Bus Operations Optimization: A Literature Review on Bus Holding, Rescheduling and Stop-skipping

Dr Konstantinos Gkiotsalitis
Assistant Professor
University of Twente

Center for Transport Studies (CTS)
Department of Civil Engineering
P.O. Box 217
7500 AE Enschede
The Netherlands
Email: k.gkiotsalitis@utwente.nl

99th Annual Meeting of the Transportation Research Board

Paper No: 20-00414

ABSTRACT
In this literature review, we systematically review studies on public transit control with a specific focus on bus operations. In our synthesis of the relevant literature, we consider three perspectives: (1) the mathematical models of the proposed methodologies; (2) their complexity; (3) their applicability in real-time operations; and (3) their advantages/disadvantages considering their practical implications. The reviewed dynamic control methods include bus holding, rescheduling, and stop-skipping. Dynamic control methods, that have attracted more attention in recent years due to the advancements in automation and data availability, try to alleviate the negative effects of service disruptions. However, dynamic transit control tends to improve the service regularity and the operational costs at the expense of certain groups of riders. Identified problems are the inconvenience of onboard passengers when a bus is held at a stop, the long waiting times of "stranded" passengers in the case of stop-skipping, and the "sliding" of the crew and vehicle schedules in the case of rescheduling. All those critical issues are discussed and future research directions are drawn considering the complexity of the mathematical models for dynamic control and their practical implications.

Keywords: dynamic stop-skipping; bus holding; rescheduling; dynamic control; public transit optimization.

INTRODUCTION

Decisions regarding the operations of bus services are made at different planning stages. At the tactical planning stage, one has to determine the frequency Yu et al. (*1*), the timetable Sun et al., Wu et al. (*2, 3*), and the crew and vehicle schedules Wren and Rousseau, Gintner et al., Kliewer et al. (*4, 5, 6*) of every bus line. Tactical plans are communicated well in advance, and all stakeholders (i.e., public transport authorities/operators, bus drivers, passengers) are aware of them prior to the start of the daily operations Ceder (*7*). At the operational stage, which is also the focus of this study, dynamic control approaches such as bus bunching, rescheduling or stop-skipping are frequently deployed to react to operational disturbances.

Dynamic control is needed in transit operations to alleviate the adverse effects of bunching, schedule sliding, and overcrowding. Bus bunching (also known as platooning), schedule sliding and overcrowding occur due to the spatio-temporal variation of travel times and passenger demand. For instance, an increase of the inter-station travel time of a trip might increase its time headway with its preceding trip resulting in a "domino effect". In this "domino effect", the delayed bus trip falls behind schedule and has to serve more passengers at every stop Hans et al., Hans et al. (*8, 9*). More passenger boardings lead to dwell time increases for the delayed bus which will inevitably result in bus bunching. Bunching has been found to have a significant negative effect on operating costs (a first attempt to quantify such costs was made by Strathman et al. (*10*) using the computer-aided bus dispatching system of Tri-Met). Additionally, longer-term implications such as a general dissatisfaction of passengers, lost patronage and lost revenue have been quantified in Abkowitz and Tozzi, Clotfelter (*11, 12*).

Alleviating bunching, overcrowding, and the sliding of schedules is a fundamental objective of advanced transit management systems. However, given that the cause of such phenomena is the stochastic nature of inter-station travel times and passenger demand, solving such problems at the strategic or the tactical planning level is not suitable Gkiotsalitis and Alesiani (*13*). Therefore, real-time "corrective" control measures are needed to react continuously to the travel time and passenger demand variations. Because of that, there is an upsurge on research and development in this field which is supported by the recent technological advances and the high penetration rates of telematics that produce automated vehicle location (AVL) data, automatic passenger counting (APC) data, and automated fare collection (AFC) data.

The data availability is not only instrumental in observing the bus operations and performing corrective actions in real-time, but is also a monitoring tool for the transport authorities that can oversee and assess the performance of operations. This has strong implications and has resulted in new business models where transport authorities provide monetary incentives to bus operators that perform well (and vice versa). For instance, the Land Transport Authority (LTA) in Singapore offers 6,000 Singaporean dollars for each 0.1-minute improvement of the excessive passenger waiting times Leong et al. (*14*). Consequently, the data availability that enables the monitoring of the transit operations has brought new challenges to the transit management which bears the responsibility of ensuring a seamless service. Under this context, transit management teams have exercised real-time control measures to improve their services Trompet et al. (*15*). Do, however, real-time control measures always improve the operational and passenger-related costs? Is the inflicted cost to external actors acceptable? Is it possible to develop models that provide an accurate representation of reality and can be solved in real-time for obtaining the appropriate control measures?

Two strands of research have tried to answer the aforementioned questions following two different approaches. One tries to incorporate the stochastic nature of the travel times and passenger demand at the dynamic control, e.g. Hickman, Berrebi et al. (*16, 17*). The other tries to limit the stochastic elements of the travel times and passenger demand by solving deterministic optimization problems in very short time-horizons, e.g. Eberlein et al. (*18*). Independent of which strand is chosen, the actual performance of the selected control measures depends on the travel time and passenger demand variability levels which, in most cases, are case study-specific.

Developing real-time control methods for bus operations has been the focus of numerous research papers over the past few years. The related literature has compelling evidence to support the implementation of real-time control measures. Notwithstanding, each control measure (i.e., holding, speed control, rescheduling, stop-skipping, traffic signal priority) inflicts different negative effect(s) to the actors of the system. Besides, the application of real-time control measures does not always guarantee the improvement of operations. Evidence from the related literature shows that if the travel time and passenger demand estimates are unreliable, the suggested control measures might have a negative impact Fu and Yang, Gkiotsalitis and Maslekar (*19, 20*). In addition, little is known about whether the inclusion of classic traffic theory models into bus control methods can limit the unpredictability of inter-station travel times and increase the chances of improving the bus operations Chow et al., Sirmatel and Geroliminis (*21, 22*).

This paper reviews and synthesizes works that cover the aforementioned research areas. Through a systematic literature review, we perform a comprehensive search of related works, identify contradictory research, and discuss under which conditions one method is expected to perform better than another. In this review we seek to answer the following questions:

- which are the side-effects of common real-time control methods on the actors of the system?
- is there potential to combine different real-time control methods?
- which are the most common models of the different real-time control methods? do they offer an accurate representation of reality and can they be efficiency solved (i.e., to global optimality) in real-time?
- which are the most common solution methods?
- what should be the direction of future research?

The remainder of this paper is as follows. In section 2, we provide a general overview of common control methods for bus operations. In section 3, we review and synthesize the works on bus holding. In section 4, we review the works on rescheduling. In section 5, we review the works on stop-skipping and mixed control approaches that integrate stop-skipping with bus holding or short-turning. Finally, in section 6 we conclude the paper and provide future research directions.

GENERAL OVERVIEW OF DYNAMIC CONTROL METHODS

Common corrective actions during the daily operations include stop-skipping (Sun and Hickman, Chen et al., Yu et al., Liu et al. (*23, 24, 25, 26*)), bus holding at specific stops (Berrebi et al., Newell, Hernández et al., Wu et al., Gavriilidou and Cats (*17, 27, 28, 29, 30*)) or rescheduling (Strathman et al., Adamski and Turnau (*10, 31*)). A distinct line of works tried to combine stop-skipping and bus holding Eberlein, Lin et al., Cortés et al., Sáez et al. (*32, 33, 34, 35*). In addition, Gkiotsalitis et al. (*36*) proposed a genetic algorithm that recommends short-turning and interlining options using the concept of virtual lines, whilst Gkiotsalitis and Maslekar (*37*) proposed a stochastic optimization method for re-scheduling and bus holding. Muñoz et al. (*38*) intertwined speed control with bus holding. Lastly, Cortés et al. (*39*) integrated short turning and deadheading. Despite the above, given the computational complexity of each problem and the requirement of computing an optimal control strategy in quasi-real-time, different operational control approaches are typically applied in isolation.

Apart from bus holding, rescheduling and stop-skipping, inter-station control can also be an option. Typical inter-station control strategies are traffic signal priority Skabardonis, Liu et al., Koehler and Kraus Jr, van Oort et al. (*40, 41, 42, 43*) and speed control Muñoz et al., Daganzo and Pilachowski, Wang et al., Ampountolas and Kring (*38, 44, 45, 46*). We note though that inter-station control strategies have received relatively little attention compared to holding, rescheduling and stop-skipping methods in the public transport literature. Therefore, they will not be our main priority in this literature review.

Considering bus holding, stop-skipping and rescheduling, we note that each control method has its own adverse effects. Stop-skipping increases the inconvenience of passengers who cannot board the bus that skips their stop Liu et al., Fu et al. (*26, 47*). Bus holding increases the inconvenience of onboard passengers who wait while the bus is held at the stop(s) Fu and Yang (*19*). Finally, rescheduling can affect the crew/vehicle schedules and result in schedule sliding.

In terms of key performance indicators, the main challenge in high-frequency services is to maintain the planned headways among buses at every bus stop Trompet et al., Cats (*15, 48*), whereas in low-frequency services is to adhere to scheduled arrival times at stops Trompet et al., Randall et al. (*49, 50*). If the demand and the travel times of all bus trips that operate in a service line are equal and stable, bus trips will maintain their even headways at all downstream stops. This will result in a regular service where the actual waiting times at stops meet the passengers' expectations. Nevertheless, travel time and passenger demand variations during the actual operations result in unreliable and inconsistent services Chen et al., Daganzo (*51, 52*). Knoppers and Muller, Berrebi et al. (*53, 54*) and Knoppers and Muller (*53*) have shown that the fixed service intervals cannot be maintained at all stops. Indeed, even if buses are dispatched according to their planned headways, their headways are expected to deviate from their scheduled values as they are moving towards downstream stops Hans et al. (*9*).

To address the adverse effects of the demand and travel time variability, several periodic optimization approaches have emerged over the past 40 years. Periodic optimization approaches of the bus operations consider multiple decision variables and are based on iterative, finite-horizon optimization(s) of the bus operations. At time t, the current state of the bus operations (i.e., current positions of running trips) is used as input and, together with the expected travel times within a relatively short time horizon $t+T$, the control measures (i.e., holding, stop-skipping, rescheduling) of multiple trips are determined. We note here that there are two main issues with the periodic optimization methods:

- if the number of bus trips, $i = \{1,2,...,\}$, that belong to the periodic optimization time period $t+T$ is too big, determining the appropriate control measures of all those trips results in complex, multivariable optimization problems that cannot be solved in real-time Hickman, Sánchez-Martínez et al. (*16, 55*). Additionally, as in model predictive control (MDP), if we determine the control measures of multiple trips many of them will not have the chance to be implemented in practice by the time new information becomes available Nikolaou (*56*);

- if the optimization horizon is too short and we only control one trip at a time, our decisions are myopic. Hence, we might improve the performance of one trip, but deteriorate the performance of the overall operations Eberlein et al. (*18*).

When defining the control measures of bus operations in real-time, a balance should be established between considering multiple future trips or focusing on one trip at a time. This is prevalent in the literature review of bus holding, rescheduling, and stop-skipping methods that is presented in the following sections. Finally, because the focus of this literature review is on mathematical models for real-time control, we briefly present some basic terminology with regards to mathematical optimization:

- continuous convex program: it is a mathematical program with convex objective function and convex feasible set. In continuous optimization, a convex program can be typically solved to global optimality with conventional exact optimization solvers because every locally optimal solution derived by a solver is also a globally optimal one;
- continuous non-convex program: it is a mathematical program that has a non-convex objective function or non-convex feasible set. Such program may have multiple feasible regions and multiple locally optimal points within each region; thus, it is not always possible to guarantee the convergence to a globally optimal solution;
- discrete (or integer) program: it is a mathematical program with discrete decision variables (very common in rescheduling and stop-skipping problems). To solve such programs to global optimality might require the exploration of the entire solution space;
- mixed-integer program: it is a mathematical program which has both discrete and continuous decision variables (i.e., a mixed-integer formulation can be used for the combined stop-skipping and bus holding problem);
- heuristic: a solution method that returns a solution to a mathematical program by exploring only a fraction of the solution space without guaranteeing global optimality.

BUS HOLDING

Control methods for bus holding have been studied since the early 1970s (see Newell, Osuna and Newell (*27, 57*)). Nevertheless, the bus holding problem remains a prominent research topic because of its inherent complexity. Newell (*27*) considered only one control point at which buses can be intentionally delayed, and devised a strategy for holding a bus to minimize the average waiting time of the passengers.

Typical objectives of bus holding methods are headway adherence to the schedule Rossetti and Turitto, Gkiotsalitis and Cats (*58, 59*), headway regularity Daganzo, Bartholdi and Eisenstein (*52, 60*), and the minimization of passenger waiting and in-vehicle times Sáez et al., Delgado et al., Delgado et al. (*35, 61, 62*). It should be noted here that, as a general practice, buses are not held at every stop because this will increase the passenger inconvenience. In contrary, bus holding is only allowed at a pre-determined sub-set of important bus stops, known as intermediate time points (ITPs) or control points Cats et al. (*63*).

In bus holding, two different directions of research have emerged. One direction models the bus holding problem as a periodic optimization, rolling horizon problem where decisions about the holding times concern the entirety of trips that will operate in a short horizon, $t+T$. To achieve that, information about the current trajectories of bus trips and their predicted values in the short future is incorporated in the respective mathematical programs Eberlein et al., Gkiotsalitis and Maslekar (*18, 37*). In this line of research, the bus holding problem is typically modeled as a mathematical program with multiple decision variables, and collective decisions are made.

The second direction of research determines the holding time of a single trip when it arrives at a control point stop. Because every time they decide the holding time of a single trip, such approaches yield single variable optimization problems. Such single variable optimization problems lead to closed-form expressions that can determine the holding time of a trip in real-time without the need of solving a complex mathematical program Hickman, Fu and Yang, Van Oort et al. (*16, 19, 64*). With such approaches, when determining the holding time of a single trip, n, the decision is typically made when it arrives at a control point stop s based on:

- its actual time headway with its preceding trip, $n-1$, (one-headway-based control logic)
- its actual time headway with its preceding trip, $n-1$, and its expected time headway with its following trip, $n+1$, (two-headway-based control logic)

This can be understood with the use of the time-space diagram in Fig.1, where:

- H_s is the target headway (i.e., scheduled headway that we need to adhere to),
- t the time when trip n has completed all its boardings/alightings at stop s and is ready to depart (in case no holding is applied),
- $d_{n-1,s}$ the actual departure time of trip $n-1$ from stop s,
- $d_{n,s}$ the potential departure time of trip n from stop s,
- $\tilde{d}_{n+1,s}$ the expected departure time of trip $n+1$ from stop s,
- $(d_{n,s}-t)$ the potential holding time of trip n at stop s.

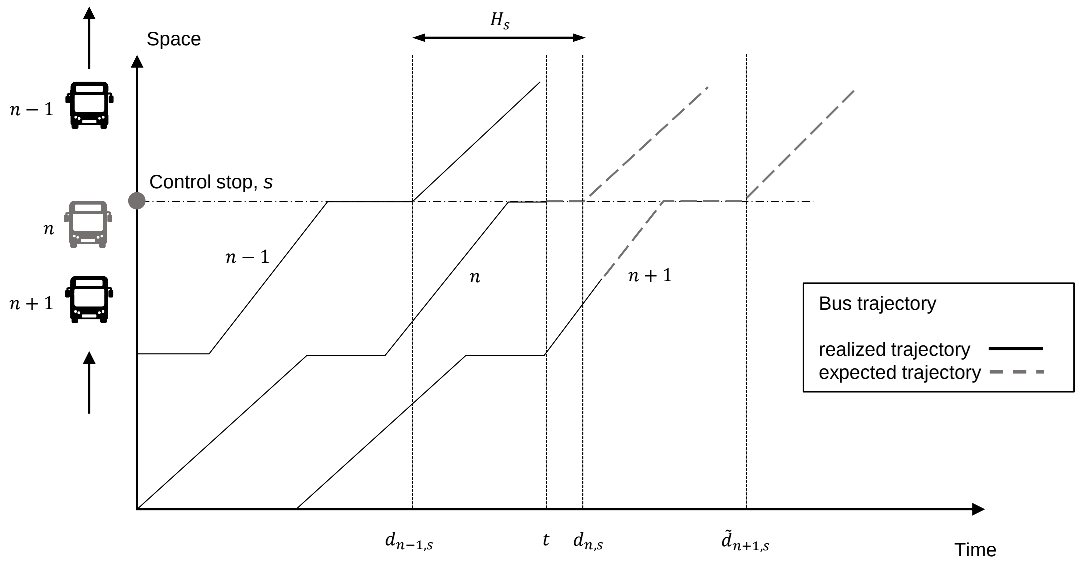

FIGURE 1 : Time-space diagram of the realized and expected trajectories of three successive bus trips

In the one-headway-based control logic, the holding time of trip n when it arrives at stop s depends on the target headway, H_s, the time when it finishes its boardings/alightings and is ready to depart, t, and the departure time of its preceding trip from stop s, $d_{n-1,s}$. A simple closed-form expression that can determine the holding time $h_{n,s}$ of trip n at stop s is:

$$h_{n,s} = \begin{cases} 0 & \text{if } t - d_{n-1,s} \geq H_s \\ H_s - (t - d_{n-1,s}) & \text{otherwise.} \end{cases} \tag{1}$$

This simple closed-form expression allows bus n to depart immediately after finishing its boardings/alightings (no holding) if $t - d_{n-1,s} \geq H_s$ which indicates that trip n is left behind and needs to catch up with its preceding trip. On the contrary, if trip n is closer to its preceding trip than the scheduled time headway H_s when it finishes its boardings/alightings, it is held at stop s for time $H_s - (t - d_{n-1,s})$ to meet again the target headway. Given that holding trip n for time $H_s - (t - d_{n-1,s})$ might not always be desirable because it increases the travel time of trip n, a weight $0 \leq a \leq 1$ can be applied to $H_s - (t - d_{n-1,s})$ resulting in $a\big(H_s - (t - d_{n-1,s})\big)$. Intuitively, if $a = 0$ bus n is never held, whereas if $a = 1$ bus n is held for time $H_s - (t - d_{n-1,s})$ to adhere again to the target headway. Any other value of a in the range $[0,1]$ determines how strong is our holding control.

Considering now the two-headway-based control logic, the holding time of trip n depends on H_s, $d_{n-1,s}$, and the expected departure time of its following trip $n+1$ at stop s, $\tilde{d}_{n+1,s}$. For instance, a two-headway-based control logic can be expressed as follows:

Algorithm 1 Two-headway-based holding control logic Fu and Yang (*19*)

0: If $t < d_{n-1,s} + H_s$, then:

1: If $\frac{1}{2}(\tilde{d}_{n+1,s} - d_{n-1,s}) < H_s$, then:

2: $d_{n,s} = d_{n-1,s} + H_s$

3: Else:

4: $d_{n,s} = d_{n-1,s} + [\frac{1}{2}(\tilde{d}_{n+1,s} - d_{n-1,s}) + H_s]/2$

5: Else:

6: $d_{n,s} = t$

In the remainder of this section, we discuss the past literature on the multivariable bus holding optimization approaches and the single variable optimization approaches in sub-sections 3.1 and 3.2. In 3.1 and 3.2, a distinction is made between approaches that consider the vehicle capacity in the optimization process and the ones that do not.

Bus Holding without considering the vehicle capacity

We hereby discuss the main multivariable and single variable bus holding optimization methods that do not consider the vehicle capacity limits in the optimization process. Multivariable bus holding approaches that try to determine the holding times of multiple bus trips might not be able to provide a solution in real-time due to the complexity of the respective mathematical programs. For this reason, we report past works on multivariable and single variable mathematical optimization in the separate sub-sections 3.1.1 and 3.1.2.

Multivariable Mathematical Programs

Examples of multivariable bus holding optimization methods are the periodic optimization mathematical programs of Eberlein et al., Eberlein, Sánchez-Martínez et al., Shen and Wilson (*18, 32, 55, 65*). Such mathematical programs determine simultaneously the holding times of all buses that are expected to operate within a rolling horizon. The optimized holding times are updated in rolled rolling horizons when new information becomes available.

Eberlein et al. (*18*) considered real-time information and assumed that travel times and passenger arrival rates remain constant in rolling horizons with short time duration. The holding problem of all running buses was modeled as a quadratic program that minimizes the total passenger waiting times. Gkiotsalitis and Maslekar (*37*) used a metaheuristic from the area of evolutionary optimization to solve an NP-Hard program that returns holding times which minimize the waiting times of passengers under regulatory constraints.

Zolfaghari et al. (*66*) developed a mathematical control model for holding using real-time information of locations of buses along a specified route and the resulting mathematical program was solved with metaheuristics (specifically, simulated annealing). Alesiani and Gkiotsalitis (*67*) followed a different approach that used reinforcement learning to determine the holding times of multiple trips instead of solving a mathematical program. Gkiotsalitis and Cats (*59*) and Gkiotsalitis and Cats (*68*) used alternating optimization and branch and bound to determine the holding times of multiple trips in a rolling horizon.

Sun and Hickman (*69*) discretized the holding times by allowing 5-sec increments and returned a solution with simple enumeration. Later, Sun and Hickman (*70*) formulated the bus holding problem considering the passenger waiting times and in-vehicle delays as a convex quadratic programming problem and used a decomposition-based heuristic for its solution. The effectiveness of the heuristic was tested in a hypothetical numerical example.

Most of the above-mentioned models resort to heuristics to solve the mathematical programs due to the complexity of the formulations and the non-polynomial computational costs. Typically, models solved with exact methods exhibit scalability issues when the number of problem variables increases, whereas models solved with heuristics result in large optimality gaps.

Single variable approaches: Rule-based methods and Analytic Solutions

In this sub-section, we report works that determine the holding time of a single bus at a time and do not consider the vehicle capacity in the optimization process. Fu and Yang (*19*) tested two of the most common rule-based bus holding strategies: (i) the one-headway-based control where a bus is held at a control point stop if its time headway with its preceding bus is lower than a pre-defined threshold; and (ii) the two-headway-based control that considers the time headway of a bus with its preceding and following bus.

Xuan et al. (*71*) proposed a simple control strategy that has a closed-form expression for holding one trip at a time. Even if its focus was on speed control, we also report the work of Daganzo and Pilachowski (*44*). Daganzo and Pilachowski (*44*) proposed an adaptive control scheme that adjusts a bus cruising speed in real-time based on both its front and rear spacings. In line with other closed-form approaches, it had a simple and decentralized logic enabling to correct the effect of traffic disruptions in real-time.

Bartholdi and Eisenstein (*60*) proposed an analytic bus holding solution which changes the headway of each newly arrived bus at a stop to the weighted average of its former headway and that of the trailing bus. This approach tends to re-equalize the headways after any disturbance. Its major difference from other works is that its objective is to maintain the headway regularity and not to adhere to a scheduled (target) headway. Thus, a rule-based, headway threshold is not triggering the bus holding. Instead, the holding decisions constantly adjust and re-equalize the headways.

Berrebi et al., Berrebi et al. (*17, 54*) proposed a method consisting of identifying probabilistically the bus that will arrive the latest to a particular point. Then, each preceding bus is held to prevent the lagging bus from departing with a big gap. Van Oort et al. (*64*) also tested schedule-based and headway-based holding strategies where the solution was expressed as a closed-form expression of arrival times and scheduled headways. They tested the importance of setting a maximum holding time and a reliability buffer time in tram line 9 in The Hague.

Wu et al. (*29*) incorporated the passenger demand into the estimation of bus trajectories and addressed the single variable bus holding problem with the use of the one-headway-based holding logic. In the one-headway-based holding of Wu et al. (*29*), a bus is held if the headway with its preceding bus is less than the scheduled headway - otherwise, it is dispatched immediately. Although Wu et al. (*29*) incorporates the demand and the capacity of vehicles in the calculation of bus dwell times, the objective of the simplistic, one-headway-based control logic does not consider the improvement of busloads and focuses merely on the service regularity. For this reason, this study is assigned to the category of studies that do not use the violation of capacity limits as an optimization objective.

Hickman (*16*) used the stochastic model developed by Marguier (*72*) for deriving the trajectories of buses on a single route. Using Marguier's model, Hickman (*16*) developed a bus holding algorithm that is applied each time a bus arrives at the control point stop. To this end, Marguier's model was used to approximate the trajectories of all "upstream" buses. The optimal holding time was computed using a line search solution method because obtaining an analytic solution was not possible given the complexity of deriving the first-order conditions of the optimization problem.

Bus Holding Methods that consider the Vehicle Capacity limits

Previously, we reviewed bus holding methods that do not account for the passenger demand and vehicle capacity limitations in the optimization process. In this sub-section, we review past works that consider the capacity limitations in the optimization process. Although we do not focus on rail operations, we note here the work of Puong and Wilson (*73*) who developed a mixed-integer program for holding trains while considering capacity limitations.

Sánchez-Martínez et al. (*55*) formulated a mathematical model to produce a plan of holding times for all running vehicles in a rolling horizon that caters for the passenger demand. Its effectiveness was evaluated within a simulation environment. The objective function in that model was not convex and did not allow the derivation of an analytic solution. Instead, Sánchez-Martínez et al. (*55*) employed the optimization algorithm of Powell (*74*) to derive local minima of the nonlinear objective function. Computational times were also prohibitive for instances with many control points and a large fleet size.

Delgado et al. (*61*) developed a mathematical program that incorporates vehicle-capacity constraints. As in Sánchez-Martínez et al. (*55*), they calculated the holding times of all vehicles in a rolling horizon resulting in a multivariable decision problem. In a later work, Delgado et al. (*62*) also addressed the problem of determining the holding times of all running buses on a rolling horizon. Their objective was to minimize the total times experienced by all passengers in the system resulting in a non-convex, nonlinear objective function that cannot be (always) solved to global optimality. Then, they performed a simulation-based evaluation of two control policies applied within a rolling horizon framework: (i) vehicle holding that does not consider boarding limits, and (ii) holding combined with boarding limits, in which the number of boarding passengers at any stop can be limited. The respective mathematical programs were solved using MINOS as an optimization solver, similarly to the work of Hernández et al. (*28*).

Finally, Sáez et al. (*35*) proposed a hybrid control approach for both bus holding and stop-skipping resulting in a mixed-integer non-linear program. Sáez et al. (*35*) utilized a dynamic objective function and a predictive model of the bus system to make decisions on bus holding and stop-skipping. The uncertain passenger demand was included in the model as a disturbance, and the resulting NP-hard problem was solved using an ad-hoc implementation of a genetic algorithm.

The aforementioned works are the main ones that incorporate capacity limitations in the bus holding optimization process. Nevertheless, they do not offer an analytic solution, and the proposed multivariable mathematical programs cannot be solved to global optimality in real-time due to their complexity and their non-convex nature. To summarize the bus holding literature, in Table 1 we present a summary of selected bus holding works and analyze their characteristics in terms of number of decision variables, objective function, mathematical program properties, stochasticity, solution method, and consideration of vehicle capacity.

TABLE 1 : Summary of selected bus holding methods

Study	Decision Variables	Objective function	Mathematical Program	Stochasticity	Solution Method	Capacity
Sánchez-Martínez et al. (*55*)	multivariable	waiting time and in-vehicle delay	non-convex	ignored	Algorithm of Powell (*74*)	considered
Delgado et al. (*62*)	multivariable	waiting time, in-vehicle delay and extra waiting time of stranded passengers	non-convex	ignored	MINOS solver	considered
Hickman (*16*)	single variable	waiting time and in-vehicle delay	convex	considered	line search	ignored
Eberlein et al. (*18*)	multivariable	waiting time	non-convex	ignored	purpose-built heuristic	ignored
Sun and Hickman (*70*)	multivariable	waiting time and in-vehicle delay	convex	ignored	decomposition-based heuristic	ignored
Zolfaghari et al. (*66*)	multivariable	waiting time and extra waiting time of stranded passengers	convex	ignored	simulated annealing	considered
Puong and Wilson (*73*)	multivariable	waiting time, in-vehicle delay and extra waiting time of stranded passengers	non-convex	ignored	decomposition-based heuristic	considered
Zhao et al. (*75*)	single variable	waiting time and in-vehicle delay	non-convex	considered	multiagent negotiation heuristic of Sandholm (*76*)	ignored
Wu et al. (*29*)	single variable	waiting time	non-convex	ignored	first-depart-first-hold rule	considered
Bartholdi and Eisenstein (*60*)	single variable	equalize headways	no program	ignored	closed-form expression	ignored
Berrebi et al. (*17*)	multivariable	waiting time	no program	considered	dynamic programming	ignored
Fu and Yang (*19*)	single variable	waiting time	no program	ignored	closed-form expression	ignored
Gkiotsalitis and Cats (*59*)	multivariable	waiting time and in-vehicle delay	non-convex	ignored	branch and bound	ignored
Sáez et al. (*35*)	multivariable	waiting time, in-vehicle delay and extra waiting time of stranded passengers	non-convex	considered	genetic algorithm	considered
Xuan et al. (*71*)	single variable	guaranteeing a maximum standard deviation from the schedule	non-convex	considered	closed-form expression	ignored
Hernández et al. (*28*)	multivariable	waiting time, in-vehicle delay and extra waiting time of stranded passengers	non-convex	ignored	MINOS solver	considered

RESCHEDULING

The rescheduling of bus services focuses on the modifications of the dispatching times of future trips. Unlike bus holding, rescheduling is allowed to modify the departure time of a trip at a single stop (the first stop). Potential stop-skipping and/or bus holding actions can be easily applied on top of rescheduling as soon as the buses are en route Cats et al., Zhao et al. (*63, 75*).

Most works model rescheduling as an integer mathematical program where the decision variables are the dispatching times of bus trips. Due to their discrete nature, such programs cannot be easily solved and many scheduling approaches try to produce robust schedules that can perform well in case of travel time and passenger demand disruptions to avoid numerous rescheduling(s) Gkiotsalitis and Alesiani, Tang et al. (*13, 77*).

Apart from controlling the bus operations, rescheduling is also commonly applied in train operations where is a common disturbance management strategy. Rescheduling solutions in train operations commonly adopt local re-timing to adjust the timetable, see D'Ariano et al., Corman et al., Meng and Zhou (*78, 79, 80*). D'Ariano et al. (*78*) aimed at improving the punctuality of trains by routing and sequencing trains in an iterative manner - first, an optimal train sequencing was produced for the given train routes, and then this solution was improved by locally rerouting some trains. Their solution method was based on local search and branch and bound (B&B) given the discrete nature of the problem. This work was extended in Corman et al. (*79*) by incorporating effective rescheduling algorithms and local rerouting strategies in a Tabu search scheme. Corman et al. (*79*) alternated between a fast heuristic and a truncated B&B algorithm for computing train schedules within a short computation time without guaranteeing the convergence to a globally optimal solution. Pellegrini et al. (*81*) aimed at minimizing delays after an unexpected disturbance perturbs the operations by seeking the best train routing and scheduling. Pellegrini et al. (*81*) proposed a mixed-integer linear program which was solved in more than one and a half minute, even with the use of heuristics. Krasemann (*82*) introduced a greedy heuristic to ensure that a (hopefully) good-enough rescheduling solution is obtained within a short time (within 30 seconds). To this end, Krasemann (*82*) introduced directly a heuristic solution method without modeling the timetable rescheduling problem as a mathematical program.

In bus operations, several works have considered dynamic rescheduling for adjusting to the travel time and passenger demand variations. Bly (*83*) used rescheduling on depleted bus services to provide equal headways for the available buses in the schedule. Gkiotsalitis and Stathopoulos (*84*) proposed a rescheduling strategy that modifies the dispatching times of bus trips to match the passenger demand of individuals who want to participate in joint activities using a genetic algorithm. Li et al. (*85*) modeled and solved the single depot bus rescheduling problem in pseudo-polynomial time using a parallel auction algorithm. In a follow-up work, Li et al. (*86*) showed that the bus rescheduling problem is NP-hard, and used a Lagrangian relaxation-based insertion heuristic for its solution.

Given that rescheduling alone results in marginal improvements, several works have coupled rescheduling with additional control measures. Mirchandani et al. (*87*) coupled rescheduling with signal priority to improve the service regularity after a disruption. Gkiotsalitis and Maslekar (*88*) and Gkiotsalitis and Maslekar (*89*) combined rescheduling and bus holding using an integer program formulation that was solved with a sequential evolutionary optimization heuristic.

Finally, we should note that rescheduling is used for synchronization among services to reduce the transfer waiting times of passengers. Cevallos and Zhao (*90*) and Cevallos and Zhao (*91*) proposed simple perturbations by merely shifting the pre-existing timetables to solve the aforementioned problem and resorted into a genetic algorithm given the computational complexity of the problem. In addition, Coffey et al. (*92*) treated the synchronization problem as a demand-supply matching problem. In their approach, they rescheduled the timetables of public transport modes by matching the passenger demand expressed via journey planners with the public transport supply to reduce missed connections. A summary of selected rescheduling methods discussed in this section is provided in Table 2.

TABLE 2 : Summary of selected rescheduling works

Study	Problem	Mathematical Program	Stochasticity	Solution Method
Gkiotsalitis and Stathopoulos (*84*)	rescheduling	integer nonlinear	no	genetic algorithm
Li et al. (*85*)	rescheduling	integer linear	no	parallel action algorithm
Li et al. (*86*)	rescheduling	integer linear	no	Lagrangian relaxation-based heuristic
Mirchandani et al. (*87*)	rescheduling and signal priority	macroscopic model	no	heuristic
Gkiotsalitis and Maslekar (*88*)	rescheduling and bus holding	integer nonlinear	no	sequential evolutionary optimization heuristic
Cevallos and Zhao (*91*)	rescheduling considering passenger transfers	integer program	no	genetic algorithm
Coffey et al. (*92*)	rescheduling considering passenger transfers	linear program	no	CPLEX

STOP-SKIPPING

In this section, we specifically focus on stop-skipping control of bus operations. An alternative of stop-skipping is "refused boarding" where passengers are not allowed to enter the bus after a certain time Desaulniers and Hickman (*93*). Stop-skipping can correct service inconsistencies due to the inherent travel time and passenger demand variations, but might result in increased waiting times at the locations of the skipped stops Chen et al. (*24*). Thus, most stop-skipping approaches address the problem holistically considering the waiting times of passengers, their in-vehicle times, and the total bus trip travel times. The two former objectives concern the passenger-related costs, whereas the last objective concerns the cost of the operator.

Addressing the stop-skipping problem at the operational level requires to compute a stop-skipping solution in near real-time. Given the computational complexity of the stop-skipping problem, a line of research considers the stop-skipping strategy of only one trip at a time to reduce the size of the solution space Liu et al., Fu et al. (*26, 47*). Such treatment enables the computation of a stop-skipping solution for typical bus services with less than 20 stops, but results in myopic control options because it addresses every bus trip in isolation without acknowledging that it belongs to a chain of trips Bartholdi and Eisenstein (*60*).

Inherently, stop-skipping is a binary, 0-1 optimization problem where a stop can be skipped (0) or served (1). If we consider the single trip that operates in the bus line of Fig.2 and can skip/serve every stop, the solution space of potential skip/serve options is $2^{|S|}$ where $|S|$ is the total number of bus stops. This exponential increase of the solution space with the number of stops allows evaluating all solution space options with the use of brute-force for mid-size bus lines that typically do not exceed 20 stop-skipping candidate stops.

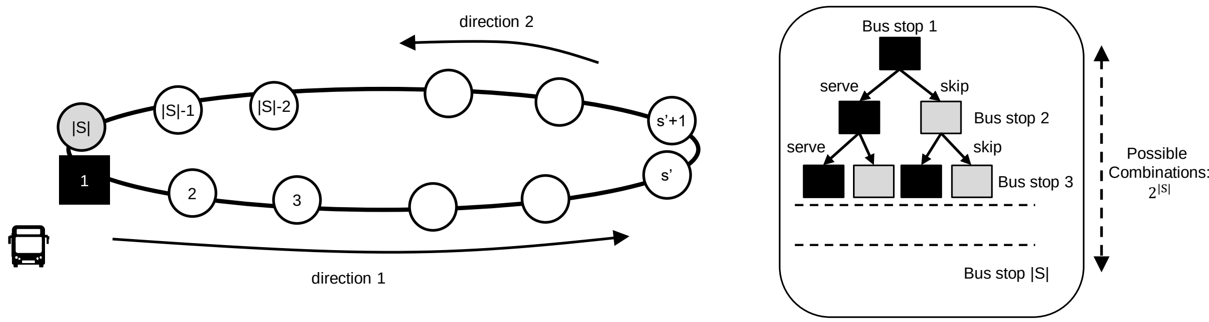

FIGURE 2 : Bus operating in a loop with $s = \langle 1, 2, ..., |S| \rangle$ stops that can be either skipped or served.

Other approaches calculate a stop-skipping plan for the entirety of daily trips or for several trips in a rolling horizon Jordan and Turnquist (*94*). Nevertheless, such approaches cannot be applied in real-time control because of the exponential increase in computational costs when considering more than one trip. In more detail, if we consider multiple trips, $|N|$, and multiple stop-skipping candidates, $|S|$, devising an optimal stop-skipping plan requires to evaluate all solutions from the solution space $2^{|N||S|}$ which grows exponentially with the number of trips and stops.

Stop-skipping strategies can be devised at the tactical planning level or the operational level (dynamic stop skipping). Depending on the level of control, the objectives of a stop-skipping strategy might differ. At the tactical planning stage, the focus is on developing reliable, resilient or robust strategies that will maintain a good performance in case of disruptions during the actual operations. In contrary, dynamic stop-skipping strategies at the operational level are reactionary and less sophisticated because they need to be simple and computationally efficient. In sub-sections 5.1 and 5.2 we provide a review of past works at the different planning levels. Note that we place more emphasis on the dynamic stop-skipping methods discussed in 5.2 because this sub-section focuses on dynamic control.

Stop-skipping at the Tactical Planning Level

Stop-skipping can be addressed at the tactical planning stage where a stop-skipping plan is devised before the start of the daily operations and is not updated ever since Jordan and Turnquist, Furth (*94, 95*). The main benefit is that the stop-skipping strategy serves as a fixed plan which can be communicated to both the bus drivers and the passengers well in advance. On the other side, it cannot be adjusted during the operational stage and cannot react to disturbances during the actual operations.

Furth and Day (*96*) and Furth (*95*) analyzed the effect of four pre-planned strategies (short-turning, restricted zonal service, semi-restricted zonal service, and stop-skipping) to bus lines with unbalanced demand between directions. The explored objectives were the minimization of the fleet size and the improvement of the passenger-related cost. Gkiotsalitis (*97*) proposed a combination of genetic algorithm and linear programming to develop a stop-skipping strategy for the entire day of operations which performs well at worst-case scenarios (robust stop-skipping plan). The approach was tested in a circular bus line in Singapore demonstrating a potential performance improvement of more than 10% at worst-case scenarios.

Jamili and Aghaee (*98*) focused on finding optimal stop-skipping patterns in railway systems. As in Gkiotsalitis (*97*), they developed robust stop-skipping plans using a decomposition-based algorithm and a simulated annealing-based algorithm. After testing their solution in an Iranian metro line, the results demonstrated that the simulated annealing meta-heuristic performs better in large-scale problems.

Dynamic Stop-Skipping

In dynamic control, several approaches determine the skipped stops of a bus trip when it is about to be dispatched Lin et al., Fu et al., Li et al., Eberlein (*33, 47, 99, 100*). Determining the skipped stops for each trip in isolation reduces the problem complexity and limits the solution space. In more detail, skipping a stop is modeled as a 0-1 decision problem, where 0 can denote a skipped stop. If only one trip is considered, the solution space comprises of $2^{|S|}$ different options where $|S|$ is the total number of stops that can be optionally skipped. Note that the solution space increases exponentially with the number of stops and cannot be explored for large values of $|S|$. Nevertheless, as we previously discussed, several works resort to exhaustive search methods (brute-force) to solve the dynamic stop-skipping problem taking advantage of the relatively small scale of the problem in bus lines with less than 20 stops Sun and Hickman, Fu et al. (*23, 47*).

Sun and Hickman (*23*) modeled the stop-skipping problem as a nonlinear integer program including assumptions of random distributions of passenger boardings and alightings. Then, the problem was solved with exhaustive search. Similarly, Fu et al. (*47*) used an exhaustive search to determine the skipped stops of one trip at a time. Fu et al. (*47*) considered the total waiting times of passengers, the in-vehicle time and the total trip travel time as problem objectives. The potential benefit was tested with a simulation of route 7D in Waterloo, Canada.

While in Fu et al. (*47*) two consecutive bus trips were not allowed to skip the same stop, Liu et al. (*26*) used a more strict rule. In Liu et al. (*26*), if a bus trip skips one (or more) stops its preceding and following trip should not skip any stops. The formulation of Liu et al. (*26*) resulted in a mixed-integer nonlinear program with a non-convex objective function. Hence, Liu et al. (*26*) used a genetic algorithm incorporating Monte Carlo simulations for the solution of the problem. Contrary to the more sophisticated models, Eberlein (*32*) developed a simplified transit operation environment to derive the stop-skipping solutions. In this simplification, the stop-skipping problem was modeled as an integer nonlinear program with quadratic objective function and constraints.

Eberlein et al. (*101*) modeled the stop-skipping problem as an integer nonlinear program considering the passenger waiting times as an objective function and solving the stop-skipping problem in rolling horizons. In each rolling horizon, the skipped stops of all trips $i \in I_m$, where I_m is the set of all trips that belong to the rolling horizon, were determined. Given the complexity of the integer non-linear program, Eberlein et al. (*101*) simplified the model formulation and proposed an analytic solution that can be applied to the simplified problem. This approach was tested at the Green Line of the Massachusetts Bay Transportation Authority.

Other approaches have considered the stop-skipping problem in combination with short-turning. Li et al. (*99*) considered both the stop-skipping and short-turning problems formulating them as a single 0-1 stochastic programming model accounting for both the deviations from the schedule and the unsatisfied passenger demand. Given the problem complexity, Li et al. (*99*) used heuristic approaches and tested the solution performance with sample data from the Shanghai Transit

Company.

Stop-skipping has also been combined with bus holding Eberlein, Lin et al., Cortés et al., Sáez et al. (*32, 33, 34, 35*). In Cortés et al. (*34*), the control decisions were applied when buses arrived at stops. Given the disproportionate increase of the problem complexity when accounting for both stop-skipping and bus holding, the problem was solved with a genetic algorithm-based multi-objective optimization solution method. Lin et al. (*33*) and Sáez et al. (*35*) integrated also the two aforementioned strategies. Finally, Lin et al. (*33*) measured the system performance in terms of passenger in-vehicle time and waiting time and Sáez et al. (*35*) considered uncertain passenger demand by formulating it as a hybrid predictive control problem.

Typical objectives of the stop-skipping problem are the following:

- O1: waiting time of passengers;
- O2: in-vehicle time of passengers;
- O3: vehicle travel time;
- O4: reduction of control actions;
- O5: deviation from the planned schedule;
- O6: unsatisfied demand;
- O7: inconvenience of passengers that need to alight because the bus trip skips their stop.

To summarize the stop-skipping literature, in Table 3 we present a summary of stop-skipping works and analyze their characteristics in terms of problem attributes, number of decision variables, optimization horizon, objective function, mathematical program properties, stochasticity, and solution method.

TABLE 3 : Summary of selected stop-skipping works

Study	Problem	Trips considered	Real-time	Objective function	Mathematical Program	Stochasticity	Solution Method
Fu et al. (*47*)	stop-skipping	one	yes	O1+O2+O3	integer nonlinear	no	brute-force
Cortés et al. (*34*)	stop-skipping and bus holding	multiple	yes	O1+O4	mixed-integer nonlinear	no	genetic algorithm
Gkiotsalitis (*97*)	stop-skipping	multiple	no	O1+O2+O3	integer linear	yes	genetic algorithm
Liu et al. (*26*)	stop-skipping	one	yes	O1+O2+O3	integer non-linear	yes	genetic algorithm
Li et al. (*99*)	stop-skipping and short-turning	one	yes	O5+O6	integer nonlinear	yes	heuristic
Lin et al. (*33*)	stop-skipping and bus holding	one	yes	O1+O2+O3 +O5	mixed-integer	no	–
Eberlein et al. (*101*)	stop-skipping	multiple	yes	O1	integer non-linear	no	analytic solution for a simplified model
Sáez et al. (*35*)	stop-skipping and bus holding	multiple	yes	O1+O2	mixed integer non-linear	yes	genetic algorithm
Sun and Hickman (*23*)	stop-skipping	one	yes	O1+O7	integer non-linear	no	brute-force
Jamili and Aghaee (*98*)	stop-skipping	multiple	no	O3	integer non-linear	yes	decomposition and simulated annealing

DISCUSSION AND FUTURE RESEARCH DIRECTIONS

In this paper, we presented a literature review on dynamic optimization approaches used for bus control. The control approaches studied were divided into bus holding, rescheduling, and stop-skipping since those control methods have received more attention in the literature. We also discussed briefly other dynamic control measures, such as speed control, refused boardings and signal priority. In our literature review, we presented the mathematical models, the decision variables, the objectives and constraints, and the solution methods of prominent decision support techniques in bus operations.

Although there has been a fruitful development of models and solution techniques to address relevant decision problems in bus transport systems, there are still many open future research directions, such as the following:

- there is a lack of mathematical models for stop-skipping control in rolling horizons since most works determine the skipped stops of each bus when it is about to be dispatched;
- integration of control methods, such as rescheduling and bus holding, has shown promising results; however, only a few works have considered such integrations;
- most works derive dynamic control measures with the use of single-objective functions that combine multiple objectives with the use of weight factors; however, there is little attention on multi-objective approaches that derive Pareto frontiers;
- the capacity limits of buses are typically not considered in dynamic control methods for simplifying the solution of the problem; thus, there is a lack of models that offer a realistic representation of the system dynamics;
- there is a lack of bus holding, rescheduling and stop-skipping works that consider stochastic travel times and passenger demand in real-time control.

REFERENCES

[1] Yu, B., Z. Yang, and J. Yao, Genetic algorithm for bus frequency optimization. *Journal of Transportation Engineering*, Vol. 136, No. 6, 2009, pp. 576–583.

[2] Sun, D. J., Y. Xu, and Z.-R. Peng, Timetable optimization for single bus line based on hybrid vehicle size model. *Journal of Traffic and Transportation Engineering (English Edition)*, Vol. 2, No. 3, 2015, pp. 179–186.

[3] Wu, Y., H. Yang, J. Tang, and Y. Yu, Multi-objective re-synchronizing of bus timetable: Model, complexity and solution. *Transportation Research Part C: Emerging Technologies*, Vol. 67, 2016, pp. 149–168.

[4] Wren, A. and J.-M. Rousseau, Bus driver schedulingâĂŤan overview. In *Computer-aided transit scheduling*, Springer, 1995, pp. 173–187.

[5] Gintner, V., N. Kliewer, and L. Suhl, Solving large multiple-depot multiple-vehicle-type bus scheduling problems in practice. *OR Spectrum*, Vol. 27, No. 4, 2005, pp. 507–523.

[6] Kliewer, N., T. Mellouli, and L. Suhl, A time–space network based exact optimization model for multi-depot bus scheduling. *European journal of operational research*, Vol. 175, No. 3, 2006, pp. 1616–1627.

[7] Ceder, A., Public Transit Planning and Operation: Theory. *Modeling and practice. 0xford: Elsevier*, 2007.

[8] Hans, E., N. Chiabaut, and L. Leclercq, Investigating the irregularity of bus routes: highlighting how underlying assumptions of bus models impact the regularity results. *Journal of Advanced Transportation*, Vol. 49, No. 3, 2015, pp. 358–370.

[9] Hans, E., N. Chiabaut, L. Leclercq, and R. L. Bertini, Real-time bus route state forecasting using particle filter and mesoscopic modeling. *Transportation Research Part C: Emerging Technologies*, Vol. 61, 2015, pp. 121–140.

[10] Strathman, J., K. Dueker, T. Kimpel, R. Gerhart, K. Turner, P. Taylor, S. Callas, D. Griffin, and J. Hopper, Automated bus dispatching, operations control, and service reliability: Baseline analysis. *Transportation Research Record: Journal of the Transportation Research Board*, , No. 1666, 1999, pp. 28–36.

[11] Abkowitz, M. and J. Tozzi, Research contributions to managing transit service reliability. *Journal of advanced transportation*, Vol. 21, No. 1, 1987, pp. 47–65.

[12] Clotfelter, C., The private life of public economics. *Southern economic journal*, Vol. 59, No. 4, 1993, pp. 579–596.

[13] Gkiotsalitis, K. and F. Alesiani, Robust timetable optimization for bus lines subject to resource and regulatory constraints. *Transportation Research Part E: Logistics and Transportation Review*, Vol. 128, 2019, pp. 30–51.

[14] Leong, W., K. Goh, S. Hess, and P. Murphy, Improving bus service reliability: The Singapore experience. *Research in Transportation Economics*, Vol. 59, 2016, pp. 40–49.

[15] Trompet, M., X. Liu, and D. Graham, Development of key performance indicator to compare regularity of service between urban bus operators. *Transportation Research Record: Journal of the Transportation Research Board*, , No. 2216, 2011, pp. 33–41.

[16] Hickman, M. D., An analytic stochastic model for the transit vehicle holding problem. *Transportation Science*, Vol. 35, No. 3, 2001, pp. 215–237.

[17] Berrebi, S. J., K. E. Watkins, and J. A. Laval, A real-time bus dispatching policy to minimize passenger wait on a high frequency route. *Transportation Research Part B: Methodological*, Vol. 81, 2015, pp. 377–389.

[18] Eberlein, X. J., N. H. Wilson, and D. Bernstein, The holding problem with real–time information available. *Transportation science*, Vol. 35, No. 1, 2001, pp. 1–18.

[19] Fu, L. and X. Yang, Design and implementation of bus-holding control strategies with real-time information. *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 1791, No. 1, 2002, pp. 6–12.

[20] Gkiotsalitis, K. and N. Maslekar, Multiconstrained Timetable Optimization and Performance Evaluation in the Presence of Travel Time Noise. *Journal of Transportation Engineering, Part A: Systems*, Vol. 144, No. 9, 2018, p. 04018058.

[21] Chow, A. H., S. Li, and R. Zhong, Multi-objective optimal control formulations for bus service reliability with traffic signals. *Transportation Research Part B: Methodological*, Vol. 103, 2017, pp. 248–268.

[22] Sirmatel, I. I. and N. Geroliminis, Mixed logical dynamical modeling and hybrid model predictive control of public transport operations. *Transportation Research Part B: Methodological*, Vol. 114, 2018, pp. 325–345.

[23] Sun, A. and M. Hickman, The real–time stop–skipping problem. *Journal of Intelligent Transportation Systems*, Vol. 9, No. 2, 2005, pp. 91–109.

[24] Chen, X., B. Hellinga, C. Chang, and L. Fu, Optimization of headways with stop-skipping control: a case study of bus rapid transit system. *Journal of advanced transportation*, Vol. 49, No. 3, 2015, pp. 385–401.

[25] Yu, Y., Z. Ye, and C. Wang, Study of Bus Stop Skipping Scheme Based on Modified Cellular Genetic Algorithm. In *CICTP 2015*, 2015, pp. 2397–2409.

[26] Liu, Z., Y. Yan, X. Qu, and Y. Zhang, Bus stop-skipping scheme with random travel time. *Transportation Research Part C: Emerging Technologies*, Vol. 35, 2013, pp. 46–56.

[27] Newell, G. F., Control of pairing of vehicles on a public transportation route, two vehicles, one control point. *Transportation Science*, Vol. 8, No. 3, 1974, pp. 248–264.

[28] Hernández, D., J. C. Muñoz, R. Giesen, and F. Delgado, Analysis of real-time control strategies in a corridor with multiple bus services. *Transportation Research Part B: Methodological*, Vol. 78, 2015, pp. 83–105.

[29] Wu, W., R. Liu, and W. Jin, Modelling bus bunching and holding control with vehicle overtaking and distributed passenger boarding behaviour. *Transportation Research Part B: Methodological*, Vol. 104, 2017, pp. 175–197.

[30] Gavriilidou, A. and O. Cats, Reconciling transfer synchronization and service regularity: real-time control strategies using passenger data. *Transportmetrica A: Transport Science*, 2018, pp. 1–29.

[31] Adamski, A. and A. Turnau, Simulation support tool for real-time dispatching control in public transport. *Transportation Research Part A: Policy and Practice*, Vol. 32, No. 2, 1998, pp. 73–87.

[32] Eberlein, X. J., *Real-time control strategies in transit operations: Models and analysis*. Ph.D. thesis, Massachusetts Institute of Technology, Department of Civil and Environmental Engineering, 1995.

[33] Lin, G., P. Liang, P. Schonfeld, and R. Larson, *Adaptive control of transit operations, Final Report*. MD-26-7002. US Department of Transportation, 1995.

[34] Cortés, C. E., D. Sáez, F. Milla, A. Núñez, and M. Riquelme, Hybrid predictive control for real-time optimization of public transport systems' operations based on evolutionary multi-objective optimization. *Transportation Research Part C: Emerging Technologies*, Vol. 18, No. 5, 2010, pp. 757–769.

[35] Sáez, D., C. E. Cortés, F. Milla, A. Núñez, A. Tirachini, and M. Riquelme, Hybrid predictive control strategy for a public transport system with uncertain demand. *Transportmetrica*, Vol. 8, No. 1, 2012, pp. 61–86.

[36] Gkiotsalitis, K., Z. Wu, and O. Cats, A cost-minimization model for bus fleet allocation featuring the tactical generation of short-turning and interlining options. *Transportation Research Part C: Emerging Technologies*, Vol. 98, 2019, pp. 14–36.

[37] Gkiotsalitis, K. and N. Maslekar, Improving Bus Service Reliability with Stochastic Optimization. In *Intelligent Transportation Systems (ITSC), 2015 IEEE 18th International Conference on*, IEEE, 2015, pp. 2794–2799.

[38] Muñoz, J. C., C. E. Cortés, R. Giesen, D. Sáez, F. Delgado, F. Valencia, and A. Cipriano, Comparison of dynamic control strategies for transit operations. *Transportation Research Part C: Emerging Technologies*, Vol. 28, 2013, pp. 101–113.

[39] Cortés, C. E., S. Jara-Díaz, and A. Tirachini, Integrating short turning and deadheading in the optimization of transit services. *Transportation Research Part A: Policy and Practice*, Vol. 45, No. 5, 2011, pp. 419–434.

[40] Skabardonis, A., Control strategies for transit priority. *Transportation Research Record*, Vol. 1727, No. 1, 2000, pp. 20–26.

[41] Liu, H., A. Skabardonis, and W.-b. Zhang, A dynamic model for adaptive bus signal priority. In *82nd Transportation Research Board Annual Meeting, Washington, DC*, 2003.

[42] Koehler, L. A. and W. Kraus Jr, Simultaneous control of traffic lights and bus departure for priority operation. *Transportation Research Part C: Emerging Technologies*, Vol. 18, No. 3, 2010, pp. 288–298.

[43] van Oort, N., J. Boterman, and R. van Nes, The impact of scheduling on service reliability: trip-time determination and holding points in long-headway services. *Public Transport*, Vol. 4, No. 1, 2012, pp. 39–56.

[44] Daganzo, C. F. and J. Pilachowski, Reducing bunching with bus-to-bus cooperation. *Transportation Research Part B: Methodological*, Vol. 45, No. 1, 2011, pp. 267–277.

[45] Wang, Y., W. Ma, W. Yin, and X. Yang, Implementation and testing of cooperative bus priority system in connected vehicle environment: case study in Taicang City, China. *Transportation Research Record*, Vol. 2424, No. 1, 2014, pp. 48–57.

[46] Ampountolas, K. and M. Kring, Mitigating bunching with bus-following models and bus-to-bus cooperation. In *2015 IEEE 18th International Conference on Intelligent Transportation Systems*, IEEE, 2015, pp. 60–65.

[47] Fu, L., Q. Liu, and P. Calamai, Real-time optimization model for dynamic scheduling of transit operations. *Transportation Research Record: Journal of the Transportation Research Board*, , No. 1857, 2003, pp. 48–55.

[48] Cats, O., Regularity-driven bus operation: Principles, implementation and business models. *Transport Policy*, Vol. 36, 2014, pp. 223–230.

[49] Trompet, M., R. J. Anderson, and D. J. Graham, Variability in comparable performance of urban bus operations. *Transportation Research Record*, Vol. 2111, No. 1, 2009, pp. 177–184.

[50] Randall, E. R., B. J. Condry, M. Trompet, and S. K. Campus, International bus system benchmarking: Performance measurement development, challenges, and lessons learned. In *Transportation Research Board 86th Annual Meeting, 21st-25th january*, 2007.

[51] Chen, X., L. Yu, Y. Zhang, and J. Guo, Analyzing urban bus service reliability at the stop, route, and network levels. *Transportation research part A: policy and practice*, Vol. 43, No. 8, 2009, pp. 722–734.

[52] Daganzo, C. F., A headway-based approach to eliminate bus bunching: Systematic analysis and comparisons. *Transportation Research Part B: Methodological*, Vol. 43, No. 10, 2009, pp. 913–921.

[53] Knoppers, P. and T. Muller, Optimized transfer opportunities in public transport. *Transportation Science*, Vol. 29, No. 1, 1995, pp. 101–105.

[54] Berrebi, S. J., E. Hans, N. Chiabaut, J. A. Laval, L. Leclercq, and K. E. Watkins, Comparing bus holding methods with and without real-time predictions. *Transportation Research Part C: Emerging Technologies*, Vol. 87, 2018, pp. 197–211.

[55] Sánchez-Martínez, G., H. Koutsopoulos, and N. Wilson, Real-time holding control for high-frequency transit with dynamics. *Transportation Research Part B: Methodological*, Vol. 83, 2016, pp. 1–19.

[56] Nikolaou, M., Model predictive controllers: A critical synthesis of theory and industrial needs. *Advances in Chemical Engineering*, Vol. 26, 2001, pp. 131–204.

[57] Osuna, E. and G. Newell, Control strategies for an idealized public transportation system. *Transportation Science*, Vol. 6, No. 1, 1972, pp. 52–72.

[58] Rossetti, M. D. and T. Turitto, Comparing static and dynamic threshold based control strategies. *Transportation Research Part A: Policy and Practice*, Vol. 32, No. 8, 1998, pp. 607–620.

[59] Gkiotsalitis, K. and O. Cats, Multi-constrained bus holding control in time windows with branch and bound and alternating minimization. *Transportmetrica B: Transport Dynamics*, Vol. 7, No. 1, 2019, pp. 1258–1285.

[60] Bartholdi, J. J. and D. D. Eisenstein, A self-coördinating bus route to resist bus bunching. *Transportation Research Part B: Methodological*, Vol. 46, No. 4, 2012, pp. 481–491.

[61] Delgado, F., J. C. Muñoz, R. Giesen, and A. Cipriano, Real-time control of buses in a transit corridor based on vehicle holding and boarding limits. *Transportation Research Record: Journal of the Transportation Research Board*, Vol. 2090, No. 1, 2009, pp. 59–67.

[62] Delgado, F., J. C. Munoz, and R. Giesen, How much can holding and/or limiting boarding improve transit performance? *Transportation Research Part B: Methodological*, Vol. 46, No. 9, 2012, pp. 1202–1217.

[63] Cats, O., A. N. Larijani, H. N. Koutsopoulos, and W. Burghout, Impacts of holding control strategies on transit performance: Bus simulation model analysis. *Transportation Research Record*, Vol. 2216, No. 1, 2011, pp. 51–58.

[64] Van Oort, N., N. Wilson, and R. Van Nes, Reliability improvement in short headway transit services: Schedule-and headway-based holding strategies. *Transportation Research Record: Journal of the Transportation Research Board*, , No. 2143, 2010, pp. 67–76.

[65] Shen, S. and N. H. Wilson, An optimal integrated real-time disruption control model for rail transit systems. In *Computer-aided scheduling of public transport*, Springer, 2001, pp. 335–363.

[66] Zolfaghari, S., N. Azizi, and M. Y. Jaber, A model for holding strategy in public transit systems with real-time information. *International Journal of Transport Management*, Vol. 2, No. 2, 2004, pp. 99–110.

[67] Alesiani, F. and K. Gkiotsalitis, Reinforcement Learning-Based Bus Holding for High-Frequency Services. In *2018 21st International Conference on Intelligent Transportation Systems (ITSC)*, IEEE, 2018, pp. 3162–3168.

[68] Gkiotsalitis, K. and O. Cats, Bus holding control of running buses in time windows. In *98th Annual Meeting of the TRB Transportation Research Board 2019*, 2019.

[69] Sun, A. and M. Hickman, Scheduling considerations for a branching transit route. *Journal of advanced transportation*, Vol. 38, No. 3, 2004, pp. 243–290.

[70] Sun, A. and M. Hickman, The holding problem at multiple holding stations. In *Computer-aided systems in public transport*, Springer, 2008, pp. 339–359.

[71] Xuan, Y., J. Argote, and C. F. Daganzo, Dynamic bus holding strategies for schedule reliability: Optimal linear control and performance analysis. *Transportation Research Part B: Methodological*, Vol. 45, No. 10, 2011, pp. 1831–1845.

[72] Marguier, P., *Bus route performance evaluation under stochastic conditions*. Ph.D. thesis, Massachusetts Institute of Technology, Cambridge, MA, 1985.

[73] Puong, A. and N. H. Wilson, A train holding model for urban rail transit systems. In *Computer-aided systems in public transport*, Springer, 2008, pp. 319–337.

[74] Powell, M. J., The BOBYQA algorithm for bound constrained optimization without derivatives. *Cambridge NA Report NA2009/06, University of Cambridge, Cambridge*, 2009, pp. 26–46.

[75] Zhao, J., S. Bukkapatnam, and M. M. Dessouky, Distributed architecture for real-time coordination

of bus holding in transit networks. *Intelligent Transportation Systems, IEEE Transactions on*, Vol. 4, No. 1, 2003, pp. 43–51.

[76] Sandholm, T., An implementation of the contract net protocol based on marginal cost calculations. In *AAAI*, 1993, Vol. 93, pp. 256–262.

[77] Tang, X., X. Lin, and F. He, Robust scheduling strategies of electric buses under stochastic traffic conditions. *Transportation Research Part C: Emerging Technologies*, Vol. 105, 2019, pp. 163–182.

[78] D'Ariano, A., F. Corman, D. Pacciarelli, and M. Pranzo, Reordering and local rerouting strategies to manage train traffic in real time. *Transportation science*, Vol. 42, No. 4, 2008, pp. 405–419.

[79] Corman, F., A. DâĂŹAriano, D. Pacciarelli, and M. Pranzo, A tabu search algorithm for rerouting trains during rail operations. *Transportation Research Part B: Methodological*, Vol. 44, No. 1, 2010, pp. 175–192.

[80] Meng, L. and X. Zhou, Simultaneous train rerouting and rescheduling on an N-track network: A model reformulation with network-based cumulative flow variables. *Transportation Research Part B: Methodological*, Vol. 67, 2014, pp. 208–234.

[81] Pellegrini, P., G. Marlière, and J. Rodriguez, Optimal train routing and scheduling for managing traffic perturbations in complex junctions. *Transportation Research Part B: Methodological*, Vol. 59, 2014, pp. 58–80.

[82] Krasemann, J. T., Design of an effective algorithm for fast response to the re-scheduling of railway traffic during disturbances. *Transportation Research Part C: Emerging Technologies*, Vol. 20, No. 1, 2012, pp. 62–78.

[83] Bly, P., *Depleted bus services: the effect of rescheduling*, 1976.

[84] Gkiotsalitis, K. and A. Stathopoulos, Demand-responsive public transportation re-scheduling for adjusting to the joint leisure activity demand. *International Journal of Transportation Science and Technology*, Vol. 5, No. 2, 2016, pp. 68–82.

[85] Li, J.-Q., P. B. Mirchandani, and D. Borenstein, Parallel auction algorithm for bus rescheduling. In *Computer-aided Systems in Public Transport*, Springer, 2008, pp. 281–299.

[86] Li, J.-Q., P. B. Mirchandani, and D. Borenstein, A Lagrangian heuristic for the real-time vehicle rescheduling problem. *Transportation Research Part E: Logistics and Transportation Review*, Vol. 45, No. 3, 2009, pp. 419–433.

[87] Mirchandani, P. B., J.-Q. Li, and M. Hickman, A macroscopic model for integrating bus signal priority with vehicle rescheduling. *Public Transport*, Vol. 2, No. 3, 2010, pp. 159–172.

[88] Gkiotsalitis, K. and N. Maslekar, Sequential evolutionary optimization for improving regularity-based bus services. In *Transportation Research Board 96th Annual Meeting*, 2016, 17-00463.

[89] Gkiotsalitis, K. and N. Maslekar, Dynamic Bus Operations Optimization with REFLEX. *NEC Tech. J*, Vol. 11, No. 1, 2016.

[90] Cevallos, F. and F. Zhao, Minimizing transfer times in public transit network with genetic algorithm. *Transportation Research Record: Journal of the Transportation Research Board*, , No. 1971, 2006, pp. 74–79.

[91] Cevallos, F. and F. Zhao, A genetic algorithm for bus schedule synchronization. In *Applications of Advanced Technology in Transportation*, 2006, pp. 737–742.

[92] Coffey, C., R. Nair, F. Pinelli, A. Pozdnoukhov, and F. Calabrese, Missed connections: quantifying and optimizing multi-modal interconnectivity in cities. In *Proceedings of the 5th ACM SIGSPATIAL International Workshop on Computational Transportation Science*, ACM, 2012, pp. 26–32.

[93] Desaulniers, G. and M. D. Hickman, Public transit. *Handbooks in operations research and management science*, Vol. 14, 2007, pp. 69–127.

[94] Jordan, W. C. and M. A. Turnquist, Zone scheduling of bus routes to improve service reliability. *Transportation science*, Vol. 13, No. 3, 1979, pp. 242–268.

[95] Furth, P. G., Zonal route design for transit corridors. *Transportation Science*, Vol. 20, No. 1, 1986, pp. 1–12.

[96] Furth, P. G. and F. B. Day, Transit routing and scheduling strategies for heavy-demand corridors (Abridgment). *Transportation Research Record*, , No. 1011, 1985.

[97] Gkiotsalitis, K., Robust stop-skipping at the tactical planning stage with evolutionary optimization. *Transportation research record*, Vol. 2673, No. 3, 2019, pp. 611–623.

[98] Jamili, A. and M. P. Aghaee, Robust stop-skipping patterns in urban railway operations under traffic alteration situation. *Transportation Research Part C: Emerging Technologies*, Vol. 61, 2015, pp. 63–74.

[99] Li, Y., M. Gendreau, et al., Real time scheduling on a transit bus route: a 0-1 stochastic programming model, 1991.

[100] Eberlein, X. J., Real-time control strategies in transit operations: Models and analysis. *Transportation Research Part A*, Vol. 1, No. 31, 1997, pp. 69–70.

[101] Eberlein, X. J., N. H. Wilson, C. Barnhart, and D. Bernstein, The real-time deadheading problem in transit operations control. *Transportation Research Part B: Methodological*, Vol. 32, No. 2, 1998, pp. 77–100.