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ACOUSTIC TIME-OF-FLIGHT MEASUREMENTS IN A REFLECTIVE ROOM

Ferdinand van der Heijden, *Gloria Túquerres, Paul Regtien*

University of Twente, Enschede, The Netherlands

Abstract – In this paper the problem how to estimate the time-of-flight of a received acoustic tone burst is addressed. In indoor applications, reflections cause interference patterns that are hardly predictable and can lead to large estimation errors. A generalisation of the well-known matched filter based on a non-stationary autocovariance model of the reflections allows us to develop a new estimator. Experiments show that the application of the new estimator can reduce these errors by about a factor four.

Keywords: Reflection model, time-of-flight, acoustic TOF estimation

1. INTRODUCCION

A straightforward approach to measure the distance between an acoustic transmitter and a receiver is based on the time-of-flight (ToF) of an acoustic tone burst. The approach is applied mainly in position measurement systems and sonar systems [1]. The determination of the moment of the arrival of a tone burst is difficult. Often, the signal-to-noise ratio is small, and the amplitude of the wave is unknown. In addition, due to the dynamics of the two transducers (a transmitter and a receiver) the received waveform starts slowly. The large rise time of the received waveform makes the moment of arrival indeterminate.

The most direct method to measure the moment of arrival is to determine when the received waveform exceeds a specified threshold value [2], [3]. Since the moment of threshold crossing depends both on the threshold and the intensity of the received waveform, a better method is to apply a threshold that is adapted to the intensity of the waveform (measured, for instance, by its peak value). Another approach is to apply curve fitting [2], [4]. On adoption of an error criterion between the observed waveform and a parametric model, the problem is to find the parameters that minimise the criterion. Useful criterions are the L_1 and the L_2-norm [5]. The most advanced method is to set up the problem within the framework of estimation theory. In its simplest form such an approach leads to cross correlation of the received signal with a template signal, i.e. matched filtering, and the determination of the moment of maximal correlation [2], [3], [4], [6], [7], [8] and [9].

The success of these techniques depends on whether the shape of the observed waveform is predictable or not. In open air, the shape of the waveform mainly depends on the characteristics of the tone burst, the transmitter, and the receiver. Hence, under these conditions the shape of the observed waveform is predictable and the techniques work fine. This paper addresses to the problem where there are reflective objects near the transmitter and receiver or near the path between these two. The echoes from these objects may interfere with the desired response. As a result, the observed waveform will be hardly predictable in a deterministic sense.

As an example, consider the waveforms in Fig. 1. The observed waveform on top is acquired with a transmitter and a receiver separated by a distance of *3.0 m* in face-to-face directions. Reflective objects in the vicinity of the measurement set-up cause extra reflections that interfere with the direct response. The interference pattern is hardly predictable in a deterministic sense because of the many unknown factors. The result of the (supposed) optimal matched filter/correlator is shown in the middle. Due to the interference the response achieves its maximum about *0.8 ms* after the arrival of the direct response. In this example, the interference is severe, and consequently the error of the

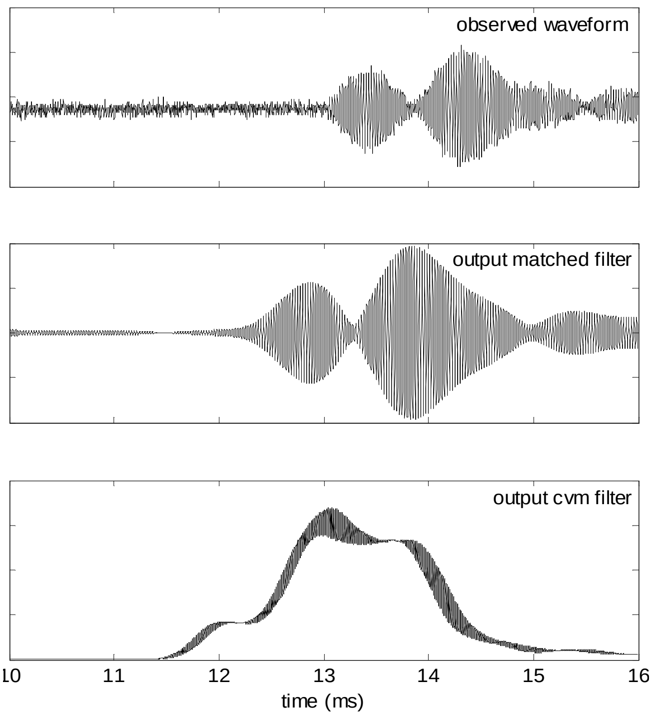

Fig. 1 Observed waveform, the output of the matched filter, and the output of the estimator based on the covariance model.

correlator is extremely large. In fact, the ad hoc methods designed without any optimality criterion, e.g. the threshold crossing method, would perform better than the 'optimal' correlator. The reason for this apparent discrepancy is that the mathematical model underlying the correlator does not apply in this case. The correlator has been designed with ignorance of possible interferences.

The paper introduces a new ToF estimator that is more robust against the presence of unwanted reflections. The base of this estimator is alike the one of the correlator, but it is extended with a model that describes the rise and fall of arriving echoes. The model is presented in [10]. The paper outline is as follows. Section 2 formalises the problem by the introduction of the mathematical framework. As such it provides the model on which the estimator will be built. Section 3 presents the actual development of the estimator. Experiments conducted to analyse and to evaluate the estimator are reported in Section 4. The paper finalises with a conclusion in section 5.

2. PROBLEM ANALYSIS

2.1 Statement of the problem

Given a waveform:

$$w(t) = \underline{a}\left(h(t-\underline{\tau}) + \underline{r}(t-\underline{\tau})\right) + \underline{n}(t) \quad (1)$$

with $\underline{a}$ the amplitude of the observed direct response, $h(t)$ the direct response of the acoustic measurement system to a tone burst but with a nominal energy (that is: $\|h(t)\|=1$), $\underline{\tau}$ the time-of-flight, $\underline{r}(t)$ the echoes due to reflections from nearby objects and $\underline{n}(t)$ white measurement noise. Underscored variables are random variables, and thus unknown.

The observed waveform $\mathbf{z} = \begin{bmatrix} z_0 & \cdots & z_{K-1}\end{bmatrix}^T$ is a sampled version of $w(t)$, i.e. $z_i = w(k\Delta)$ where Δ is the sampling period, and K is the number of samples. Hence, $K\Delta$ is the registration period. The problem is to estimate the time-of-flight $\underline{\tau}$ based on the measurements $\mathbf{z}$.

2.2 The conventional solution: matched filtering and correlation

The conventional solution to this problem is achieved by neglecting the reflections. In that case the measurements are modelled as $z_k = a\, h(k\Delta - \underline{\tau}) + \underline{n}(k\Delta)$ and the elements of the expectation of z_k are $\overline{z}_k(\tau) = a\, h(k\Delta - \underline{\tau})$. Assuming white Gaussian noise with variance σ_n^2 the vector $\mathbf{z}$ has a Gaussian conditional probability density with covariance matrix $\mathbf{C} = \sigma_n^2\mathbf{I}$.

Upon introduction of a vector $\mathbf{h}(\tau)$ with elements $h_k(\tau) = h(k\Delta - \tau)$ the conditional probability density of $\mathbf{z}$ is:

$$p(\mathbf{z}\,|\,\tau) = \frac{1}{\sqrt{(2\pi\sigma_n^2)^K}}\exp\left(-\frac{1}{2\sigma_n^2}\left(\mathbf{z}-a\mathbf{h}(\tau)\right)^T\left(\mathbf{z}-a\mathbf{h}(\tau)\right)\right) \quad (2)$$

Maximization of this expression yields the maximum likelihood estimate for τ. In order to do so, we only need to minimize the L_2 -norm of $\mathbf{z} - a\mathbf{h}(\tau)$:

$$\left(\mathbf{z}-a\mathbf{h}(\tau)\right)^T\left(\mathbf{z}-a\mathbf{h}(\tau)\right) = \mathbf{z}^T\mathbf{z} + a^2\mathbf{h}(\tau)^T\mathbf{h}(\tau) - 2a\mathbf{z}^T\mathbf{h}(\tau) \quad (3)$$

The term $\mathbf{z}^T\mathbf{z}$ does not depend on τ and can be ignored. Also, the second term can be ignored as a change of τ only causes a shift of the direct response. Hence, the maximum likelihood estimate implies finding τ that maximizes $a\mathbf{z}^T\mathbf{h}(\tau)$. A further simplification occurs if the extent of $h(t)$ is limited to, say $N\Delta$ with $N \ll K$. Then, $a\mathbf{z}^T\mathbf{h}(\tau)$ is obtained by cross correlating z_k by $a\, h(k\Delta + \tau)$:

$$y(\tau) = a\sum_{n=0}^{N-1} h(n\Delta - \tau)z_n \quad (4)$$

The value of τ which maximizes $y(\tau)$ is the best estimate. The operator expressed by (4) is called a matched filter or a correlator. Note that apart from its sign, the amplitude a does not affect the outcome of the estimate. Hence, the fact that a is usually unknown doesn't matter much.

2.3 Covariance model for the waveform with reflections

We return to the case of having reflections $\underline{a}\,\underline{r}(t)$. In a previous paper [10] we discussed a covariance model that provides a statistical description of the reflections. We assume for a moment that $\tau = 0$. The model postulated in [10] is that $\underline{r}(t)$ is a zero mean, Gaussian random process with a non-stationary autocovariance:

$$R_{rr}(t,u) \stackrel{def}{=} \mathrm{E}\left[\underline{r}(t)\underline{r}(u)\right] \approx \sigma_r(t,\mathbf{p})\hat{\sigma}_r(u,\mathbf{p})\int_{s=-\infty}^{\infty} h(s+t)h(s+u)ds \quad (5)$$

Here, $\sigma_r(t,\mathbf{p})$ is a function that modulates the standard deviation of the reflections so as to describe the rise and fall of the echoes arriving at the receiver. The vector $\mathbf{p}$ contains the parameters that describe this process. The numerical value of $\mathbf{p}$ is obtained by fitting $\sigma_r(t,\mathbf{p})$ to the standard deviation estimated from a number of observed waveforms.

For arbitrary τ, the reflections are shifted correspondingly. The sampled version of the reflections is $\underline{r}(k\Delta - \underline{\tau})$. With that, the elements of the covariance matrix $\mathbf{C}(\tau)$ of these terms, conditioned on τ, become:

$$C_{n,m}(\tau) = R_{rr}(n\Delta - \tau, m\Delta - \tau) \quad (6)$$

If the registration period is sufficiently large, the determinant $|\mathbf{C}(\tau)|$ does not depend on τ. This is so, because a change of τ merely causes a shift of elements in the matrix in the direction along the diagonal.

Besides $C_{n,m}(\tau)$, statistics of the amplitude $\underline{a}$ should be found to determine the probabilistic model of the full waveform. The observed waveform expressed in (1) involves two unknown factors, $\underline{a}$ and $\underline{\tau}$. The prior probability density of the latter is not important because the maximum likelihood estimator that we will apply does not require it. However, the first factor $\underline{a}$ is just a nuisance parameter. We deal with it by regarding $\underline{a}$ as a random variable with its own density $p(a)$. We conveniently assume $p(a)$ to be a zero mean, Gaussian density. With that, all information in $\underline{a}\, h(t-\underline{\tau})$ about τ is integrated in a conditional covariance matrix $\mathbf{B}(\tau)$ with elements:

$$B_{n,m}(\tau) = \mathrm{E}\left[\underline{a}^2 h(n\Delta-\tau)h(m\Delta-\tau)\right] = \sigma_a^2 h(n\Delta-\tau)h(m\Delta-\tau) \quad (7)$$

where σ_a^2 is the variance of the amplitude $\underline{a}$. The matrix $\mathbf{B}(\tau)$ can be written as $\mathbf{B}(\tau) = \sigma_a^2\mathbf{h}(\tau)\mathbf{h}^T(\tau)$. The advantage of modelling $\underline{a}$ as a zero mean Gaussian random variable is that the dependence of τ to $\mathbf{z}$ is now captured in a concise model, i.e. a single covariance matrix $\mathbf{D}(\tau)$:

$$\begin{aligned}\mathbf{D}(\tau) &= \mathbf{B}(\tau) + \sigma_a^2\mathbf{C}(\tau) + \sigma_n^2\mathbf{I} \\ &= \sigma_a^2\left(\mathbf{h}(\tau)\mathbf{h}^T(\tau) + \mathbf{C}(\tau)\right) + \sigma_n^2\mathbf{I}\end{aligned} \tag{8}$$

In the following section this matrix will be used to derive the estimator.

3. MAXIMUM LIKELIHOOD ESTIMATION OF THE TIME-OF-FLIGHT

With the signal modelled as a zero mean, Gaussian random vector with the covariance matrix given in (8), the likelihood function for τ becomes:

$$p(\mathbf{z}\,|\,\tau) = \frac{1}{\sqrt{(2\pi)^K\left|\mathbf{D}(\tau)\right|}}\exp\left(-\frac{1}{2}\mathbf{z}^T\mathbf{D}^{-1}(\tau)\mathbf{z}\right) \tag{9}$$

The maximization of this probability with respect to τ yields the maximum likelihood estimate of τ. Unfortunately, this solution as such is not practical because it involves the inversion of the matrix $\mathbf{D}(\tau)$. The size of $\mathbf{D}(\tau)$ is $K\times K$ where K is the number of samples of the registration (which can easily be in the order of 10^4). Thus, an economical solution is the main topic of the succeeding sections.

3.1 Principal components

If the registration period is sufficiently large, the determinant $\left|\mathbf{D}(\tau)\right|$ will not depend on τ. With that, we can safely ignore the factor $\left|\mathbf{D}(\tau)\right|^{-0.5}$. What remains is the maximization of the argument of the exponential:

$$\Lambda(\tau) = -\mathbf{z}^T\mathbf{D}^{-1}(\tau)\mathbf{z} \tag{10}$$

The functional $\Lambda(\tau)$ is a sufficient statistic. It reduces the measurement vector $\mathbf{z}$ to a single variable while retaining all information about τ that is captured in $\mathbf{z}$. For obvious reasons, $\Lambda(\tau)$ is called the "log-likelihood function".

The first computational savings can be achieved if we apply a principal component analysis to $\mathbf{D}(\tau)$. The matrix can be decomposed as $\mathbf{D}(\tau) = \sum_{k=0}^{K-1}\lambda_k(\tau)\mathbf{v}_k(\tau)\mathbf{v}_k^T(\tau)$. Here, λ_k and $\mathbf{v}_k$ are eigenvalues and eigenvectors of $\mathbf{D}$, that is $\mathbf{D}\mathbf{v}_k = \lambda_k\mathbf{v}_k$ with $\left\|\mathbf{v}_k\right\| = 1$. The inverse matrix takes the form:

$$\mathbf{D}^{-1}(\tau) = \sum_{k=0}^{K-1}\frac{\mathbf{v}_k(\tau)\mathbf{v}_k^T(\tau)}{\lambda_k(\tau)} \tag{11}$$

Using (11) the expression $\Lambda(\tau)$ becomes:

$$\Lambda(\tau) = -\mathbf{z}^T\left(\sum_{k=0}^{K-1}\frac{\mathbf{v}_k(\tau)\mathbf{v}_k^T(\tau)}{\lambda_k(\tau)}\right)\mathbf{z} = -\sum_{k=0}^{K-1}\frac{\left(\mathbf{z}^T\mathbf{v}_k(\tau)\right)^2}{\lambda_k(\tau)} \tag{12}$$

The computational savings are obtained by ignorance of all terms in (12) that do not capture much information about the true value of τ. If we suppose that the λ_k's and $\mathbf{v}_k$'s are arranged according to their importance with respect to the estimation, and that above some value of k, say J, the importance is negligible then the number of terms in (12) is reduced from K to J. In practice, a speed up with a factor 10^3 is feasible because K (on the order of 10^4) is replaced by J that might be on the order of 10.

3.2 Selection of good components

The problem addressed in this section is how to order the eigenvectors in (12) such that the most useful components come first, and thus will be selected. For that purpose, we rewrite (12) as follows:

$$\Lambda(\tau) = \sum_{k=0}^{K-1}\frac{\left(\lambda_k - \sigma_n^2\right)\left(\mathbf{z}^T\mathbf{v}_k(\tau)\right)^2}{\lambda_k(\tau)\sigma_n^2} - \sum_{k=0}^{K-1}\frac{\left(\mathbf{z}^T\mathbf{v}_k(\tau)\right)^2}{\sigma_n^2} \tag{13}$$

However, since the set of eigenvectors $\mathbf{v}_k(\tau)$ forms an orthonormal basis in the space of $\mathbf{z}$, the second term is just $\left\|\mathbf{z}\right\|^2/\sigma_n^2$. It does not depend on τ. The maximum likelihood estimate for τ appears to be equivalent to the one that maximizes:

$$\sum_{k=0}^{K-1}\gamma_k(\tau)\left(\mathbf{z}^T\mathbf{v}_k(\tau)\right)^2 \text{ with } \gamma_k(\tau) = \frac{\lambda_k(\tau) - \sigma_n^2}{\lambda_k(\tau)\sigma_n^2} \tag{14}$$

Each factor $\gamma_k(\tau)$ can be regarded as a weight for the corresponding term $\mathbf{z}^T\mathbf{v}_k(\tau)$. Since $\lambda_k(\tau) \geq \sigma_n^2$, and thus $\gamma_k(\tau) \geq 0$, this weight is a good criterion to measure the importance of an eigenvector. Hence, a plot of $\gamma_k(\tau)$ versus k is helpful to find a reasonable value of J. Hopefully, $\gamma_k(\tau)$ is large for the first few k, and then drops down rapidly to zero.

3.3 The computational structure of the estimator

When (14) is implemented a practical problem arises. The eigenvectors and eigenvalues must be calculated for varying values of τ. Since the dimension of the $\mathbf{v}_k$'s is very large, this is not computationally feasible. We solve this problem by limiting the inner products $\mathbf{z}^T\mathbf{v}_k(\tau)$ arising in (14) to a window of $\mathbf{z}$. The window starts at m and ends at $m+N-1$. Thus, it comprises N samples. We stack these samples into a vector $\mathbf{x}(m)$. Each value of m corresponds to a hypothesized value $\tau = m\Delta$. Instead of applying operation (14) for varying τ, we fix the value of τ to zero and replace $\mathbf{z}$ by the moving window $\mathbf{x}(m)$.

$$y(m) = \sum_{k=0}^{J-1}\gamma_k(0)\left(\mathbf{x}(m)^T\mathbf{v}_k(0)\right)^2 \tag{15}$$

If $\hat{m}$ is the index that maximizes $y(m)$, then the estimate for τ is found as $\hat{m}\Delta$.

The operation $\mathbf{x}(m)^T\mathbf{v}_k(0)$ is just a FIR filter. The computational structure (Fig. 2) consists of a parallel bank of J filters/correlators, one for each eigenvector $\mathbf{v}_k(0)$. The results of the filters are squared, multiplied by weight factors γ_k, and then accumulated to yield the signal

$$y(m) = \sum_{k=0}^{J-1}\gamma_k(0)\left(\mathbf{x}(m)^T\mathbf{v}_k(0)\right)^2 \tag{16}$$

Then the estimate for τ is found as $\hat{m}\Delta$ if $\hat{m}$ is the index that maximizes $y(m)$.

The procedure to get the eigenvectors and weight factors is as follows: First, calculate the $N\times N$ matrix $\mathbf{D}(0)$ according to (8) using (5) and (6). Next, calculate the normalized eigenvectors and eigenvalues of $\mathbf{D}(0)$. Then, sort the eigenvectors and eigenvalues in descending order of γ_k. After that, select the first J eigenvalues and eigenvectors. The first J eigenvectors are the kernels of the filters. Ideally, the selection of J is such that $\gamma_k \approx 0$ for $k > J$.

4. EXPERIMENTS

In this section, the experiments that have been conducted to validate the proposed estimation method are introduced. 150 measurements were performed under different conditions: room, location, height above the floor, distance between transducers, etc. These measurements have been used as a benchmark to compare the performance of the new estimator with the one of the matched filter. Details of the experimental set-up are given in section 4.1. Section 4.2 presents the results and section 4.3 finalises with a discussion.

4.1 Experimental set-up

Using an acoustic measurement system, 150 data records have been acquired under various conditions. The acoustic system uses two air ultrasonic ceramic transducers mounted on pedals in a face-to-face direction. A waveform generator applied a 40 *kHz* sinusoidal tone burst consisting of twenty cycles to a transmitter. The transmitted signal was detected by a receiver. The received waveform was acquired at a sampling period Δ of $2\,\mu s$. The two transducers were piezoelectric sensors.

In addition to the 150 records, also a special record was acquired in an anechoic room. This waveform is used as a reference waveform representing the nominal response $\mathbf{h}(0)$. Before the actual experiments took place, all records were individually processed in order to identify the true ToFs of each record and to estimate the parameter vector $\mathbf{p}$, that is mentioned in (5), and that describes the covariance model. The records were also used to estimate $\hat{\sigma}_n = 0.0025$ and $\hat{\sigma}_a = 0.0026$. Details of the estimation procedure are explained in [10].

Using (8), we calculated $\mathbf{D}(0)$ with $\mathbf{h}(0)$, $\mathbf{p}$ and σ_n. After that $\lambda_k(0)$ and $\mathbf{v}_k(0)$ are obtained. The window size selected was $N = 1000$, whereas the number of samples in a record is between 5000 and 10000 samples. Fig. 3 shows the first 50 eigenvalues and their weights obtained from (17). The figure suggests that a large number of eigenvectors have weights that differ significantly from zero. Hence, a large number of filters/correlators would be needed to obtain the best result.

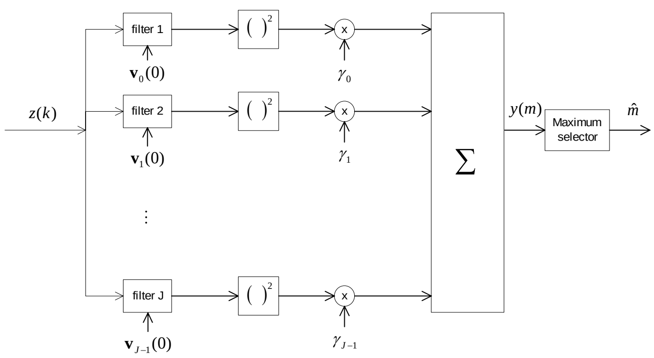

Fig. 2 Computational structure of the estimator

4.2 Results

The performance of the estimator has been assessed for varying numbers of J to determine the number of filters/correlators. The performance involves two error factors, the bias and the standard deviation that are combined in the *RMS* (root mean squared). This error measure is calculated on the 150 records. It appeared that the estimator performs best if the number of filters is restricted to seven.

Application of the estimator to the 150 records and comparison of the estimated ToFs with their true value resulted in measurement errors that are shown in Fig. 5. The figure also shows the errors made by the conventional matched filter. A statistical analysis applied to these errors yields the following result (units in *ms*):

	Bias	standard deviation	*RMS*
Cov. estimator	-0.010 ± 0.002	0.027 ± 0.002	0.029
Matched filter	0.007 ± 0.01	0.12 ± 0.01	0.12

Fig. 5 shows that the matched filter produces two outliers. One of the two waveforms that gave rise to such an outlier is shown in Fig. 1. The interaction between the direct response and the first echo is such that the maximum value of the second interfering peak is much larger than the one of the first peak. In this example, the true ToF is near 13 *ms*. The matched filter has its maximum response near the second peak at 13.8 *ms*. The covariance based filter produces a maximum near the first peak at 13.1 *ms*.

Fig. 5 shows how the covariance based estimator builds up the log-likelihood function. In this example, the observed waveform $\mathbf{z}$ is given in Fig. 1. The waveforms on the left side are the outputs $\mathbf{x}(m)^T\mathbf{v}_k(0)$. The waveforms on the right are the squared and accumulated signals, i.e. $\Sigma_{k=0}^{J-1}(\mathbf{x}(m)^T\mathbf{v}_k(0))^2$ with J running from 1 up to 7.

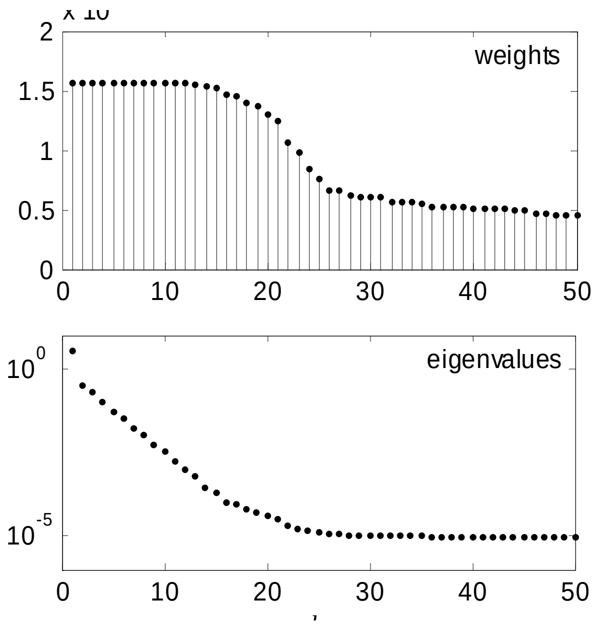

Fig. 3 The first 50 eigenvalues and weights of the covariance based estimator

4.3 Discussion

Since $\mathbf{h}(0)\mathbf{h}^T(0)$ is the dominant contribution to $\mathbf{D}(0)$, the first eigenvector $\mathbf{v}_0(0)$ is close to the direct response $\mathbf{h}(0)$. Apparently, the first filter is equivalent to the matched filter, and our estimator embeds the conventional matched filter. Since all eigenvectors are orthogonal, the contributions of the other filters are supplementary to the matched filter. In the absence of reflections, the contribution of $\mathbf{C}(0)$ to $\mathbf{D}(0)$ vanishes, and consequently all weights except the first one will be zero then. Thus, our estimator is a generalisation of the matched filter

The statement that the information conveyed by the output of the other filters is supplementary is illustrated in Fig. 5. Clearly, the matched filter (first row in the figure) is misled seriously by the presence of the reflections. The other filters catch these reflections and correct the output of the matched filter. In the example of Fig. 5, the 3rd, 4th and 7th filters provide the correction. For other reflections, as present in other records, the other filters are useful. In a waveform without reflections, the contribution of these extra filters is zero, due to the orthogonality of the filter responses.

The newly proposed estimator outperforms the matched filter by a factor of about 4. Nevertheless, we have observed that the optimal number of filters is seven whereas the theoretical number is much larger. Probably, the explanation of this apparent contradiction is that the reflection model that we have used is only approximate. These modelling errors must be responsible for the deficiency. Future research must confirm this hypothesis

5. CONCLUSION

The introduction of a model for the reflection in terms of a non-stationary covariance function leads to a new estimator for the time-of-flight of an acoustic tone burst. This estimator is a generalisation of the well-known matched filter since in the absence of reflections the new estimator and the matched filter are equivalent. In many practical circumstances, for instance indoor measurements, the ignorance of reflections can lead to large estimation errors. Application of the new estimator can reduce these errors by about a factor four.

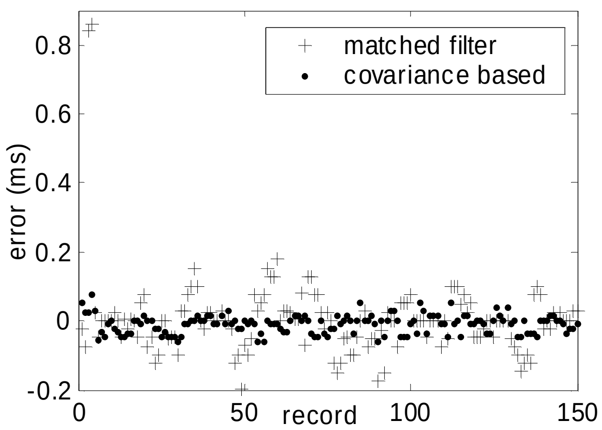

Fig. 5 Measurement errors of the ToFs of the 150 records

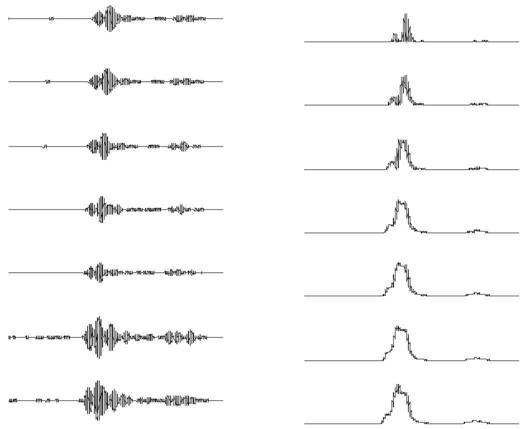

Fig. 4 The response of the first seven filters to the waveform in Fig. 1. Left: output of the filters. Right: Squared, weighted, and accumulated outputs.

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Authors: dr.ir. F. van der Heijden, M.Sc. G. Tuquerres, prof.dr.ir. P.P.L. Regtien, Laboratory for Measurement and Instrumentation, Faculty EEMCS, University of Twente, P.O. Box 217, 7500AE Enschede, The Netherlands, mailto:F.vanderHeijden@utwente.nl.