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To link to this article: https://doi.org/10.1016/j.jaubas.2017.01.002

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Published online: 27 Mar 2018.

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Semi-analytical investigation on micropolar fluid flow and heat transfer in a permeable channel using AGM

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Received 16 July 2016; revised 29 November 2016; accepted 15 January 2017
Available online 21 February 2017

KEYWORDS
Akbari–Ganji’s Method (AGM); Heat transfer; Mass transfer; Micropolar fluid; Permeable channel

Abstract In this paper, micropolar fluid flow and heat transfer in a permeable channel have been investigated. The main aim of this study is based on solving the nonlinear differential equation of heat and mass transfer of the mentioned problem by utilizing a new and innovative method in semi-analytical field which is called Akbari–Ganji’s Method (AGM). Results have been compared with numerical method (Runge–Kutte 4th) in order to achieve conclusions based on not only accuracy and efficiency of the solutions but also simplicity of the taken procedures which would have remarkable effects on the time devoted for solving processes.

Results are presented for different values of parameters such as: Reynolds number, micro-rotation/angular velocity and Peclet number in which the effects of these parameters are discussed on the flow, heat transfer and concentration characteristics. Also relation between Reynolds and Peclet numbers with Nusselts and Sherwood numbers would found for both suction and injection

Furthermore, due to the accuracy and convergence of obtained solutions, it will be demonstrating that AGM could be applied through other nonlinear problems even with high nonlinearity.

1. Introduction

Micropolar fluids are fluids with microstructure. They belong to a class of fluids with nonsymmetrical stress tensor that we shall call polar fluids, which could be mentioned as the well-established Navier–Stokes model of classical fluids. These fluids respond to micro-rotational motions and spin inertia and therefore, can support couple stress and distributed body couples. Physically, a micropolar fluid is one which contains suspensions of rigid particles. The theory of micropolar fluids was first formulated by Eringen (1966). Examples of industrially relevant flows that can be studied with accordance to this theory include flow of low concentration suspensions, liquid crystals, blood, lubrication and so on. The micropolar theory has recently been applied and considered in different aspects of sciences and engineering applications. For instance, Gorla (1989), Gorla (1988), Gorla (1992) and Araf and Gorla (1992) have considered the free and mixed convection flow of a micropolar fluid from flat surfaces and cylinders. Raptis (2000) studied boundary layer flow of a micropolar fluid through a porous medium by using the generalized Darcy
law. The influence of a chemical reaction and thermal radiation on the heat and mass transfer in MHD micropolar flow over a vertical moving plate in a porous medium with heat generation was studied by Mohamed and Abo-Dahab (2009).

It would be worthy to mention the fact that many scientists and researchers all around the world are working on the effects of using micropolar fluids and nanofluids on flow and heat transfer problems (Kelson and Desseaux, 2001; Sheikholeslami et al., 2016a,b; Rashidi et al., 2011; Sheikholeslami et al., 2015; Turkyilmazoglu, 2014c; Turkyilmazoglu, 2016b) which will lead to suitable perspective for future industrial and research applications such as: pharmaceutical processes, hybrid-powered engines, heat exchangers and so on.

In many engineering problems solving procedures will finally lead to whether mathematical formulation or modeling processes. For obtaining better understanding in both of these factors, many researchers from different fields devote their time to expand relevant knowledge. As one of the most important type of these knowledge, we could mention analytical, semi-analytical methods and numerical technics in solving nonlinear differential equations. By utilizing analytical and semi-analytical methods, solutions for each problem will approach to a unique function. Most of the heat transfer and fluid mechanics problems would engage with nonlinear equation which finding accurate and efficient solutions for these problems have been considered by many researchers recently. Therefore, for the purpose of achieving the mentioned facts, many researchers have tried to reach acceptable solution for these equations due to their nonlinearity by utilizing analytical and semi-analytical methods such as: Perturbation Method by Ganji et al. (2007), Homotopy Perturbation Method by Turkyilmazoglu (2012), Sheikholeslami et al. (2013) and Mirgolbabaei et al. (2009), Variational Iteration Method by Turkyilmazoglu (2016a), Mirgolbabaei et al. (2009) and Samee et al. (2015), Homotopy Analysis Method by Sheikholeslami et al. (2014), Sheikholeslami et al. (2012) and Turkyilmazoglu (2011), Parameterized Perturbation Method (PPM) by Ashorynejad et al. (2014), Collocation Method (CM) by Hoshyar et al. (2015), Adomian Decomposition Method by Sheikholeslami et al. (2013), Least Square Method (LSM) by Fakour et al. (2014), Galerkin Method (GM) by Turkyilmazoglu (2014a,b) so on.

It’s noteworthy to mentioned the fact that Semi-Analytical methods could be categorized into two perspectives due to their solving procedures as for simplicity we would call them as: Iterate-Base Method and Trial Function-Base Method. In Iterate-Base Method such as: HPM, VIM, ADM and etc., the important factor which affect the solving procedures is number of iterations. Although in this methods we may assume a trial functions, which are based on our in depended functions, however, in order to achieve solution in each step we have to solve previous steps at first. According to mentioned explanations, it’s obvious that whilst the iteration results in higher steps can’t be obtained by related software, we will face problem which will interrupt our solving procedures. Also these methods usually take more time for obtaining solutions. In Trial Function-Base Method such as: CM, LSM, Akbari–Ganjii’s Method (AGM) and etc., the main factor which affect the solving procedures is trial function. In this methods we will assume an efficient trial function base on the problem’s bound-

ary and initial conditions which contains different constant coefficients. Afterward, due to the basic idea of each method, we are obligated for solving the constant coefficients. In most cases the constant coefficients will be obtain easily by solving set of polynomials. Although in these methods, number of terms in our trial function could be referred as needed iterations, however, it’s essential to mention the fact that utilized constants will obtain simultaneously in solving procedures. So in these methods the iteration problems have been eliminated.

In this article attempts have been made in order to obtain approximate solutions of the governing nonlinear differential equations of micropolar fluid flow. We have utilized a new and innovative semi-analytical method calling Akbari–Ganjii’s Method which is developed by Akbari and Ganji by Akbari et al. (2014) and Rostami et al. (2014) in 2014 for the first time. Since then this method has been investigated by many authors to solve highly nonlinear equations in different aspects of engineering problems such as: Fluid Mechanics, Nonlinear Vibraton Problems, Heat Transfer Applications, Nanofluids and etc. Some of the excellence of proposed method could be referred as Ledari et al. (2015) and Mirgolbabae et al. (2016a,b).

Due to recently achievements from this method and also the Trial Function-Base characteristics of this method, it could precisely conclude that AGM has high efficiency and accuracy for solving nonlinear problems with high nonlinearity. It is necessary to mention that a summary of the excellence of this method in comparison with the other approaches can be considered as follows: Boundary conditions are needed in accordance with the order of differential equations in the solution procedure but when the number of boundary conditions is less than the order of the differential equation, this approach can create additional new boundary conditions in regard to the own differential equation and its derivatives.

2. Mathematical formulation

We consider the steady laminar flow of a micropolar fluid along a two-dimensional channel with parallel porous walls through which fluid is uniformly injected or removed with speed \( v_0 \) which is represented in Fig. 1. The geometry of problem has defined clearly in Fig. 1. By utilizing Cartesian coordinates, the governing equations for flow are Sibanda and Awad (2010):

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{\partial P}{\partial x} + (\mu + \kappa) \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \kappa \frac{\partial N}{\partial x} \tag{2}
\]

\[
\rho \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial P}{\partial y} + (\mu + \kappa) \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \kappa \frac{\partial N}{\partial y} \tag{3}
\]

\[
\rho \left( \frac{\partial N}{\partial x} + \frac{\partial N}{\partial y} \right) = - \frac{\kappa}{f} \left( 2N + \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right) + \frac{\rho v_0}{f} \left( \frac{\partial^2 N}{\partial x^2} + \frac{\partial^2 N}{\partial y^2} \right) \tag{4}
\]
reduce the following dimensionless variables: the stress tensor and concentrations and the vanishing of the antisymmetric part of microelements close to the wall. Also, the case where the microelements are free to rotate near channel walls. The celerity and brevity. The appropriate boundary conditions are: we have refused to announce these again for the purpose of

The parameters of primary interests are the buoyancy ratio \( N_t \), the Peclet numbers for the diffusion of heat \( Pe_h \) and mass \( Pe_m \) respectively, the Reynolds number \( Re \) where for suction \( Re > 0 \) and for injection \( Re < 0 \) also Grashof number \( Gr \) given by:

\[
N_t = \frac{k}{\mu}, \quad N_2 = \frac{v}{\mu R^2}, \quad N_3 = \frac{J}{R^2}, \quad Re = \frac{V_0}{v} h
\]

\[
Pr = \frac{\nu \sigma_p}{k_1}, \quad Sc = \frac{v}{D}, \quad Gr = \frac{gbT^2}{v^2}
\]

where \( Pr \) is the Prandtl number, \( Sc \) is the generalized Schmidt number, \( N_t \) is the coupling parameter and \( N_2 \) is the spin-gradient viscosity parameter. In technological processes, Nusselt and Sherwood numbers are being considered widely which are defined as follows:

\[
Nu = \frac{q''}{\frac{T_0 - T}{h x}}, \quad Sh = \frac{m''}{\frac{C_1 - C_2}{D_x}}
\]

where \( q'' \) and \( m'' \) are local heat flux and mass flux respectively.

### 3. Basic idea of Akbari–Ganjii’s method (AGM)

Physics of the problems in every fields of engineering sciences lead to set of linear or nonlinear differential equations as its governing equations. According to physics of these problems and their obtained mathematical formulation, sufficient boundary or initial conditions should be applied in order to achieve solution for considered problems. Since procedures of applying analytical methods for obtaining solution of linear and nonlinear differential equations are not an exception from mentioned fact, so we could recognize the importance of these boundary and initial conditions in determining the accuracy and efficiency of analytical methods in achieving acceptable solution due to physis of problems. In order to comprehend the given method in this paper, the entire process has been declared clearly.

In accordance with the boundary conditions, the general manner of a differential equation is as follows:

\[
p_k : f(u, u', u'', \ldots, u^{(m)}) = 0; \quad u = u(x)
\]

The nonlinear differential equation of \( \rho \) which is a function of \( u \), the parameter \( u \) which is a function of \( x \) and their derivatives are considered as follows:

\[
\left\{ \begin{array}{l}
u(x) = u_0, \quad \nu'(x) = u_1, \ldots, \nu^{(m-1)}(x) = u_{m-1} \quad \text{at} \quad x = 0 \\
u(x) = u_L, \quad \nu'(x) = u_1, \ldots, \nu^{(m-1)}(x) = u_{m-1} \quad \text{at} \quad x = L
\end{array} \right.
\]
To solve the first differential equation with respect to the boundary conditions in $x = L$ in Eq. (19), the series of letters in the $n$th order with constant coefficients which we assume as the solution of the first differential equation is considered as follows:

$$u(x) = \sum_{i=0}^{n} a_i x^i = a_0 + a_1 x^1 + a_2 x^2 + \cdots + a_n x^n$$  \hspace{1cm} (20)

The more choice of series sentences from Eq. (20) cause more precise solution for Eq. (18). For obtaining solution of differential Eq. (18) regarding the series from degree $(n)$, there are $(n + 1)$ unknown coefficients that need $(n + 1)$ equations to be specified. The boundary conditions of Eq. (19) are used to solve a set of equations which is consisted of $(n + 1)$ ones.

3.1. Applying the boundary conditions

(a) The application of the boundary conditions for the answer of differential Eq. (20) is in the form of:

$$\begin{align*}
  u(0) &= u_0 \\
u'(0) &= u_1 \\
u''(0) &= u_2 \\
&\vdots \\
  u(L) &= a_0 + a_1 L + a_2 L^2 + \cdots + a_n L^n = u_{L0} \\
u'(L) &= a_1 + 2a_2 L + 3a_3 L^2 + \cdots + na_n L^{n-1} = u_{L1} \\
u''(L) &= 2a_2 + 6a_3 L + 12a_4 L^2 + \cdots + (n-1)a_n L^{n-2} = u_{L2} \\
&\vdots \\
  \end{align*}$$  \hspace{1cm} (21)

And when $x = L$:

$$\begin{align*}
  u(L) &= a_0 + a_1 L + a_2 L^2 + \cdots + a_n L^n = u_{L0} \\
u'(L) &= a_1 + 2a_2 L + 3a_3 L^2 + \cdots + na_n L^{n-1} = u_{L1} \\
u''(L) &= 2a_2 + 6a_3 L + 12a_4 L^2 + \cdots + (n-1)a_n L^{n-2} = u_{L2} \\
&\vdots \\
  \end{align*}$$  \hspace{1cm} (22)

(b) After substituting Eq. (22) into Eq. (18), the application of the boundary conditions on differential Eq. (18) is done according to the following procedure:

$$\begin{align*}
p_0 &= f(u(0), u'(0), u''(0), \ldots, u^{(m)}(0)) \\
p_1 &= f(u(L), u'(L), u''(L), \ldots, u^{(m)}(L)) \\
&\vdots \\
  \end{align*}$$  \hspace{1cm} (23)

With regard to the choice of $n$; $(n < m)$ sentences from Eq. (20) and in order to make a set of equations which is consisted of $(n + 1)$ equations and $(n + 1)$; unknowns, we confront with a number of additional unknowns which are indeed the same coefficients of Eq. (20). Therefore, to remove this problem, we should derive $m$ times from Eq. (18) according to the additional unknowns in the afore-mentioned set of differential equations and then apply the boundary conditions on them.

$$\begin{align*}
p_k^0 &= f(u'(0), u''(0), u^{(m)}(0), \ldots, u^{(m+1)}(0)) \\
p_k^m &= f(u'(L), u''(L), u^{(m)}(L), \ldots, u^{(m+2)}(L)) \\
&\vdots \\
  \end{align*}$$  \hspace{1cm} (24)

(c) Application of the boundary conditions on the derivatives of the differential equation $p_k$ in Eq. (24) is done in the form of:

$$\begin{align*}
p_k^0 &= \begin{cases} 
  f(u'(0), u''(0), u^{(m)}(0), \ldots, u^{(m+1)}(0)) \\
  f(u'(L), u''(L), u^{(m)}(L), \ldots, u^{(m+1)}(L)) 
\end{cases} \\
  \end{align*}$$  \hspace{1cm} (25)

$(n + 1)$ equations can be made from Eq. (21) to Eq. (26) so that $(n + 1)$ unknown coefficients of Eq. (20) such as $a_0, a_1, a_2, \ldots, a_n$ will be compute. The solution of the nonlinear differential Eq. (18) will be gained by determining coefficients of Eq. (20). To comprehend the procedures of applying the following explanation we have presented the relevant process step by step in following part.


According to mentioned coupled system of nonlinear differential equations and also by considering the basic idea of the method, we rewrite Eqs. (10)–(13) in the following order:

$$\begin{align*}
  F(\eta) &= (1 + N_1)f'' - N_1 g - \text{Re}(f'' - f') = 0 \\
  G(\eta) &= N_2 g'' - N_1 (f'' - 2g) - N_3 \text{Re}(fg' - f g) = 0 \\
  \Theta(\eta) &= \theta' - P_c h (f h - \theta h') = 0 \\
  \Phi(\eta) &= \phi' + P_c j (f j - \phi j') = 0 
\end{align*}$$  \hspace{1cm} (27)

Due to the basic idea of AGM, we have utilized a proper trial function as solution of the considered differential equation which is a finite series of polynomials with constant coefficients, as follows:

$$\begin{align*}
  f(\eta) &= \sum_{i=0}^{q} a_i \eta^i \\
  &= a_0 + a_1 \eta + a_2 \eta^2 + a_3 \eta^3 + a_4 \eta^4 + a_5 \eta^5 + a_6 \eta^6 \\
  &= a_7 \eta^7 + a_8 \eta^8 + a_9 \eta^9 \\
  \end{align*}$$  \hspace{1cm} (28)

$$\begin{align*}
  g(\eta) &= \sum_{i=0}^{q} b_i \eta^i \\
  &= b_0 + b_1 \eta + b_2 \eta^2 + b_3 \eta^3 + b_4 \eta^4 + b_5 \eta^5 + b_6 \eta^6 \\
  &= b_7 \eta^7 + b_8 \eta^8 + b_9 \eta^9 \\
  \end{align*}$$  \hspace{1cm} (29)

$$\begin{align*}
  \Theta(\eta) &= \sum_{i=0}^{q} c_i \eta^i \\
  &= c_0 + c_1 \eta + c_2 \eta^2 + c_3 \eta^3 + c_4 \eta^4 + c_5 \eta^5 + c_6 \eta^6 \\
  &= c_7 \eta^7 + c_8 \eta^8 + c_9 \eta^9 \\
  \end{align*}$$  \hspace{1cm} (30)

$$\begin{align*}
  \Phi(\eta) &= \sum_{i=0}^{q} d_i \eta^i \\
  &= d_0 + d_1 \eta + d_2 \eta^2 + d_3 \eta^3 + d_4 \eta^4 + d_5 \eta^5 + d_6 \eta^6 \\
  &= d_7 \eta^7 + d_8 \eta^8 + d_9 \eta^9 \\
  \end{align*}$$  \hspace{1cm} (31)

4.1. Applying boundary conditions

In AGM, the boundary conditions are applied in order to compute constant coefficients of Eqs. (28)–(34) according to the following approaches:

(a) Applying the boundary conditions on Eqs. (28)–(34) as expressed as follows:
where BC is the abbreviation of boundary condition. According to the explanations given, the boundary conditions are applied on Eqs. (31)–(34); in the following form:

\[ f(-1) = 0 \quad \text{with} \quad -a_0 + a_8 - a_1 + a_6 + a_5 - a_3 + a_2 - a_1 + a_0 = 0 \]  

\[ f(+1) = 0 \quad \text{with} \quad a_0 + a_8 + a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0 = 0 \]  

\[ f'(-1) = 0 \quad \text{with} \quad 9a_8 - 8a_6 + 7a_7 - 6a_6 + 5a_5 - 4a_4 + 3a_3 - 2a_2 + a_1 = 0 \]  

\[ f'(+1) = -1 \quad \text{with} \quad 9a_8 + 8a_6 + 7a_7 + 6a_6 + 5a_5 + 4a_4 + 3a_3 + 2a_2 + a_1 = -1 \]  

\[ g(-1) = 0 \quad \text{with} \quad -b_9 + b_8 - b_7 + b_6 - b_5 + b_4 - b_3 + b_2 - b_1 + b_0 = 0 \]  

\[ g(+1) = 1 \quad \text{with} \quad b_9 + b_8 + b_7 + b_6 + b_5 + b_4 + b_3 + b_2 + b_1 + b_0 = 1 \]  

\[ \theta(-1) = 1 \quad \text{with} \quad -c_7 + c_6 - c_5 + c_4 - c_3 + c_2 - c_1 + c_0 = 1 \]  

\[ \theta(+1) = 0 \quad \text{with} \quad c_7 + c_6 + c_5 + c_4 + c_3 + c_2 + c_1 + c_0 = 0 \]  

\[ \phi(-1) = 1 \quad \text{with} \quad -d_7 + d_6 - d_5 + d_4 - d_3 + d_2 - d_1 + d_0 = 1 \]  

\[ \phi(+1) = 0 \quad \text{with} \quad d_7 + d_6 + d_5 + d_4 + d_3 + d_2 + d_1 + d_0 = 0 \]  

(b) Applying the boundary conditions on the main differential equations, which in this case study are Eqs. (27)–(30), and also on theirs derivatives is done after substituting Eqs. (31)–(34) into the main differential equations as follows:

\[ F'(f(\eta)) \rightarrow f(f(B.C)) = 0, \quad F'(f(B.C)) = 0, \ldots \]  

\[ G'(g(\eta)) \rightarrow g(g(B.C)) = 0, \quad G'(g(B.C)) = 0, \ldots \]  

\[ \Theta'(\theta(\eta)) \rightarrow \Theta(\theta(B.C)) = 0, \quad \Theta'(\theta(B.C)) = 0, \ldots \]  

\[ \Phi'(\phi(\eta)) \rightarrow \Phi(\phi(B.C)) = 0, \quad \Phi'(\phi(B.C)) = 0, \ldots \]  

The boundary conditions on the achieved differential equation are applied based on the above equations. In fact, due to the excellence of AGM from other methods, we have to reach to set of polynomials in the processes of solution according to the overall number of used constant coefficients in trial functions which finally we would be able to obtain these only by simple calculations. Since in the proposed problem we have engaged with four trial functions which contain 36 constant coefficients and we have 10 equations according to Eqs. (36)–(45), we have to create 26 additional equations from Eqs. (46)–(49) in order to achieve a set of polynomials which contains 36 equations and 36 constants.

According to the above explanations we have created additional equations Eqs. (46)–(49) in the following order:

I. 6 equations have been created by calculating obtained equations from \( F(-1) = 0, F(+1) = 0, F'(1) = 0, \) \( F'(1) = 0, F''(-1) = 0, F''(+1) = 0 \)

II. 8 equations have been created by calculating obtained equations from \( G(-1) = 0, G(+1) = 0, G'(-1) = 0, \) \( G'(1) = 0, G''(-1) = 0, G''(+1) = 0 \)

III. 6 equations have been created by calculating obtained equations from \( \Theta(-1) = 0, \Theta(+1) = 0, \) \( \Theta'(-1) = 0, \Theta'(1) = 0 \)

IV. 6 equations have been created by calculating obtained equations from \( \Phi(-1) = 0, \Phi(+1) = 0, \Phi'(1) = 0, \) \( \Phi'(+1) = 0 \)

The mentioned equations in (I)–(IV) subsections are too large to be displayed graphically. So by utilizing the above procedures we have obtained a set of polynomials containing 36 equations and 36 constants which by solving them we would be able to obtain Eqs. (31)–(34). For instance, when \( Re = 0.1, N_1 = 0.1, N_2 = 0.1, N_3 = 0.1, Pe_a = 0.1, Pe_m = 0.1 \), by substituting obtained constant coefficients from mentioned procedures Eqs. (31)–(34) could easily be yielded as follows:

\[ f(\eta) = 0.0000011226\eta^9 + 0.00007769\eta^8 - 0.0016237\eta^7 + 0.004568\eta^5 - 0.0321635\eta^4 - 0.0609659\eta^3 - 0.306566\eta^2 - 0.341988\eta^1 + 0.1596354\eta + 0.0924003 \]  

\[ g(\eta) = 0.00001106\eta^9 + 0.0000777\eta^8 + 0.0016237\eta^7 + 0.004568\eta^5 + 0.0321635\eta^4 + 0.0609659\eta^3 + 0.306566\eta^2 + 0.341988\eta^1 + 0.1596354\eta + 0.0924004 \]  

\[ \theta(\eta) = -0.000001704\eta^7 - 0.00001289\eta^6 - 0.0012362\eta^5 + 0.0021367\eta^4 + 0.00413266\eta^3 - 0.0126272\eta^2 - 0.50289\eta + 0.510503 \]  

\[ \phi(\eta) = -0.000001704\eta^7 - 0.00001289\eta^6 - 0.0012362\eta^5 + 0.0021367\eta^4 + 0.00413266\eta^3 - 0.0126272\eta^2 - 0.50289\eta + 0.510503 \]  

5. Result and discussion

In this paper, Akbari–Ganji’s Method (AGM) has been utilized in order to solve the nonlinear differential equation of heat and mass transfer equation of steady laminar flow of a micropolar fluid along a two-dimensional channel with porous walls. The geometry of the problem has been shown in Fig. 1. Although the processes of obtaining analytical solution for the proposed problem have been explained clearly in the previous sections, it is noteworthy to mention the fact that the mentioned trial functions have been chosen in logical order in which applications of boundary conditions due to basic idea of AGM can be satisfied and also symmetric condition would be able to applied in both boundary points which are startpoint = -1 and endpoint = +1 in this case. We have shown AGM efficiency and accuracy through proper figures and table.

Fig. 2 shows the difference between obtained solution by AGM and numerical method (Runge–Kutte 4th) in which we have introduced error percentage as follow:
\[
\text{Error} = \frac{|u(\eta)_{\text{NM}} - u(\eta)_{\text{AGM}}|}{u(\eta)_{\text{NM}}} \times 100
\]  

Eq. (54)

where \( u(\eta)_{\text{NM}} \) is value obtained by numerical method (Runge–Kutte 4th) and \( u(\eta)_{\text{AGM}} \) is value obtained by AGM. Eq. (54) has been applied through functions of Eqs. (31)–(34) so the parameter \( u \) has only been defined as a symbol of data in this case.

In Fig. 3(a)–(d), the convergence issue has been considered which shows that by increasing steps in our assumed trial functions we will obtain more accurate solutions. In these figures we have obtained our results due to critical points in Fig. 2. Which are shown as follows:

Comparison between AGM and numerical results for different values of active parameters is shown in Figs. 4–6 and Table 1. The obtained results in comparison with numerical results represented that AGM has enough accuracy and efficiency so it would be applicable for solving nonlinear equations of coupled system.

Afterward, effect of different parameters such as: Reynolds number, micro rotation/angular velocity and Peclet number on the flow, heat transfer and concentration characteristics are discussed. Fig. 7 shows set of figures which in each of these effects of on parameter has been represented. Generally values of micro rotation profile \( g(\eta) \) decrease with increase of Re, \( N_1 \), \( N_2 \), however, it increases when \( N_2 \) increases. It is noteworthy to mention that when \( N_1 > 1 \) and \( N_2 < 1 \) the behavior of the angular velocity is oscillatory and irregular.

Since Nusselt and Sherwood numbers have great usage in technological processes, we have shown changes of these dimensionless numbers in Figs. 8 and 9. The effects of Peclet number on the fluid temperature and concentration profile are shown in Figs. 8(a) and 9(a). As shown in Fig. 8(a) the fluid temperature increases with increase of Peclet number and also due to Fig. 9(a) concentration profile increases while Peclet number increases. On the other hand, according to Fig. 8(b), increase in Peclet number and Reynolds number leads to increase in Nusselt number. Also according to Fig. 9(b) the same could be concluded for Sherwood number which increase in Peclet number and Reynolds number leads to increase in Sherwood number.
Fig. 4 Comparison between numerical and AGM solution results for $f(\eta)$.

Fig. 5 Comparison between numerical and AGM solution results for $g(\eta)$.

Fig. 6 Comparison between numerical and AGM solution results for $\theta(\eta)$.

Fig. 7 Effects of Re, $N_1$, $N_2$, $N_3$ on micro rotation profile ($g$) when (a) $N_1 = N_2 = N_3 = 1$ (b) Re = $N_2 = N_3 = 1$; (c) $N_1 = \text{Re} = N_3 = 1$ (d) $N_1 = N_2 = \text{Re} = 1$. 
6. Conclusion

In this study, AGM has been utilized in order to solve non-linear differential equation of heat and mass transfer equation of steady laminar flow of a micropolar fluid along a two-dimensional channel with porous walls. Comparisons have been done among AGM and numerical method (Runge–Kutta 4th) by different parameters values. Data from error figure represent that obtained solutions with AGM has minor differences with exact solutions and also convergence figure represent that by applying more terms of AGM we would be able to obtain more accurate solutions. Furthermore,

**Table 1** Comparison between the numerical results and AGM solution for $\phi(\eta)$ at various $Re$, $Pe_m$ when $Pe_h = 0.2$, $N_1 = N_2 = N_3 = 0.1$.

<table>
<thead>
<tr>
<th>$\eta$</th>
<th>$Re = 1$, $Pe_m = 0.5$</th>
<th></th>
<th>$Re = 0.5$, $Pe_m = 0.2$</th>
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![Fig. 8](image_url) (a) Effects of $Pe_h$ on temperature profile at $Re = N_1 = N_2 = N_3 = Pe_m = 1$, (b) effects of $Re$ and $Pe_m = Pe_h$ on Nusselt number when $N_1 = N_2 = N_3 = 1$.

![Fig. 9](image_url) (a) Effects of $Pe_m$ on concentration profile at $Re = N_1 = N_2 = N_3 = Pe_h = 1$, (b) effects of $Re$ and $Pe_h = Pe_m$ on Sherwood number when $N_1 = N_2 = N_3 = 1$. 

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according to achieved results, Reynolds number has direct relationship with Nusselt number and Sherwood number, however, Peclet number has reverse relationship with them.

Finally, it will be obvious that AGM is a convenient analytical method and due to its accuracy, efficiency and convergence it could be applied for solving nonlinear problems.

**Nomenclature**

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<tr>
<th>Symbol</th>
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</tr>
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<td>C</td>
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<td>thermal conductivity and molecular diffusivity</td>
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<td>(f)</td>
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<td>(g)</td>
<td>dimensionless micro rotation</td>
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<td>(\psi)</td>
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**References**


