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A superfluid gyrometer: quantized currents and fractional trapped circulation

R. Aarts[a*], G.G. Ihas[a†], O. Avenel [a] and E. Varoquaux[b]

[a]Service de Physique de l'Etat Condensé, Centre d'Etudes de Saclay, 91191 Gif-sur-Yvette Cedex (France)

[b]Laboratoire de Physique des Solides, Bât. 510, Université Paris-Sud, 91405 Orsay (France)

A detailed analysis of phase-slip observations in superfluid ^{4}He in a double-hole hydromechanical resonator fitted with a micro-aperture yields the remanent current trapped in the superfluid loop threading the two holes. The value of this bias current remains constant in each given cool-down below the λ point. Its value at each cool-down appears to be fixed at random. It can be determined to about a hundredth of the circulation quantum. This is the present practical resolution of the double-hole resonator used as an absolute gyrometer.

Double-hole hydromechanical resonators fitted with a microscopic orifice and a long channel, as shown in fig.1, are close analogs, for superfluids, of the well-known *rf*-SQUIDS in superconductivity [1]. Their operation can be analyzed in detail [2], and so can their output. This output consists of the positive and negative going peak amplitude of the membrane deflection d at successive half cycles of the resonant motion. Its remarkable features are sudden jumps, from one half cycle to the next, of the resonance amplitude by quantized amounts: these discrete dissipation events signal the occurrence of phase slips by 2π of the quantum phase difference $\theta = \varphi_{out} - \varphi_{in}$ of the superfluid wavefunction outside and inside the resonator. We briefly describe the effect of trapped circulation on the resonator output.

We start with the continuity equation expressing the fact that the volume (compressibility is neglected) swept by the membrane per unit time equals the sum of the flows through the weak link and the long channel:

$$S\dot{d} = s_w v_w + s_l v_l. \tag{1}$$

The velocity is related to the gradient of the phase, $v = (\hbar/m)\nabla\varphi$ or, by definition of the hydraulic length l of an orifice [3], $v = \hbar\theta/ml$. A

*Permanent address: Physics Department, Eindhoven University of Technology - 5600 MB Eindhoven - The Netherlands

†Permanent address: Department of Physics, University of Florida, Gainesville - FL 32611 - USA

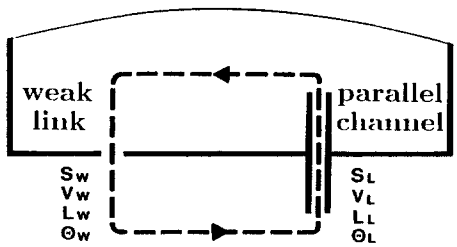

Figure 1. Two-hole resonator. The flexible membrane has area S and deflection d. The dashed line marks the superfluid loop threading the two holes.

given orifice i is then characterized by its cross section s_i and its length l_i. If a persistent current is present in the loop threading the two holes, it will give rise to flow velocities in the holes v_{0w} and v_{0l} which persist even when the membrane is at rest and which obey eq.(1) with $\dot{d} = 0$. This loop current can be caused either by a rotation with respect to the inertial frame of reference of the resonator with angular velocity Ω, or by vorticity randomly pinned in the flow path. This current amounts to a trapped circulation κ_b which is not necessarily quantized. The total circulation threading the loop is given by

$$\kappa = \oint v_s dl = v_w l_w - v_l l_l = \kappa_b + n\kappa_4. \tag{2}$$

It changes by multiples of the quantum of cir-

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culation κ_4 when the velocity reaches v_c in the weak link: a phase slip by $2j\pi$ occurs, θ_w changes by $2\pi j$, the quantum state of the loop n by j, the flow velocity v_w by $\kappa_4 j/l_w$, which is always a decrease in absolute value, and the membrane velocity $\dot{d}$ by $s_w \kappa_4 j/S l_w$, again a decrease in absolute terms. [1] The changes in v_{0w} and v_{0l} are found, to be $j\kappa_4 R/(1+R)l_w$ $and - j\kappa_4/(1+R)l_l$ respectively, with $R = l_w s_l/l_l s_w$.

The absolute value of membrane velocity at which v_c is reached in the weak link depends on the magnitude and direction of the current trapped in the loop, i.e. on n, and is given by:

$$S\dot{d}(\epsilon, n)_c = s_w v_c(1+R) - \epsilon(n\kappa_4 + \kappa_b)s_l/l_l. \quad (3)$$

The index ϵ is ± 1 according to whether the current induced by the membrane adds to or subtracts from the persistent current in the loop, *i.e.* ϵ and n have the same sign: the apparent membrane critical velocity is always reduced by trapped currents.

This effect is monitored on the experimental traces as follows. A strict (computerized) accounting is kept of the slips which take place over time, as well as of the changes in the quantum state of the loop and of the membrane amplitudes at which the slips occur for each quantum state. Membrane amplitudes and velocities are assumed to be proportional to one another because the quality factor of the resonator is very high. The most frequently met quantum state is assigned, by definition, quantum number n zero. The next most populated states have $n = \pm 1$ which are reached by single phase slips ($j = \pm 1$). The states with higher quantum numbers are reached when multiple slips occur. These events are most frequent at low temperature (*i.e.* when the critical velocity is high). They have been noted [4] [5] to take place preferentially in one flow direction. A plot of d_c versus n, as in fig. 2, yields the quantity R, the ratio of the hole inertances, and κ_b/κ_4, the fractional part of the bias. In our set-up, the superfluid loop has too small an area to give rise to measurable effects due to rotation and the bias is entirely due to remanent vortices. The value of

[1]The critical threshold v_c need not, in principle, be the same for both directions of flow but the experimental evidence [4] indicates no measurable difference.

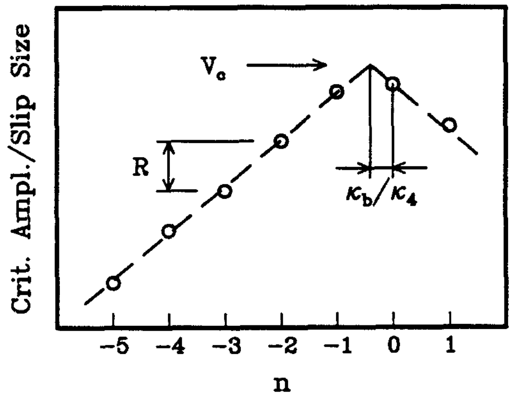

Figure 2. Determination of the bias.

this bias is apparently random for each cool-down below $T\lambda$, but it remains stable unless the membrane is severely overdriven. Such perturbations can cause a rearrangement of the remanent vorticity in the cell, but no spontaneous vortex motion takes place, apart from the one causing the phase slip. The resolution which is achieved at present is of the order of 10^{-2} κ_4 for data records of 20 minutes. The statistics on slip counts could be increased by two orders of magnitude by using suitable driving signals, thus boosting the resolution to 10^{-3} κ_4, a value which fixes the present limit of the device used as a gyrometer. The method described here yields κ_b directly in units of κ_4 and is inherently more stable and sensitive than that based on staircase patterns.

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