parameter to be designed.

The main result based on the above design can be stated as follows.

Theorem 6. *Consider a MAS described by agents (9) and (10) where (A, BG_1, G_2C) is squared-down minimum phase and with relative degree one. Let any $\gamma > \beta > 0$ be given; hence, a set of network graphs $\mathbb{G}_{\beta,\gamma}^{\tau,N}$ be defined.*

The state synchronization problem stated in Problem 1 is solvable for the set of graphs $\mathbf{G} = \mathbb{G}_{\beta,\gamma}^{\tau,N}$. In particular, there exists a $\rho_* > 0$ such that for any $\rho > \rho_*$, protocol (25) solves the state synchronization problem for any graph $\mathcal{G}_t \in \mathbb{G}_{\beta,\gamma}^{\tau,N}$.

Before we prove the above theorem, we need a preliminary lemma.

Lemma 4. *There exists $P_d > 0$ and $\nu > 0$ such that, for any upper triangular matrix $U_t \in \mathbb{C}^{(N-1)\times(N-1)}$ with $\|U_t\| < \tilde{\gamma}$, whose eigenvalues satisfy $Re(\lambda_i) > \beta$ for all $i = 1, \ldots, N-1$, we have*

$$P_dU_t + U_t^*P_d \geq \nu P_d + 2I. \tag{34}$$

Proof. We define

$$P = \frac{1}{\beta}I.$$

We define, for $k = 1, \dots, N-1$,

$$\bar{P}_k = \begin{pmatrix} \alpha^{i_1}P & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \alpha^{i_k}P \end{pmatrix}, \qquad \bar{U}_k = \begin{pmatrix} \lambda_1 & \mu_{1,2} & \dots & \mu_{1,k} \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mu_{k-1,k} \\ 0 & \dots & 0 & \lambda_k \end{pmatrix},$$

where $\lambda_1, \dots, \lambda_{N-1}$ are the eigenvalues of U_t and $\mu_{i,j} = [U_t]_{ij}$ for $j > i$ are the bounded upper triangular elements of U_t.

We will next use a recursive argument. For $k = 1$ and $\alpha^{i_1} = N$ we have

$$\bar{P}_1\bar{U}_1 + \bar{U}_1^*\bar{P}_1 = 2\alpha^{i_1}\mathrm{Re}(\lambda_1)P > 2NI.$$

Assume that for $k = j$, we have $\alpha^{i_1}, \dots \alpha^{i_j}$ such that

$$\bar{U}_{11} = \bar{P}_j\bar{U}_j + \bar{U}_j^*\bar{P}_j > 2(N-j+1)I. \tag{35}$$

We will show that, for $k = j+1$, there exists $\alpha^{i_{j+1}}$ such that

$$\bar{P}_{j+1}\bar{U}_{j+1} + \bar{U}_{j+1}^*\bar{P}_{j+1} > 2(N-j)I \tag{36}$$

Note that

$$\bar{P}_{j+1}\bar{U}_{j+1} + \bar{U}_{j+1}^*\bar{P}_{j+1} = \begin{pmatrix} \bar{U}_{11} & \bar{U}_{12} \\ \bar{U}_{12}^* & 2\alpha^{i_{j+1}}I \end{pmatrix}$$

where $\bar{U}_{11}$ is defined by (35) while $\bar{U}_{12}$ is given by:

$$\bar{U}_{12} = \begin{pmatrix} \alpha^{i_1}\mu_{1,j+1}P \\ \vdots \\ \alpha^{i_j}\mu_{j,j+1}P \end{pmatrix}.$$

Note that the coefficients $\mu_{1,j+1}$ are unknown but bounded since the norm of U_t is bounded and hence there exists M such that $\|\bar{U}_{12}\| < M$. Via Schur complement, it is easy to verify that, for given bound M, there exists $\alpha^{i_{j+1}}$ sufficiently large such that the matrix

$$\begin{pmatrix} \bar{U}_{11} & \bar{U}_{12} \\ \bar{U}_{12}^* & 2\alpha^{i_{j+1}}I \end{pmatrix} > 2(N-j)I$$

for all matrices $\bar{U}_{11}$ and $\bar{U}_{12}$ such that $\bar{U}_{11} > 2(N-j+1)I$ and $\|\bar{U}_{12}\| < M$. This guarantees that (36) is satisfied.

Using a recursive argument, we find that there exist $\alpha^{i_1}, \dots, \alpha^{i_{N-1}}$ such that

$$\bar{P}_{N-1}U_t + U_t^*\bar{P}_{N-1} \geq 4I$$

since $U_t = \bar{U}_{N-1}$. This obviously implies that for v that is small enough, we have (34) for $P_d = \bar{P}_{N-1}$. □

Proof of Theorem 6. Since all agents are squared-down minimum phase with relative degree of 1, we have (26) and (27) after the input transformation matrix T_u and state transformation matrix T_x.

The closed-loop system of (26) and (27) is written as (28). Since A_{11} is Hurwitz stable, there exists a $P_a > 0$ and small enough v such that

$$P_aA_{11} + A_{11}^{\mathrm{T}}P_a = -vP_a - I. \tag{37}$$

We have $\bar{L}(t) = T_1L(t)T_2$, where T_1 and T_2 are given by (15). As noted in the beginning of this section, we have $\bar{Q}_t^{-1}\bar{L}(t)\bar{Q}_t = \bar{U}_t$, where $\bar{U}_t$ is the Schur form of $\bar{L}(t)$. Let

$$\eta_a = (T_1 \otimes I)\tilde{x}_a, \qquad \tilde{x}_a = \begin{pmatrix} \tilde{x}_{11} \\ \tilde{x}_{12} \\ \vdots \\ \tilde{x}_{1N} \end{pmatrix}, \qquad \eta_d = (U_tQ_t^{-1}T_1 \otimes I)\tilde{x}_d, \qquad \tilde{x}_d = \begin{pmatrix} \tilde{x}_{21} \\ \tilde{x}_{22} \\ \vdots \\ \tilde{x}_{2N} \end{pmatrix}.$$

We obtain

$$\begin{aligned} \dot{\eta}_a &= (I \otimes A_{11})\eta_a + W_{ad,t}\eta_d, \\ \varepsilon\dot{\eta}_d &= W_{da,t}^{\varepsilon}\eta_a + W_{dd,t}^{\varepsilon}\eta_d \end{aligned}$$

where $\varepsilon = \rho^{-1}$ while

$$W_{ad,t} = \bar{Q}_t \bar{U}_t^{-1} \otimes A_{12},$$
$$W_{da,t}^{\varepsilon} = \varepsilon \bar{U}_t \bar{Q}_t^{-1} \otimes A_{21},$$
$$W_{dd,t}^{\varepsilon} = \varepsilon I \otimes A_{22} - \bar{U}_t \otimes I.$$

We define

$$V_a = \varepsilon^2 \eta_a^* (I_{N-1} \otimes P_a) \eta_a, \qquad V_d = \varepsilon \eta_d^* (P_d \otimes I) \eta_d$$

where P_a and P_d are defined by (37) and (34), respectively.

The derivative of V_a is obtained by

$$\begin{aligned}\dot{V}_a &= \varepsilon^2 \eta_a^* (I_{N-1} \otimes (P_a A_{11} + A_{11}^{\mathrm{T}} P_a)) \eta_a + 2\varepsilon^2 \mathrm{Re}(\eta_a^* (I \otimes P_a) W_{ad,t} \eta_d) \\ &= -\varepsilon^2 \eta_a^* (I_{N-1} \otimes (\nu P_a + I)) \eta_a + 2\varepsilon^2 \mathrm{Re}(\eta_a^* (I \otimes P_a) W_{ad,t} \eta_d) \\ &= -\nu V_a - \varepsilon^2 \|\eta_a\|^2 + 2\varepsilon^2 \mathrm{Re}(\eta_a^* (I \otimes P_a) W_{ad,t} \eta_d)\end{aligned}$$

for (37). Meanwhile, when ε is small enough, we have

$$2\mathrm{Re}(\eta_a^* (I \otimes P_a) W_{ad,t} \eta_d) \leq 2 r_4 \|\eta_a\| \|\eta_d\| \leq \|\eta_a\|^2 + r_4^2 \|\eta_d\|^2 \leq \|\eta_a\|^2 + \frac{\nu}{2\varepsilon^2 \sqrt{\varepsilon}} V_d$$

with

$$r_4 \geq \|\bar{Q}_t \bar{U}_t^{-1} \otimes P_a A_{12}\|.$$

Note that we can choose r_4; hence, ε is independent of the network graph but only depending on our bounds on the eigenvalues and on the norm of our expanded Laplacian $\bar{L}(t)$. Thus, we have the bound of $\dot{V}_a$

$$\dot{V}_a \leq -\nu V_a + \frac{\nu}{2\sqrt{\varepsilon}} V_d. \tag{38}$$

Next, the derivative of V_d is obtained by

$$\begin{aligned}\dot{V}_d &= 2\mathrm{Re}\left(\eta_d^* \left(P_d \otimes W_{da,t}^{\varepsilon}\right) \eta_a\right) + \eta_d^* \left[(P_d \otimes I) W_{dd,t}^{\varepsilon} + \left(W_{dd,t}^{\varepsilon}\right)^* (P_d \otimes I)\right] \eta_d \\ &= 2\mathrm{Re}\left(\eta_d^* \left(P_d \otimes W_{da,t}^{\varepsilon}\right) \eta_a\right) + \varepsilon \eta_d^* \left[P_d \otimes \left(A_{22} + A_{22}^{\mathrm{T}}\right)\right] \eta_d - \eta_d^* \left[\left(P_d \bar{U}_t + \bar{U}_t^* P_d\right) \otimes I\right] \eta_d \\ &\leq 2\mathrm{Re}\left(\eta_d^* \left(P_d \otimes W_{da,t}^{\varepsilon}\right) \eta_a\right) + 2\varepsilon \mathrm{Re}\left(\eta_d^* (P_d \otimes A_{22}) \eta_d\right) - \eta_d^* \left[(\nu P_d + 2I) \otimes I\right] \eta_d \\ &= -\nu \varepsilon^{-1} V_d - 2\|\eta_d\|^2 + 2\mathrm{Re}\left(\eta_d^* (P_d \otimes I) W_{da,t}^{\varepsilon} \eta_a\right) + 2\varepsilon \mathrm{Re}\left(\eta_d^* (P_d \otimes A_{22}) \eta_d\right)\end{aligned}$$

for (34). Meanwhile, we have

$$2\varepsilon \mathrm{Re}\left(\eta_d^* (P_d \otimes A_{22}) \eta_d\right) \leq \|\eta_d\|^2$$

for small enough ε, and

$$2\mathrm{Re}\left(\eta_d^* (P_d \otimes I) W_{da,t}^{\varepsilon} \eta_a\right) \leq 2\varepsilon r_1 \|\eta_a\| \|\eta_d\| \leq \varepsilon^2 r_1^2 \|\eta_a\|^2 + \|\eta_d\|^2 \leq \frac{\nu}{2\sqrt{\varepsilon}} V_a + \|\eta_d\|^2,$$

provided r_1 is such that we have

$$r_1 \geq \|P_d \bar{U}_t \bar{Q}_t^{-1} \otimes A_{21}\|,$$

and ε sufficiently small. Note that we can choose r_1 independent of the network graph but only depending on our bounds on the eigenvalues and on the norm of our expanded Laplacian $\bar{L}_t(t)$. Thus, we can obtain the bound of $\dot{V}_d$

$$\dot{V}_d \leq \frac{\nu}{2\sqrt{\varepsilon}} V_a - \nu \varepsilon^{-1} V_d. \tag{39}$$

We define V_a^u and V_d^u via

$$\begin{pmatrix} \dot{V}_a^u \\ \dot{V}_d^u \end{pmatrix} = A_e \begin{pmatrix} V_a^u \\ V_d^u \end{pmatrix}, \qquad \begin{pmatrix} V_a^u \\ V_d^u \end{pmatrix} \left(t_{k-1}^+\right) = \begin{pmatrix} V_a \\ V_d \end{pmatrix} \left(t_{k-1}^+\right) \tag{40}$$

where

$$A_e = \nu \begin{pmatrix} -1 & \frac{1}{2\sqrt{\varepsilon}} \\ \frac{1}{2\sqrt{\varepsilon}} & -\varepsilon^{-1}\nu \end{pmatrix}.$$

It is then easy to check that if $V_d^u(t) = V_d(t)$ and $V_a^u(t) \geq V_a(t)$, then $\dot{V}_d^u(t) \geq \dot{V}_d(t)$. Similarly, if $V_a^u(t) = V_a(t)$ and $V_d^u(t) \geq V_d(t)$, then $\dot{V}_a^u(t) \geq \dot{V}_a(t)$. It is then easily verified that

$$V_d^u(t) \geq V_d(t), \quad V_a^u(t) \geq V_a(t)$$

for $t \in [t_{k-1}^+, t_k^-]$.

The matrix A_e has eigenvalues $\hat{\lambda}_1 = -\frac{3}{4}\nu$ and $\hat{\lambda}_2 = -\varepsilon^{-1}\nu - \nu$. We find

$$e^{A_e t} = \frac{1}{4+\varepsilon} \begin{pmatrix} 4e^{\hat{\lambda}_1 t} + \varepsilon e^{\hat{\lambda}_2 t} & 2\sqrt{\varepsilon}\left(e^{\hat{\lambda}_1 t} - e^{\hat{\lambda}_2 t}\right) \\ 2\sqrt{\varepsilon}\left(e^{\hat{\lambda}_1 t} - e^{\hat{\lambda}_2 t}\right) & 4e^{\hat{\lambda}_2 t} + \varepsilon e^{\hat{\lambda}_1 t} \end{pmatrix}.$$

We find

$$V_a\left(t_k^-\right) + V_d\left(t_k^-\right) \leq V_a^u\left(t_k^-\right) + V_d^u\left(t_k^-\right) = \begin{pmatrix} 1 & 1 \end{pmatrix} e^{A_e(t_k - t_{k-1})} \begin{pmatrix} V_a \\ V_d \end{pmatrix} \left(t_{k-1}^+\right). \tag{41}$$

This yields, given that $t_k - t_{k-1} > \tau$, that

$$V_a\left(t_k^-\right) + V_d\left(t_k^-\right) \leq e^{\hat{\lambda}_3(t_k - t_{k-1})}\left[V_a\left(t_{k-1}^+\right) + \sqrt{\varepsilon} V_d\left(t_{k-1}^+\right)\right]$$

where $\hat{\lambda}_3 = -\nu/2$ provided ε is small enough.

We have a potential jump at time t_{k-1} in V_d. However, there exists an M such that $V_d(t_{k-1}^+) \leq M V_d(t_{k-1}^-)$. On the other hand, V_a is continuous. We find that

$$\begin{aligned} V_a\left(t_k^-\right) + V_d\left(t_k^-\right) &\leq e^{\hat{\lambda}_3(t_k - t_{k-1})}\left[V_a\left(t_{k-1}^-\right) + M\sqrt{\varepsilon} V_d\left(t_{k-1}^-\right)\right] \\ &\leq e^{\hat{\lambda}_3(t_k - t_{k-1})}\left[V_a\left(t_{k-1}^-\right) + V_d\left(t_{k-1}^-\right)\right] \end{aligned}$$

for ε that is small enough. Combining these time intervals, we get

$$V_a\left(t_k^-\right) + V_d\left(t_k^-\right) \leq e^{\hat{\lambda}_3 t_k}\left[V_a(0) + V_d(0)\right].$$

Assume $t_{k+1} > t > t_k$. Since we do not necessarily have $t - t_k > \tau$, we use the bound

$$\begin{aligned} V_a(t) + V_d(t) \leq V_a^u(t) + V_d^u(t) &\leq 2e^{\hat{\lambda}_3(t - t_k)}\left[V_a\left(t_k^+\right) + V_d(t_k^+)\right] \\ &\leq 2Me^{\hat{\lambda}_3(t - t_k)}\left[V_a\left(t_k^-\right) + V_d\left(t_k^-\right)\right]. \end{aligned}$$

Putting everything together, we get

$$V(t) \leq 2Me^{\hat{\lambda}_3 t} V(0)$$

where $V = V_a + V_d$. Hence,

$$\lim_{t\to\infty} \eta_a(t) = 0, \quad \text{and} \quad \lim_{t\to\infty} \eta_d(t) = 0. \tag{42}$$

We have

$$T_1 \bar{x}_a = \eta_a,$$

and

$$T_1 \bar{x}_d = \left(\bar{Q}_t \bar{U}_t^{-1} \otimes I_{m\rho}\right) \eta_d.$$

We know that for graphs in $\mathbb{G}_{\beta,\gamma}^{\tau,N}$ the matrices $\bar{Q}_t$ and $\bar{U}_t^{-1}$ are bounded. Therefore, (42) yields that

$$\lim_{t\to\infty} T_1 \bar{x}_a(t) = 0 \quad \text{and} \quad \lim_{t\to\infty} T_1 \bar{x}_d(t) = 0,$$

which guarantees that we achieve synchronization. □

Remark 6. In this paper, we explicitly design static controllers that require almost zero computational effort for the agents. The design of the protocol is also straightforward and requires almost zero computational effort (see the protocols (14) and (25)).

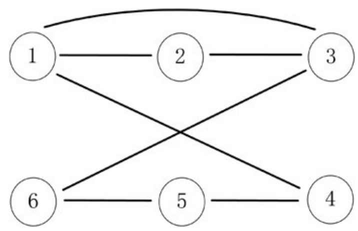

FIGURE 4 The undirected communication topology

6 | EXAMPLES

In this section, we will provide three examples to verify our state synchronization results.

6.1 | Time-varying graph

Example 1 (Squared-down passifiable via static input feedforward).
Consider a MAS with six identical agents, which are squared-down passifiable via static input feedforward. The agent model is of the form of (9) with communication given by (11) where

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \end{pmatrix}, \quad D = \begin{pmatrix} 2 & 1 \\ 4 & 5 \end{pmatrix}.$$

This agent model is squared-down passifiable via static input feedforward, which can be seen by choosing

$$G_1 = \begin{pmatrix} 1 & 0 \\ -3 & 1 \\ 0 & 0 \end{pmatrix}, \quad \text{and} \quad G_2 = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}.$$

Meanwhile, we have $b = 12.831$. Then, we define a set of undirected network $\mathbb{G}_{t,\gamma}^{u,N}$. We consider a communication network as shown in Figure 4 with a time-varying Laplacian matrix

$$L(t) = \frac{\sin(t) + 2.6}{4.4142} \begin{pmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 3 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{pmatrix}$$

with $\|L(t)\| \leq 3.6$, ie, it belongs to the network graph set $\mathbb{G}_{t,\gamma}^{u,N}$ with $\gamma = 3.6$.

Thus, we have $\rho < \frac{2}{b\gamma} = 0.0433$. According to Theorem 3 and (24), our design just has three parameters to be calculated or chosen, ie, ρ with a bound, G_1, and G_2. Therefore, we obtain the following static protocol:

$$u_i = -\begin{pmatrix} 0.08 & 0.04 \\ -0.2 & -0.12 \\ 0 & 0 \end{pmatrix} \zeta_i$$

by choosing $\rho = 0.04$. The error and synchronized trajectories of the state of the agents in the MAS are given in Figure 5 when $\rho = 0.04$. We see that all six agents achieve state synchronization, where Figures 5A to C show the responses of state error $x_{i1} - x_{61}, x_{i2} - x_{62}, x_{i3} - x_{63}$ $(i = 1, \dots, 5)$ which converge to zero in roughly 500 s. Meanwhile, system state x_i will synchronize to the final state trajectory x_s which is shown in Figure 5D.

Example 2 (Squared-down minimum phase with relative degree one).
Consider a MAS with six identical agents, which are squared-down minimum phase with relative degree one. The agent model is of the form of (9) with communication given by (11) with

$$A = \begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \\ 0 & -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \end{pmatrix}.$$

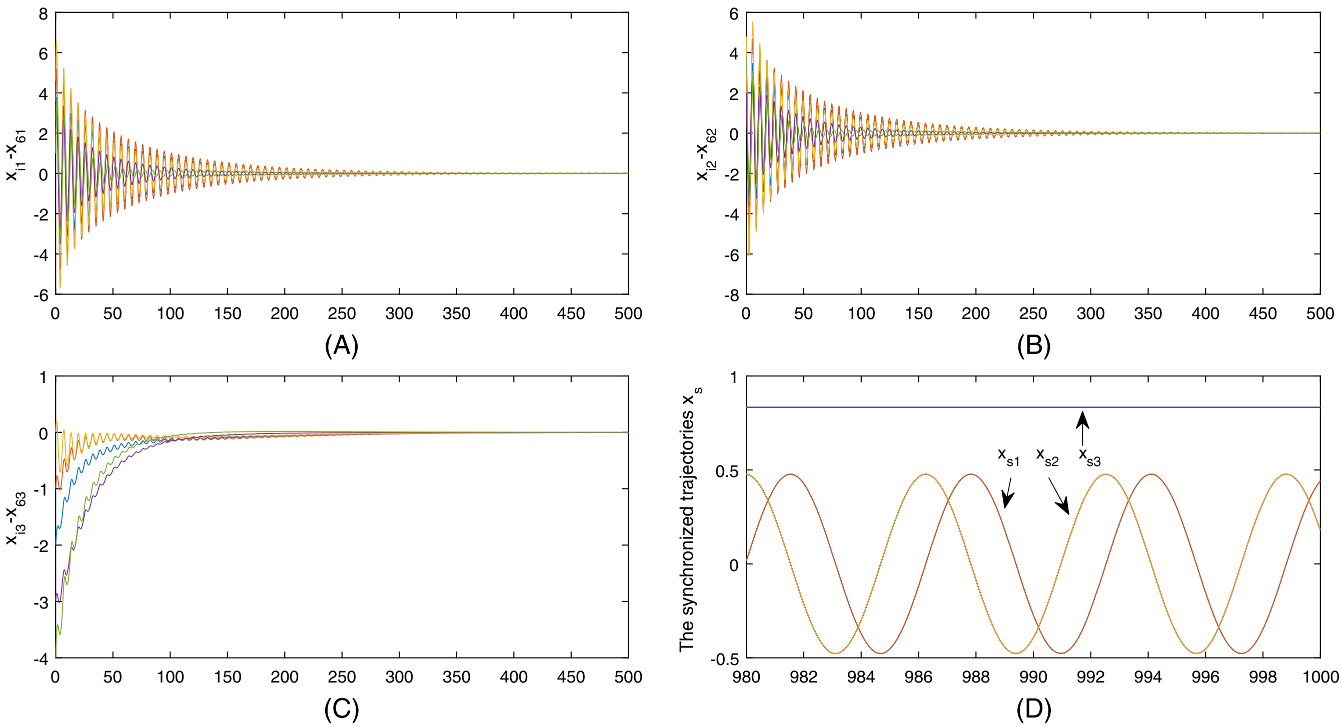

FIGURE 5 Squared-down passifiable agent via static input feedforward. A, Response of error state $x_{i1} - x_{61}$ $(i = 1, \ldots, 5)$; B, Response of error state $x_{i2} - x_{62}, (i = 1, \ldots, 5)$; C, Response of error state $x_{i3} - x_{63}$ $(i = 1, \ldots, 5)$; D, Synchronized state trajectory x_s [Colour figure can be viewed at wileyonlinelibrary.com]

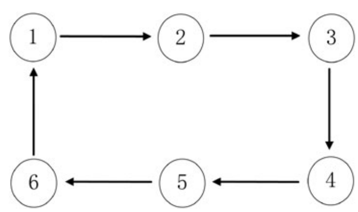

FIGURE 6 The balanced communication topology

This agent model is squared-down minimum phase with a relative degree of 1, which can be seen by choosing

$$G_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \text{and} \quad G_2 = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

Then, we have $T_u = T_x = I$ and $A_{22} + A_{22}^{\mathrm{T}} = b = 4$. We define a set of balanced network $\mathbb{G}_{t,\beta}^{b,N}$. A network is assumed as shown in Figure 6 with time-varying Laplacian matrix

$$L(t) = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} (0.6\sin(10t) + 1)$$

with

$$\bar{L}(t) + \bar{L}^{\mathrm{T}}(t) \geq 0.4.$$

That is, it belongs to the network graph set $\mathbb{G}_{t,\beta}^{b,N}$ with $\beta = 0.4$.

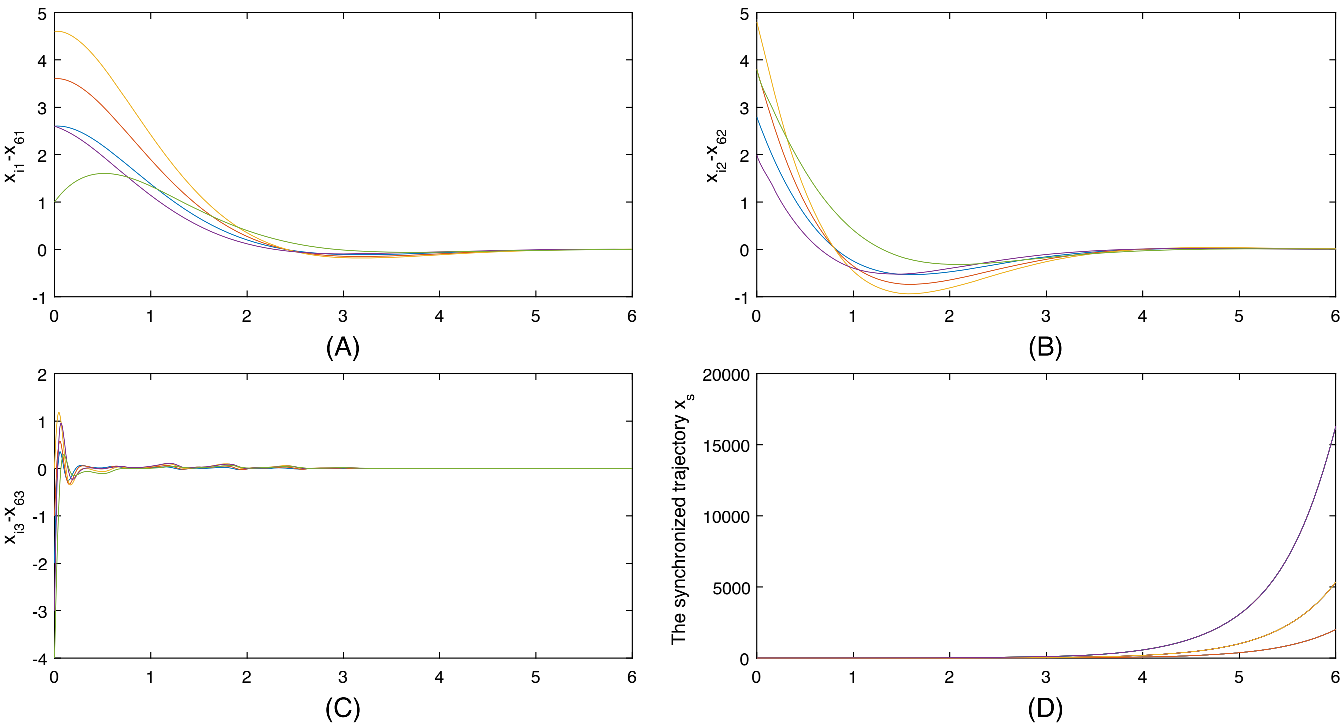

FIGURE 7 Squared-down minimum phase with relative degree one. A, Response of error state $x_{i1} - x_{61}$ $(i = 1, \dots, 5)$; B, Response of error state $x_{i2} - x_{62}$ $(i = 1, \dots, 5)$; C, Response of error state $x_{i3} - x_{63}$ $(i = 1, \dots, 5)$; D, Synchronized state trajectory x_s [Colour figure can be viewed at wileyonlinelibrary.com]

Thus, we have $\rho > \frac{b}{\beta} = 10$. According to (25), we can obtain

$$u_i = -\begin{pmatrix} 0 & 0 \\ 10 & 0 \\ 0 & 0 \end{pmatrix} \zeta_i$$

by choosing $\rho = 10$, G_1, G_2 and T_u. The error and synchronized trajectories of the state of the agents in the MAS are given in Figure 7 when $\rho = 10$. We see that all six agents achieve state synchronization in Figure 7, where Figure 7A to C show the trajectory of the state error $x_{i1} - x_{61}, x_{i2} - x_{62}, x_{i3} - x_{63}$ $(i = 1, \dots, 5)$, which converge to zero in roughly 5 s. Meanwhile, system state x_i goes to infinity which is shown in Figure 7D because the agent has an unstable pole.

6.2 | Switching graph

Example 3 (Squared down minimum phase with relative degree one).
Consider a MAS with six identical agents, which are squared-down minimum phase with relative degree one. The agent model is of the form of (9) with communication given by (11) with

$$A = \begin{pmatrix} -1 & 1 & 0 \\ -1 & -1 & 1 \\ 0 & -1 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 2 & 1 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -2 \end{pmatrix}.$$

We can know that this agent model is squared-down minimum phase with relative degree one by choosing

$$G_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \text{and} \quad G_2 = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

Thus, we have $T_u = T_x = I$ and $A_{22} + A_{22}^{\mathrm{T}} = b = 4$. We define a set of directed networks $\mathbb{G}_{\beta,\gamma}^{\tau,N}$ and assume a network with a switching Laplacian matrix $L(t)$, which includes four modes, ie, $\sigma = 4$, and dwell time $\tau \geq 0.3s$. The

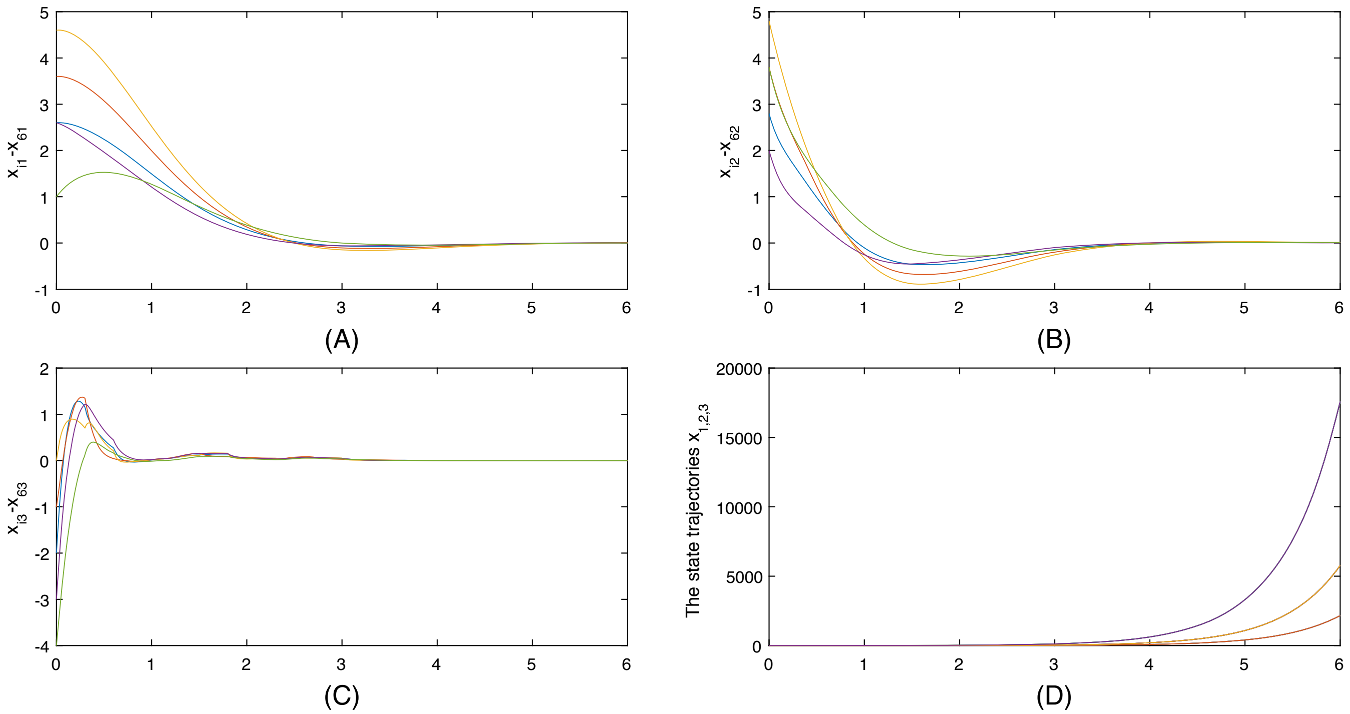

FIGURE 8 Squared-down minimum phase with relative degree one under switching graph. A, Response of error state $x_{i1} - x_{61}$ $(i = 1, \dots, 5)$; B, Response of error state $x_{i2} - x_{62}$ $(i = 1, \dots, 5)$; C, Response of error state $x_{i3} - x_{63}$ $(i = 1, \dots, 5)$; D, Synchronized trajectory [Colour figure can be viewed at wileyonlinelibrary.com]

corresponding Laplacian matrices are

$$L_1 = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & -1 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{pmatrix},$$
$$L_3 = \begin{pmatrix} 3 & 0 & -1 & -1 & -1 & 0 \\ -1 & 2 & 0 & 0 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}, \quad L_4 = \begin{pmatrix} 3 & -1 & -1 & -1 & 0 & 0 \\ 0 & 2 & -1 & -1 & 0 & 0 \\ -1 & -1 & 3 & 0 & 0 & -1 \\ -1 & 0 & -1 & 2 & 0 & 0 \\ 0 & -1 & 0 & -1 & 3 & -1 \\ 0 & 0 & -1 & 0 & -1 & 2 \end{pmatrix}$$

which belongs to the network graph set provided $\beta = 0.6482$. Obviously, L_1–L_4 are unbalanced directed Laplacian matrices; hence, the result cannot be obtained by viewing the switched graph as a general time-varying graph and applying Theorem 5.

Thus, we have $\rho = 7 > \frac{b}{\beta}$ with $\beta = 0.6482$. Similarly, according to (25), we can obtain

$$u_i = -\begin{pmatrix} 0 & 0 \\ 7 & 0 \\ 0 & 0 \end{pmatrix} \zeta_i$$

by choosing $\rho = 7$. The global state error and synchronized state trajectories of MAS are given in Figure 8 when $\rho = 7$. We see that all six agents achieve state synchronization by Figure 8A to C, even though the system state x_i go to infinity (see the curves in Figure 8D). Meanwhile, the switching behaviors can be found in Figure 8, especially for the subgraph in 8C.

7 | CONCLUSION

In this paper, we have studied state synchronization of homogeneous MAS with partial-state coupling in the presence of time-varying communication topology. A static protocol design has been design for two classes of time-varying networks, general time-varying graphs, and switching graphs. Meanwhile, we have defined four classes of agents for which we can find a static protocol to achieve synchronization given general time varying but balanced graphs. For one class of systems, we have shown that we can find a static protocol to achieve synchronization, given switching graphs which are not necessarily balanced.

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