A SEMI-MARKOV MODEL OF A HOME NETWORK ACCESS PROTOCOL

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1 Abstract

Home networks provide a means for interconnecting consumer products in home environments. In this paper the performance characteristics of the medium access protocol—a slotted persistent CSMA/CD protocol—of a twisted pair home network are investigated. Using a dedicated semi-Markov model throughput results are obtained by analyzing an overload situation.

2 Context and Introduction

Nowadays we observe a still growing interest in having greater control of domestic appliances and of interconnecting several kinds of electronic devices in the home (and similar environments) so as to obtain additional features (IEEE Special Issue 1991). Typical applications are energy management, safety, security, home automation and entertainment. Thus the conventional trend of viewing consumer products in isolation is shifting towards a more system-oriented approach—*The Home System*.

Several home systems are currently candidate for world-wide standardization: the Home Bus System (Japan), Consumer Electronic Bus (USA), Smart-House (USA), and Esprit Home Systems (Europe). The latter system (EHSA 1991) emerged from the cooperation of several European manufacturers and its architecture is based on the well-known OSI model (OSI 1984). The Home System has adopted four OSI layers: the Physical layer, the Data Link layer, the Network layer and the Application layer (Vlot and Katoen 1991). The Data Link layer has been subdivided into the Logical Link Control (LLC) sublayer and Medium Access Control (MAC) sublayer. The medium access protocol, that determines which station is allowed to use the medium when there is a competition for using it, is the major function of the MAC layer.

A transmission medium offers two communication services to products connected to it: a control channel provides a means for exchanging "commands" and (one or more) information channels offer real-time connections of various capacities for transmitting e.g. speech, audio and video signals. Commands play an important role in the inter-operability of domestic appliances. Therefore, the performance characteristics of the control channel are of vital importance. It is well-known from the computer network field that MAC protocols have a great impact on the performance of networks (Hammond and O'Reilly 1986) (Tobagi 1980).

In this paper we investigate the throughput of the MAC protocol of a twisted pair home network by means of a semi-Markov model. The model considers the network under a high load situation. The protocol was originally described in (Tritton 1991).

The paper is organized as follows. In Section 3 we describe the medium access protocol in detail. A motivation for defining the Markov model is given in Section 4. The Markov model itself is defined in Section 5. In subsequent sections the transition probabilities of the model are determined, some performance measures related to the model are defined, and, finally, some results are given which are compared to results obtained by simulation.

3 The Access Protocol

In this paper we consider a slotted persistent CSMA/CD protocol (Carrier Sense Multiple Access with Collision Detection). Slotted means that the time axis is divided into slots of a fixed duration. All stations are forced to start transmissions only at the

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beginning of a slot.

The basic idea is as follows: a ready station, that is, a station that has a packet ready for transmission, checks whether the medium is free prior to transmission (*carrier sensing*). When the medium is free the station immediately transmits the packet and otherwise it persists on transmitting as soon as the medium becomes free. Notice that two (or more) ready stations may sense the medium free simultaneously, and, hence, decide to transmit in the same slot. In that case both transmitted packets are corrupted (*collision*). Each station is able to detect a collision while transmitting. At detection it immediately aborts its transmission and repeats the transmission after a *back-off period*.

When a packet is not successfully transmitted within k attempts, where k is a multiple of 7, an error notification is given to the LLC layer which immediately initiates a new transmission of the packet, restarting the MAC procedure. Note that a packet may thus be transmitted infinitely often. In order to avoid recurrent collisions the back-off period length both depends on the station's address and the number of (re-)transmissions the packet already experienced. Formally, let $B_\alpha(k)$ be the length in slots of the back-off period of station α $(0 \leq \alpha < 256)$ prior to the kth $(k \geq 0)$ retransmission of a packet. Then we have

$$B_\alpha(k) = \begin{cases} 0 & \text{if } k \,\mathbf{mod}\, 7 = 0 \\ \alpha \,\mathbf{mod}\, 2^{k \,\mathbf{mod}\, 7 + 2} & \text{if } k \,\mathbf{mod}\, 7 \neq 0. \end{cases}$$

Because B is a function of the address of the transmitting station we have a kind of 'priority' function among stations.

As said before, the time axis is slotted. During a slot 2 bytes can be transmitted. Slots are organized into frames. A frame consists of 16 time slots; 14 slots are reserved for information channels, one slot for control channel, and one for overhead (frame synchronization and the like). There is no fixed allocation of slots to stations. Access to an information channel is requested via the control channel, while access to the control channel is regulated by the CSMA/CD protocol as described above.

A packet is of varying size, from 16 bytes up to 261 bytes, including overhead information. At most 40 stations may be connected to the network. The transmission rate is 1,536 Mbit/s, resulting in a transmission rate per slot of 96 Kbit/s. Taking slot synchronization delays (guard bands) and overhead bits like parity and start/stop bits into account, an effective transmission rate (per slot) of 64 Kbit/s results. In the following we consider the control channel in isolation. That is, we consider a single slotted medium of 64 Kbit/s. Note that a slot now corresponds to a frame.

Each transmission period, that is, a period during which either a successful transmission or a collision takes place, is followed by 1 free slot so as to enable stations to detect the availability of the medium. So, the transmission of a minimum length packet takes 9 slots, while a collision occupies 2 slots—assuming that detecting a collision takes less than one frame delay.

4 Motivation

Performance of slotted persistent CSMA/CD protocols is well-studied in literature (see e.g. (Takagi and Kleinrock 1985) (Takagi and Murata 1987a) (Tobagi and Hunt 1980)0. Several models are developed so as to analyze both throughput and transfer delay. We started our analysis by adopting a simple "traditional" model (Takagi and Kleinrock 1985) to obtain throughput results. These analytical results were validated by simulation, and it turned out that both results are quite close for low loads, whereas for high loads they differ considerably. For the sake of brevity we here omit both the analysis and the simulation experiments. A full description of these can be found in (Katoen 1991).

The main cause of the discrepancy of results seemed to be the assumption in (Takagi and Kleinrock 1985) that the time between two successive transmission attempts of a station is geometrically distributed. In fact, this assumes a memoryless back-off period length. This violates the definition of B_α as a (deterministic) function of the number of retransmissions already made. This observation motivated us to develop a more realistic—and more complex—model of the protocol, a semi-Markov model modeling the system under a high load. In this way we hope to have more insight in the protocol's operation in an overload situation and to obtain results comparable to the simulation results.

5 The Markov Model

The system is modeled as a set of N equal stations and a shared medium. Each station has an infinite queue of ready packets, such that after a successful transmission a new ready packet is immediately available. We assume a constant packet length whose

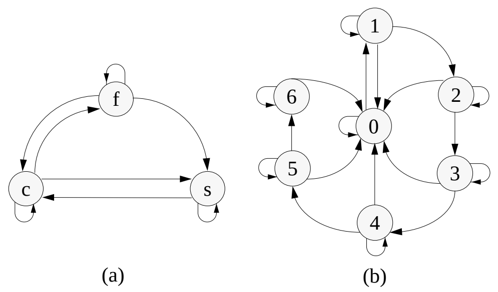

Figure 1: Markov chain of both medium and station behavior.

transmission time is $1/a$ slots. a is usually called the normalized packet transmission time. The medium is considered to be error-free in the sense that failed transmissions are due to collisions only. It is assumed that overlap of two (or more) packets is destructive, such that all collided packets need to be retransmitted.

The details of the model are as follows. The medium can be in any of 3 states as depicted in Figure 1(a). States are represented by circles and transitions are represented by arrows. The medium is considered to be free (**f**) when there is no transmission going on, it is in the successful state (**s**) when exactly one transmission is going on, and in the collided state (**c**) otherwise. There is no direct transition from **s** to **f**, since after a successful transmission of station n, say, n immediately wants to transmit its next packet—due to a high load [2]. So, the medium can either stay in **s** (when n is the only station that transmits) or move to **c**.

A station can be in any of 7 states, numbered from 0 to 6, as shown in Figure 1(b). A state represents the number of collisions (modulo 7) the current packet experienced before. When a station in state i ($i{\neq}0$) transmits a packet successfully it returns to state 0, when it causes a collision it goes to state $i{+}1$ (modulo 7), and when it transmits no packet it stays in i. A station in state 0 wants to transmit as soon as the medium becomes free. It stays in 0 when the transmission is successful and goes to state 1 otherwise. (Note that the model may be extended to normal load situations—without considering queueing of packets—by adding an idle state to the station's model of which the residence time is geometrically distributed.)

The time during which the medium (and thus the system) remains in a certain state is deterministically distributed. Namely, the system stays in state **s** for $1{+}\frac{1}{a}$ slots, in state **c** for 2 slots, and in state **f** for 1 slot. After staying in a state for some slots (depending on that state) the system makes a transition to a next state. Let $w(k)$ denote the state of the system at the kth transition instant ($k{\geq}0$). The system state $w(k)$ involves two components: the state of the medium at instant k and the number of stations in state i ($0{\leq}i{<}7$) at instant k, $N_i(k)$. Formally, we have

$$\begin{aligned} w(k) &= (d_w(k), e_w(k)) \\ d_w(k) &\in \{\mathbf{c}, \mathbf{f}, \mathbf{s}\} \\ e_w(k) &= (N_0(k), \ldots, N_6(k)) \quad , \end{aligned}$$

where $\sum_{i=0}^{6} N_i(k){=}N$. The total number of states is $\mathcal{O}(N^6)$. The model is an enhanced variant of the model presented by Akhtar and Sood (Akhtar and Sood 1988).

In the sequel we adopt the following conventions. The state space is denoted by W and v and w denote elements of W. When it is clear from the context which transition instant is meant we omit the argument k from the system state (and its (sub)components). Moreover, let $0{\leq}i{<}7$.

6 Transition Probabilities

First observe the system without taking the (deterministic) residence times into consideration, and define π_w as the stationary probability of this system being in state w. In fact, π_w denotes the (stationary) probability of the system being in state w at some transition instant. Stated otherwise, π_w is the fraction of *instants* at which the system is in state w, when an infinite amount of instants is considered. In order to obtain the fraction of *time* the system is in some state w the deterministic residence times must be taken into account. Let r_w be the average residence time (in slots) in state w. In our model r_w is only determined by the time the medium remains in a certain state. More precisely,

$$r_w = \begin{cases} 2 & \text{if } d_w = \mathbf{c} \\ 1 & \text{if } d_w = \mathbf{f} \\ 1 + \frac{1}{a} & \text{if } d_w = \mathbf{s} \end{cases} \quad .$$

[2] Recall that each transmission period is followed by a free slot. So, station n must obey one slot between two successive transmissions, in order to give other stations the opportunity to make an attempt.

Let ϕ_w denote the fraction of time the system is in state w, or, equivalently, ϕ_w is the (stationary) probability of the system being in state w. ϕ_w and π_w are then related by

$$\phi_w = \frac{\pi_w * r_w}{\sum_{v \in W} \pi_v * r_v} \quad . \tag{1}$$

(Aside: from a theoretical point of view the model is a so-called semi-Markov chain (Kleinrock 1975). A semi-Markov chain is a weak kind of Markov chain, since it permits an arbitrary distribution of residence times. At transition instants the semi-Markov chain behaves identical to an ordinary Markov chain, in fact, when only transition instants are considered, we have an "embedded" Markov chain. π_w corresponds to the stationary probability of this embedded Markov chain, while ϕ_w corresponds to the stationary probability of the semi-Markov chain.)

In the following we consider the system at transition instants. Define $p_w(v)$ as the conditional probability of the system making a transition from state w to v at instant k,

$$p_w(v) = Pr[\, w(k) = v \mid w(k-1) = w \,] \quad .$$

Furthermore, let q_i be the probability that a station in state i leaves state i after staying in that state for one slot. Observe that a station in state 0 wants to transmit its ready packet at once, hence, q_0 is equal to 1. In order to facilitate the analysis it is assumed that the residence time of a station in state i ($i \neq 0$) is geometrically distributed with mean $2^{i+1} - \frac{1}{2}$. It should be noticed that this differs from the deterministic distribution of the back-off period in the protocol [3]. Due to the memoryless property of the geometrical distribution we now have

$$q_i = \begin{cases} 1 & \text{if } i=0 \\ (2^{i+1} - \frac{1}{2})^{-1} & \text{if } i \neq 0 \end{cases} \quad .$$

In the sequel we let p_i denote $1-q_i$.

Consider the system at some transition instant moving from state w to state v with $e_w = (N_0, \ldots, N_6)$ and $e_v = (N'_0, \ldots, N'_6)$. First we focus our attention on the relation between e_v and e_w. To that end, let m_i denote the number of stations that move from state i to $i+1$ and let l_i be the number of stations that return to state 0 from state i. Then

$$N'_i = \begin{cases} N_i - m_i + m_6 + \sum_{j, j \neq 0}^{6} l_j & \text{if } i=0 \\ N_i - m_i + m_{i-1} - l_i & \text{if } i \neq 0 \end{cases} \quad .$$

Recall that a station returns to state 0 when it successfully transmits a packet. Hence, at most one station can return to state 0 by successfully transmitting a packet, or $0 \leq \sum_j l_j \leq 1$.

Now consider the first component of the system's next state, d_v. In the following the total number of stations leaving state i, is denoted by k_i. k_i is equal to $m_i + l_i$. Obviously, d_v depends on the number of stations that want to transmit at the transition instant, that is, the total number of stations that leave their station state ($\sum_i k_i$). The probability that a station leaves state i at the transition from w to v is $1 - p_i^{r_w}$. The probability of k_i stations leaving state i at the transition from w to v has a binomial distribution

$$Q_w(k_i, i) = \binom{N_i}{k_i} (1 - p_i^{r_w})^{k_i} * p_i^{r_w (N_i - k_i)} \; .$$

The possible transitions are as follows. The medium moves to the free state when no station wants to transmit ($\sum_i k_i = 0$), to the successful state when exactly one station wants to transmit ($\sum_i k_i = 1$), and to the collision state otherwise ($\sum_i k_i > 1$). The transition probabilities $p_w(v)$ are given in Table 1. In (2) $k_i = 0$ for all i, in (3) $k_i = 1$ and $k_j = 0$ for $j \neq i$, and in the last case $\sum_i k_i > 1$.

Since the Markov chain is aperiodic and irreducible—any state can be reached from every other state in a finite, but not a priori fixed number of transitions—the stationary probabilities π_w can be calculated in a traditional way (Kleinrock 1975), using (2), (3), and (4). These results can subsequently be used to solve (1). Here we will not elaborate on these calculations.

7 Performance Measures

From the stationary probabilities ϕ_w several interesting performance measures can be obtained. Two important performance measures are the average number of successful transmissions per packet transmission time, S, and the average number of attempted transmissions per packet transmission time G. Note that G includes both successful and failed transmissions, and thus, $G \geq S$. S is equivalent to the probability of the system being in state $\mathbf{s}$. However, since a successful transmission period is followed by a free

[3] In fact, we model the back-off period length as a function of the number of collisions a packet caused before, and we do not take into account the relation with the originator's address. In the latter case the number of states would be excessive.

$$p_w(v) = \prod_{i=0}^{6} Q_w(0,i) \qquad \text{if } d_v = \mathbf{f} \tag{2}$$

$$p_w(v) = Q_w(1,i) * \prod_{j=0, j\neq i}^{6} Q_w(0,j) \qquad \text{if } d_v = \mathbf{s} \tag{3}$$

$$p_w(v) = \prod_{i=0}^{6} Q_w(k_i,i) \qquad \text{if } d_v = \mathbf{c} \quad . \tag{4}$$

Table 1: Transition probabilities of Markov chain.

slot, S is obtained by normalizing this probability as follows

$$S = \frac{1}{1+a} * \sum_{w\in W, d_w=\mathbf{s}} \phi_w \quad .$$

Furthermore, the expected number of stations in state i is given by

$$\sum_{n=0}^{N} n * R_i(n) \quad ,$$

where $R_i(n)$ denotes the probability of having n stations in state i, that is,

$$R_i(n) = \sum_{w\in W, N_i=n} \phi_w \quad .$$

Deriving a relation for G is less straightforward. Let G' be the average number of attempts per transition (instant) of the system. Again consider the system in state w. The average number of attempts at the system's transition from w to v, G_w, is defined by

$$G_w = \sum_{k=0}^{N} k * P_w(k) \quad ,$$

where $P_w(k)$ is the probability that k attempts are made ($0 \leq k \leq N$), given the system is in state w. By definition, G' is then equal to

$$G' = \sum_{w\in W} G_w * \pi_w \quad .$$

The average number of attempts per slot is obtained by dividing G' by the average time between two transitions of the system which is equal to $\sum_{w\in W} \pi_w * r_w$. The average number of attempts per packet transmission time then is

$$G = \sum_{w\in W} \frac{G_w * \phi_w}{a * r_w} \quad .$$

It remains to derive a relation for $P_w(k)$. $P_w(k)$ is equal to the probability that k stations leave their state at a system's transition from w to some other state. k is equal to $\sum_i k_i$. The probability that k_i stations leave their state (i) is $Q_w(k_i,i)$. The probability of some combination $k_0,\ldots,k_6$ such that $\sum_i k_i = k$ is

$$\prod_i Q_w(k_i,i) \quad \text{with} \quad \sum_i k_i = k \quad .$$

Considering all possible combinations we obtain

$$P_w(k) = \sum_{\sum_i k_i = k} \left[\prod_i Q_w(k_i,i) \right] \quad .$$

8 Results

The stationary probabilities π_w are obtained by applying iterative techniques. Starting from state w with $d_w = \mathbf{c}$ and $N_0 = N$, π_w could be calculated in 25 iterations. Results obtained by making 100 iterations differed less than 2%. The stationary probabilities are used to compute ϕ_w, and subsequently to compute G and S. We have taken a to be 1/8, the minimal packet length. Remark that the optimal achievable throughput S is 8/9, since each transmission is followed by a free slot. Results for some values of N are given in Table 2 and compared to simulation results S^* and G^*. The simulation results were obtained by using the LANSF (Local Area Network Simulation Facility) tool (Gburzyńsky and Rudnicki 1991). The simulation is based on a deterministic distribution of the back-off period as opposed to the geometrical distributed residence time of a station in some state i assumed in the Markov model. All other assumptions are kept equal (Katoen 1991). Results are only obtained for at most 13 stations. The results are almost identical to the simulation results.

N	1	3	5	7	9	11	13
G	0.89	1.49	2.02	2.50	2.94	3.37	3.79
S	0.89	0.76	0.69	0.66	0.63	0.61	0.60
G^*	0.89	1.46	2.02	2.49	2.89	3.31	3.67
S^*	0.89	0.76	0.69	0.66	0.65	0.64	0.63

Table 2: Markov model results.

So, the specific (semi-) Markov model gives quite realistic values for overload situations. Remark that the throughput decreases only slightly as N is increased. Data for large values of N would be needed to judge the full merits of the model.

9 Conclusions

In this paper we consider the performance of a particular slotted persistent CSMA/CD protocol. Experiences with a simple model taken from literature (Takagi and Kleinrock 1985) showed unreliable results at high loads. Therefore, a more realistic—and more complex—model of the protocol, a semi-Markov model, was developed. The model is described in this article in all details. Validation by means of simulation shows that the model generates rather reliable results. This indicates the usefulness of the model. Furthermore, the author feels that the model can quite easily be extended to cover e.g. other packet length distributions (rather than constant length packets) and other station behaviors.

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11 Bibliography

Joost-Pieter Katoen took his Master's degree in Computing Science (with honours) in the fall of 1987 from the University of Twente. At the Eindhoven University of Technology he was involved in research concerning the use of calculational programming, that is, a way of constructing programs and their correctness proofs hand in hand, in different programming styles (parallel, functional, and imperative styles). He joined the Philips Research Laboratories at 1990 and worked mainly on home systems, in particular protocol design and performance evaluation. In the beginning of 1992 he joined the Tele-Informatics and Open Systems group at the University of Twente where he is involved in the development of 3rd generation mobile telecommunication systems. His current interests are protocol design, verification, performance evaluation, and design methods for distributed systems.

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