A GLOBAL-LOCAL OPTIMIZATION METHOD FOR PROBLEMS IN STRUCTURAL DYNAMICS

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Abstract

The optimization of complex structures involving many design variables and constraints can be performed using a multi-level approach: a structure consisting of several components is optimized as a whole (global) and on the component level (local). Earlier work [1], [2], [3], described a multi-level technique developed for the optimization the Airbus A380 vertical tail plane. In this application, a global model is used to calculate the loads on each of the components. These components are then optimized using the prescribed loads, followed by a new global calculation to update the loads. The component optimization strategy is based on Neural Networks (NN) and Genetic Algorithms (GA).

This paper describes a strategy that makes this global-local optimization method possible for problems in structural dynamics. It is established that a parametrization of the component interactions (e.g. component loads) is problematic due to frequency dependence. Hence, a modified method is proposed in which the speed of Component Mode Synthesis (CMS) is used to avoid this parametrization. The effectiveness of this method is demonstrated in a test case concerning the placement of sensor and actuator locations in Active Structural Acoustic Control (ASAC). Special attention is paid to the behavior of the optimization strategy.

1 Introduction

1.1 Global-local strategy

The global-local strategy discussed in this paper consists of a *structure evaluation*, where a global model is evaluated to calculate the interactions between the components, and a *component optimization* where the components are optimized separately while taking into account the interactions (see figure 1). Together, these two steps are named a *structure iteration*. As the name suggests, the results of the component optimization are subjected to a structure evaluation again, making this an iterative process.

In earlier research, this strategy has been ap-

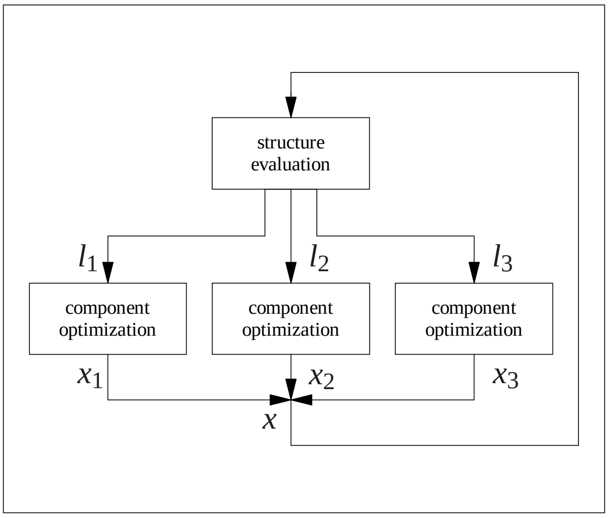


Fig. 1 Global-local strategy. l: interactions, x: design variables

plied to a weight minimization for the carbon fibre re-enforced panels of the Airbus A380 vertical tail plane. In that case, the interactions calculated during the structure evaluation consist of boundary forces. In the component optimization, each panel is optimized for minimum weight subject to static strength and buckling constraints, while taking into account the boundary forces calculated during the structure evaluation. A Finite Element (FE) model is used to calculate the static stress and buckling load.

The component optimization strategy has been developed to be efficient in a global-local setting: as the global-local strategy converges, the interactions between the components change ever more slightly from one structure evaluation to the next. Hence, the efficiency of the component optimization benefits if the information from FE calculations performed in earlier structure iterations is maintained. An approximate model (or response surface) is used to interpolate between FE results of both the current and earlier structure iterations. This approximate model consists of a Neural Network (NN) architecture known as a backpropagating feedforward network. In order to find the global minimum in the NN approximation, a genetic algorithm (GA) is applied. Although the GA requires a relatively large number of function evaluations to converge, the time needed to find the optimum is small because a function evaluation of the NN requires very little computational effort. The optimal result obtained with the GA is subjected to FE analysis and the FE result is incorporated in the NN approximation. This leads to a new optimum in the approximate model (see figure 2). Again, this optimum is identified using the GA and subjected to FE analysis. This iterative process is continued until convergence is reached. Finally, it is noted that unconditionally evaluating the global optimum of the Neural Network can cause the optimization to stagnate. Hence, some heuristical selection methods are applied to recognize or avoid this stagnation.

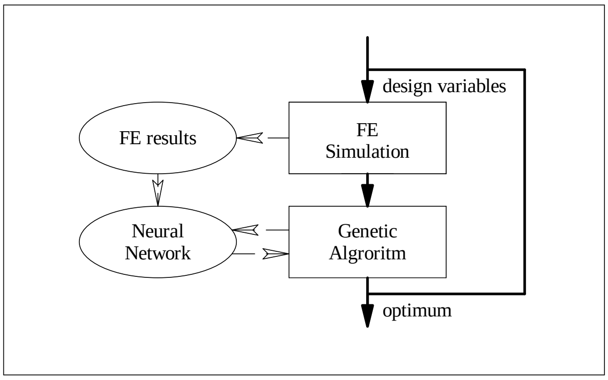
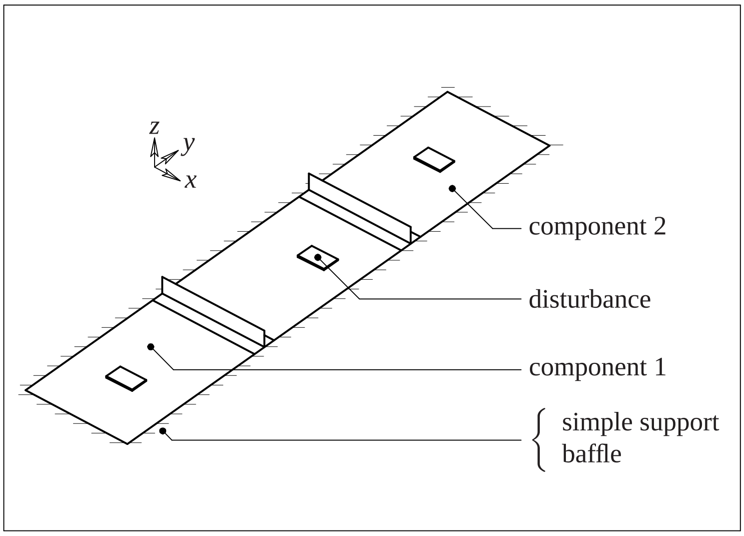


Fig. 2 Component optimization (without selection method)

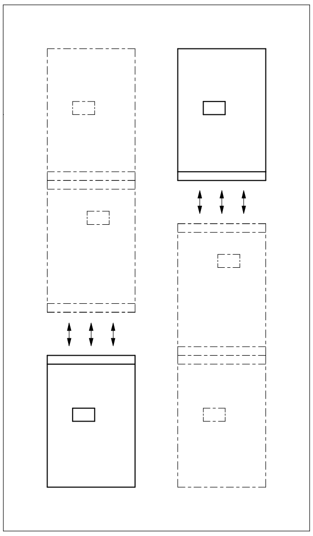
1.2 Active Structural Acoustic Control

In this paper, the optimization strategy is applied to problems in structural dynamics. The test case involves a noise reduction strategy referred to as Active Structural Acoustic Control (ASAC) [5], which is applicable when the source of noise is a vibrating plate-like structure such as an aircraft trim panel. In ASAC, sensors and actuators are connected to the noise source such that shape and amplitude of flexure can be controlled to minimize sound radiation. Generally, several sensors and actuators are placed on the noise source. Simple design rules for placing the sensor-actuator pairs exist, but these have a number of distinct drawbacks [5]. Hence, a numerical optimizer is applied to select the sensor and actuator locations.

As a test problem, we use a structure consisting of three rectangular aluminium plates, separated by transverse stiffeners (see figure 3(a)). The disturbance is generated by a rectangular patch of piëzoelectric material placed on the middle plate. On the other two plates, a sensor-actuator pair is placed to reduce the noise. The actuators are piëzoelectric patches and the sensors are accelerometers placed on the center of each actuator. The plate is placed in an infinite baffle – an acoustically hard surface that does not vibrate – which means the Rayleigh integral can be used for acoustical calculations. The reader is referred to [4] for the exact properties of the test problem.



(a) global design of the test problem



(b) Global-local problem. Solid line: component. Dashed: remaining structure.

Fig. 3 The test problem: global design and global-local structure.

The optimization problem consists of finding locations for the sensor-actuator pairs that minimize radiated sound power under a broadband disturbance in a frequency range that includes the first nine eigenfrequencies of the plate. In the global-local setting, the structure has two components consisting of a plate and a sensor-actuator pair (see figure 3(b)). The two plates that are not part of component i are called the *remaining structure* of component i.

2 Convergence and optimality

The behavior of the global-local optimization strategy depends strongly on the relations between the objective functions of the components. Here, we give a brief summary of a theoretical study with respect to this behavior. The problem definition is as follows. Let the design variables of component i at the beginning of structure iteration k be denoted as a vector: $\mathbf{x}_i^k \in \mathbb{R}^{n_i}$, where n_i is the number of design variables of this component. The design variables of the global problem can then be defined as the concatenation of all vectors $\mathbf{x}_i^k$:

$$\mathbf{x}^k \in \mathbb{R}^n \equiv \left\{ \begin{array}{c} \mathbf{x}_1^k \\ \mathbf{x}_2^k \\ \vdots \\ \mathbf{x}_N^k \end{array} \right\} \tag{1}$$

where $\mathbf{x}^k \in \mathbb{R}^n$ is the vector of design variables in the global problem in iteration k, N is the number of components and $n = \sum_{i=1}^{N} n_i$ is the number of design variables in the global problem.

Next, the component objective functions are defined. Instead of using an explicit parametrization of the interactions between the components, such as the boundary forces, each component objective function is defined to depend explicitly on the design variables of all components:

$$f_i : \mathbb{R}^n \to \mathbb{R} \equiv f_i(\left\{ \begin{array}{c} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{array} \right\}) = f_i(\mathbf{x}) \tag{2}$$

Where f_i is the objective function of component i. It is recalled that the component objective function may differ from one component to the next. For example, each component optimization problem may be the minimization of the radiated sound power of that component only.

2.1 Convergence

In order to study the convergence of the optimization strategy, the concept of a component optimization is formalized. Since each component is optimized while leaving the interactions (e.g. the boundary forces) unchanged, this can be modeled as an optimization of each component while leaving the design variables of the other components constant. The result of this optimization for component i can be written as follows:

$$g_i(\mathbf{x}^k) = \arg\min_{\hat{\mathbf{x}}_i} f_i(\begin{Bmatrix} \mathbf{x}_1^k \\ \mathbf{x}_2^k \\ \vdots \\ \hat{\mathbf{x}}_i^k \\ \vdots \\ \mathbf{x}_N^k \end{Bmatrix}) \tag{3}$$

Where $\arg\min$ denotes the argument of the minimum: the design variables for belonging to optimal result. $\hat{x}_i$ is a dummy variable needed to find the minimum. The function $g_i : \mathbb{R}^n \to \mathbb{R}^{n_i} = g_i(\mathbf{x})$ is named the component optimization function. The result of a structure iteration is a vector containing the results of all component optimizations:

$$g(\mathbf{x}^k) \equiv \begin{Bmatrix} g_1(\mathbf{x}^k) \\ g_2(\mathbf{x}^k) \\ \vdots \\ g_N(\mathbf{x}^k) \end{Bmatrix} \tag{4}$$

Where $g : \mathbb{R}^n \to \mathbb{R}^n = g(\mathbf{x})$ is named the *structure iteration operator*. The global design $g(\mathbf{x})$ is used to calculate the interactions between the components that are subsequently used in the component optimization. These interactions are not modeled. Hence, the design variables themselves are used as input for the next structure iteration:

$$\mathbf{x}^{k+1} = g(\mathbf{x}^k) \tag{5}$$

It can be seen that the result in the next structure iteration is simply a function of the current result. Convergence to a point $\mathbf{x}^\star \in \mathbb{R}^n$ occurs when results in the vicinity of $\mathbf{x}^\star$ are attracted to that point. Assuming that the function $g(\mathbf{x})$ is sufficiently smooth, we may linearize equation 5 near the point $\mathbf{x}^\star$:

$$\mathbf{x}^{k+1} \approx g(\mathbf{x}^\star) + \mathbf{D}_{\mathbf{x}^\star}\mathbf{g} \cdot (\mathbf{x}^k - \mathbf{x}^\star) \tag{6}$$

Where $\mathbf{D}_{\mathbf{x}^\star}\mathbf{g} \in \mathbb{R}^{n\times n}$ is the Jacobian (derivative matrix) of g on the point $\mathbf{x}^\star$. It can be proven that convergence occurs if and only if:

$$g(\mathbf{x}^\star) = \mathbf{x}^\star \tag{7}$$

$$\rho(\mathbf{D}_{\mathbf{x}^\star}\mathbf{g}) < 1 \tag{8}$$

Where $\rho(\mathbf{A})$ denotes the *spectral radius* of the matrix $\mathbf{A}$, which is equal to its largest eigenvalue in absolute value. In current problem, it is a measure for the interdependency of the location of the optima for different components. If there is no interdependency, then the spectral radius is equal to zero and convergence occurs in a single structure iteration for the linearized problem. If the interdependency is very strong, then the spectral radius is large and the optimization does not converge. The reader is referred to [4] for more a more detailed model and a discussion about its use in theory and practice.

2.2 Optimality

Each component optimization minimizes the objective function of that component. Any harm done to the other components is not taken into account. Hence, a combination of designs that are each optimal from a component point of view are not necessarily an optimum from a global perspective. The mechanism of components improving themselves by harming each-other is exemplified in the famous *prisoner's dilemma*, which is commonly used in introductions to game theory.

Consider the case where the optimization has converged to a point $\mathbf{x}^\star$. Since the optimization has converged, each of the component objective functions have a global optimum in this point. Naturally, the optimization can only have converged to an optimum from a global point of view if $\mathbf{x}^\star$ is that optimum. Hence, the global-local strategy can not converge to an optimum from a global perspective unless the optimum in the component objective functions coincides with the optimum from a global point of view.

In case the global objective function is defined as the sum of the component objective functions, a sufficient condition can be derived for the component objective functions [4]. For minimization problems, a converged result is guaranteed to be an optimum from a global point of view if for each component i:

$$\|\mathbf{D}_{\mathbf{x}_i}\mathbf{f}_i\|_2^2 \geq -\mathbf{D}_{\mathbf{x}_i}\mathbf{f}_i^T \sum_{\substack{j=1 \\ j\neq i}}^{N} \mathbf{D}_{\mathbf{x}_i}\mathbf{f}_j \qquad (9)$$

Where $\mathbf{D}_{\mathbf{x}_i}\mathbf{f}_j \in \mathbb{R}^{n_i}$ is the gradient of component j with respect to design variables of component i. In this equation, the left hand side indicates the improvement done to component i for an incremental step in the steepest descent direction of the objective function of component i. The right hand side denotes any harm done to the other components for this step. If the improvement always outweighs the harm, then a converged result is guaranteed to be a minimum.

Two cases are discussed where component objective functions trivially comply with equation 9. First, a structure is optimized where there are no interactions between the components. In that case,

$$\mathbf{D}_{\mathbf{x}_i}\mathbf{f}_j = 0 \quad \forall \quad i,j \qquad (10)$$

such that the inequality is guaranteed to hold. Second, all components share the same objective function. Hence,

$$\mathbf{D}_{\mathbf{x}_i}\mathbf{f}_i = \mathbf{D}_{\mathbf{x}_i}\mathbf{f}_j \quad \forall \quad i,j \qquad (11)$$

Hence, inequality 9 simplifies to:

$$\|\mathbf{D}_{\mathbf{x}_i}\mathbf{f}_i\|_2^2 \geq -(N-1)\cdot\|\mathbf{D}_{\mathbf{x}_i}\mathbf{f}_i\|_2^2 \qquad (12)$$

with N the number of components. This inequality holds for any objective function.

Based on the above theory, it is concluded that an optimization technique where the component objective functions are local, such as the sound power radiated by one of the three plates, is insufficient in the current global-local setting. Instead, the sound power radiated by the entire structure is to be minimized in the component optimizations. With equation 12, a converged solution is then guaranteed to be at least a local minimum.

3 Parametrization of interactions

The NN serves as an approximate model which interpolates FE results from all structure iterations and also from different components within the structure. In order to interpolate between these results the approximate model must have three sets of quantities as input (see figure 4).

- *Design variables*, such as the thickness of the carbon fibre re-enforced panel.
- *Interactions*, such as the boundary forces in the static optimization.
- *Fixed parameters*. These quantities allow the approximate model to interpolate between results of different components. By definition, they remain unchanged between structure iterations, but differ from one component to the next. An example of these quantities is the length and the width of a panel.

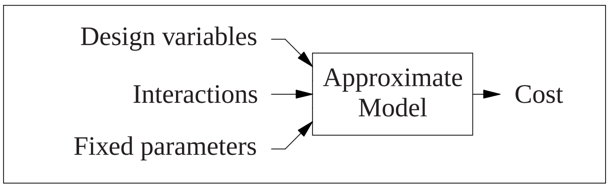


Fig. 4 Input and output of the approximate model.

Although this manner of interpolation is efficient for problems in statics a direct generalization to problems in dynamics is difficult. In many numerical strategies, the broadband response of a structure is obtained from harmonic calculations at a large number of frequency steps. Although it is possible to parameterize the interactions for one frequency at a time, using the Neural Network to approximate the solution at each frequency step is not feasible because training a

Neural Network with FE results of a large number of frequencies would require an unacceptable amount of time. Hence, an approach is needed where the parameterization of the interactions includes the frequency dependence. The approximate model must then be able to predict the way resonance frequencies are influenced by the location of the actuators and the degree of damping introduced by the control system. Although such parameterizations exist, the number of parameters needed is invariably larger than the number of dynamic modes taken into account. In practical problems, this leads to dozens of input parameters. It is expected that the number of FE simulations needed to obtain an accurate fit is so large that interpolation would be a burden instead of a benefit. Hence, a different approach is taken.

4 CMS-based optimization

In the proposed approach, the parametrization problem is avoided rather than solved. This is achieved by applying Component Mode Synthesis (CMS) [6]. CMS consists of two steps.

- Model reduction. Based on a FE model of a component or a remaining structure, a system of equations is generated that approximates the dynamical behavior predicted by the FE model, but with a very small number of degrees of freedom (DOF).

- Synthesis. Several reduced models are combined and the dynamical behavior of the structure as a whole is calculated. In the implementation of CMS for acoustical problems, it is convenient to include an acoustical calculation in this step, such that the result of synthesis is the total radiated sound power in a given frequency range.

It is convenient to explain these CMS based optimization strategies for the optimization of one component (i) and place them in the context of a global problem later. A simple way to apply CMS to such a component optimization is as follows (see figure 5). Prior to the optimization, a large number of reduced models is generated for component i, each with different design variables. In order to optimize this component, a reduced model of its remaining structure is generated and the synthesis process is applied to calculate the cost of each combination of a component model with the remaining structure. In this simple example, the synthesis result with the smallest cost is treated as optimal.

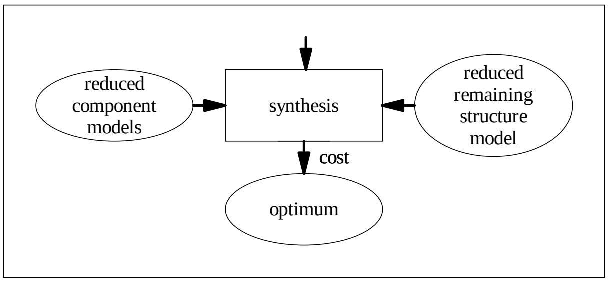


Fig. 5 CMS-based component optimization (example)

This approach is well-suited for global-local optimization. The optimization is started by assigning an some initial value to the design all variables of all components. Based on these design variables, FE models are generated for the remaining structures belonging to each component. Each component is then optimized by combining reduced models, which leads to new 'optimal' design variables for all components. Based on these designs, new FE models of the remaining structures are generated and a new component optimization is performed. This process is repeated until convergence.

It is clear that the parametrization of interactions is avoided in this technique. Instead, the component cost is re-evaluated each time the remaining structure changes. The computational cost of this strategy is quite acceptable because the amount of time and memory needed for the calculation of the component cost from two reduced models is very low.

Obviously, the component optimization strategy in this example must be refined in order to be practical. In essence, this is achieved by interpolating the component results and iteratively refining this approximation. In order to optimize one component (i), the following steps are taken.

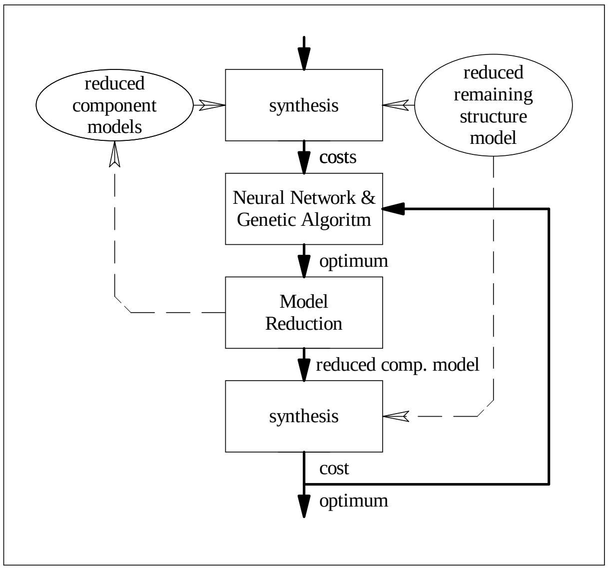


Fig. 6 CMS-based component optimization (proposed strategy)

Prior to the optimization, a small number of reduced models is generated for component *i*, each with different design variables. A reduced model of the remaining structure is generated and component optimization (see figure 6) begins. The reduced model of the remaining structure is synthesized with each available reduced component model. Based on the results, a NN-based approximate model is generated and the GA is used to find the optimal result in the approximate model. This 'optimal' design is subjected to model reduction and synthesis leading to the component cost. The approximate model is improved with this information and the GA is used to find the new minimum. This process is repeated until the optimum has been found.

An efficient global-local strategy is obtained by storing all reduced component models for use in the next component optimization. After the first component optimization, the second structure evaluation begins. This consists of generating a new reduced model of the remaining structure based on the optimal component designs (see figure 7). The synthesis process is then performed to combine this new reduced model with all available component models (see figure 6). Only this new data is used in the approximate model for this iteration: the approximate model

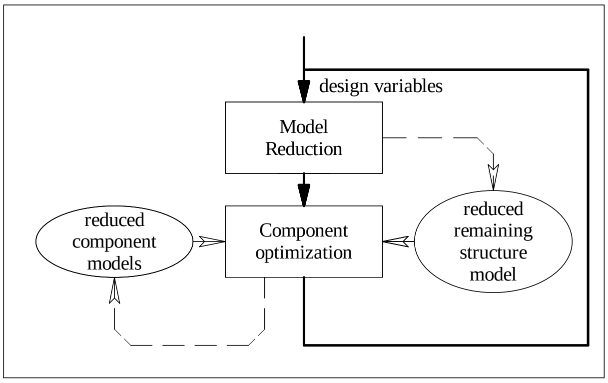


Fig. 7 CMS-based global-local strategy (for each component)

of the previous iteration is discarded. Again, the approximate model is iteratively improved. After component optimization has converged, a new structure evaluation begins. This iterative process is continued until convergence.

In conclusion, the NN may be thought of as a local model that predicts global quantities. Instead of interpolating between the results of earlier structure iterations, the approximate models are discarded after each structure iteration. However, at the beginning of a new structure iteration the approximate models are brought up-to-date with information of all reduced component models available, which includes the component design that was optimal in the previous structure iteration. Due to the speed of CMS, synthesizing all component models with the remaining structure requires less time than neural network training for the resulting data set.

5 Optimization results

The test problem introduced in section 1.2 is used to demonstrate the properties of the CMS based global-local optimization strategy. Two cases are presented. First, we present the result of an optimization where the excitation is broadband. Second: the convergence of the optimizer is studied in the case of a harmonic excitation. It will become clear that a large subset of all possible configurations can be characterized as optimal for engineering purposes. In order to test the effectiveness of the optimizer, both the design vari-

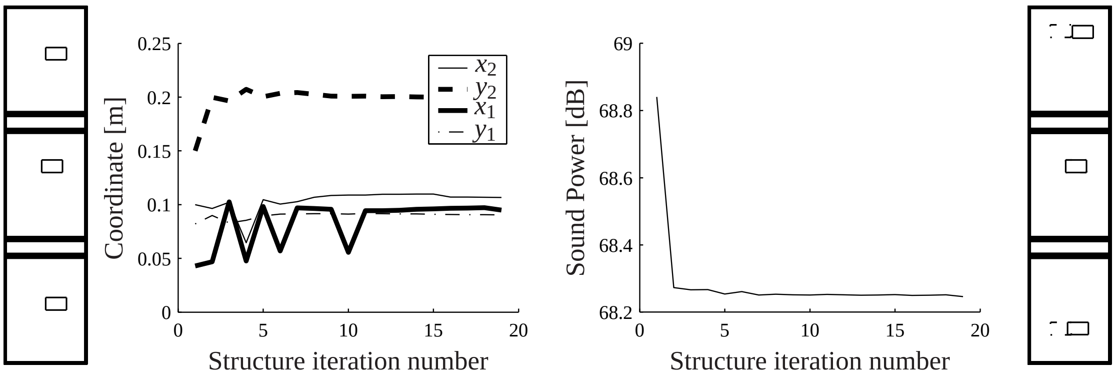


Fig. 8 Results of broadband optimization. From left to right: Initial configuration, design variables, radiated sound power and result (solid line) superimposed on reference (dash-dot).

ables and the accompanying cost must converge to the reference result. Hence, a difference of 0.1dB can be a large difference for the test result even though this difference is negligible in practice.

5.1 Broadband excitation

The results of an optimization under a broadband disturbance and a simple feedback control system are given in figure 8. The optimization is started with an initial configuration given on the left. In the second figure, the results the global-local optimization are displayed. Note that the convergence of the component optimizations themselves are not depicted. The results are depicted for component 1: the bottom plate. The figure gives the optimal values of the design variables: x_1 and y_1, and the design variables of component 2 (x_2, y_2) for which this result has been achieved. Note that x_2 and y_2 are in fact the result of the optimization of component 2 in the preceding iteration.

Initially, the x_1 and x_2 alternate between two values independently, but after iteration 11, the result has converged. In the third figure, the cost of the optimization result of component 1 is given. The radiated sound power does not change noticeably as the design variables x_1 and x_2 alternate between low and high values. This can be explained using a picture of the objective function of component 1 (see figure 10): there are two optima with a negligible difference in cost. From this and other results it is also found that the optimal value for the design variables of component 1 are almost independent of the design variables of component 2 which explains the fact that the radiated sound power has converged after the first iteration.

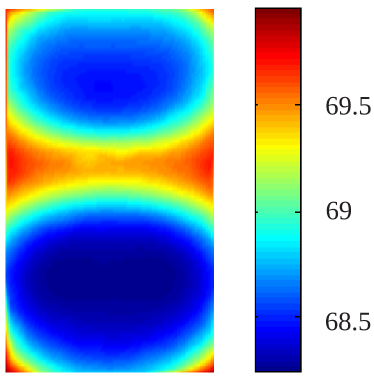


Fig. 10 objective function of component 1: radiated sound power (dB)

In order to obtain a reference result, the NN-based optimization strategy used for component optimization is applied to the structure as a whole. This reference is the mirror-image of the result of global-local optimization (dashed in figure 8). However, the global-local strategy outperformed the reference result with 68.24dB versus 68.4dB. A comment on the sub-optimal result of the reference is given in section 5.3.

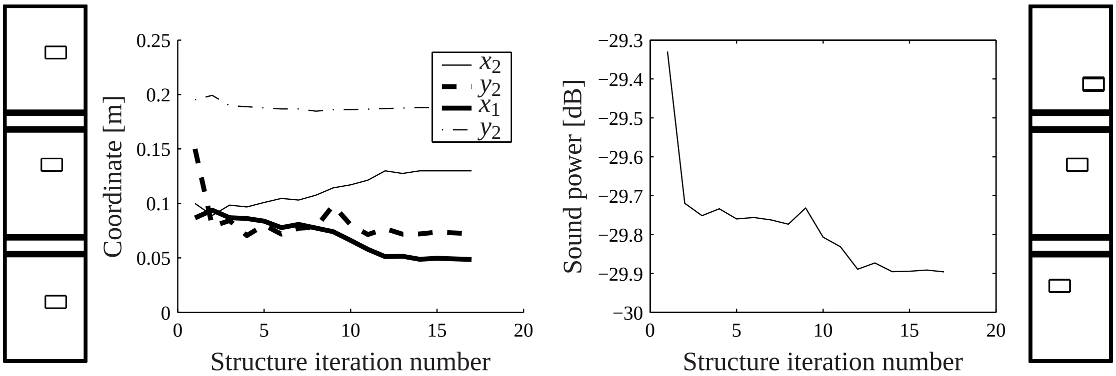


Fig. 9 Results of harmonic optimization. From left to right: Initial configuration, design variables, radiated sound power and result.

5.2 Harmonic excitation

The optimization is repeated for the same structure under a harmonic excitation and feed-forward control. Results are given in figure 9, where the radiated sound power is scaled to an excitation of one Volt. In this case, the optimal configurations are slightly more interdependent, which leads to a more gradual convergence. From the reference optimization, it is known that there is a vertical axis of symmetry in the objective function, leading to two minima in the objective function with a negligible difference in cost. The global-local optimization converged to one of these configurations and the (scaled) radiated sound power is -29.9dB, equal to the reference result.

Other results have indicated that the optimization can converge to the other optimum or not converge at all, depending on the initial design. Small deviations in the results of the earlier iterations also have a strong impact on the behavior in later iterations. Since each component optimization minimizes the *global* radiated sound power, a converged result is always at least a local optimum.

5.3 NN approximation

The reference result of the optimization under a broadband disturbance was found to be slightly worse than the result of global-local optimization. Although this NN-based optimization was continued for a large number of iterations, results in the vicinity of the true optimum were never evaluated. This is an indication that the optimization strategy is inefficient. Indeed, in some test-cases, all of the tested NN-based strategies were outperformed by a simple heuristic strategy that does not make use of any interpolation. The inefficiency is caused by the fact that an unnecessarily large data set is needed for an accurate interpolation. It is concluded that the NN interpolation technique must be further refined in order to be competitive for optimization purposes.

6 Conclusions

- A global-local optimization method for problems in structural dynamics has been proposed. The effectiveness of this method has been demonstrated for a test case in Active Structural Acoustic Control

- The objective functions and constraints on the component level must be chosen with some care because, in general, the optimization will converge to a sub-optimal result. A good choice is the case where all components optimize the same (global) objective function and only side constraints are present. In this case, a converged result is guaranteed to be at least a local optimum.

7 Recommendations

- The NN-based approximate model must be further refined in order to be competitive.

- Both the theory and the optimization method can be modified to include constraints on a global level.

8 Acknowledgements

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