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Numerical simulations of the operation of a superfluid gyrometer

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We have performed detailed numerical simulations of the operation of a double-hole hydromechanical resonator operated in superfluid ^{4}He. These simulations make use of measured parameters and can be compared in a realistic way to the actual outputs of the device. Most of the experimental observations are reproduced in minute detail and, in particular, the occurrence of staircase patterns. The simulation program can be used to determine the effect of each of the parameters governing the resonator operation and to predict its effectiveness as a gyrometer.

Miniature Helmholtz resonators, in which the displacement of a flexible membrane controls the flow of liquid helium through a small orifice and a long channel in parallel, have been used for several years to study phase slippage in superfluid ^{4}He [1]. The resonator is usually driven by repeatedly firing, in quadrature with the output signal, single sinusoidal periods of amplitude V_1 and frequency slightly above the resonance frequency ω_0. A dc bias V_0 is also applied to linearize the excitation. Phase slips occur whenever the critical velocity is exceeded in the orifice.

The operation of these devices can easily be simulated numerically by direct integration of the equations of motion [2]. Each cycle of excitation is divided into N integration steps of duration dt. In the i^{th} iteration, the membrane velocity and its displacement are updated in the simplest way as:

$$\dot{X}_i = \dot{X}_{i-1} + \omega_0^2 (2bV_0V_1 \sin\frac{2\pi i}{N} - X_{i-1} - \frac{\dot{X}_{i-1}}{\omega_0 Q})dt$$
$$X_i = X_{i-1} + \dot{X}_i \, dt \, , \qquad (1)$$

in which Q, the quality factor of the resonator, is easily measured from the free decay time, and b is a calibration constant, obtained by recording the amplitude change in response to an applied dc bias.

When a persistent circulation κ_x is trapped in the loop threading the two holes, the phase difference across the orifice can be written as:

$$\frac{\theta}{2\pi} = \frac{\dot{X}}{(1+R)\,\omega_0\,\Delta X_{2\pi}} + \frac{R}{1+R}\frac{\kappa_x}{\kappa_0} \, , \qquad (2)$$

in which R is the ratio of the hydraulic inductances of the two holes, and $\Delta X_{2\pi}$ is the recorded change in maximum amplitude after a single 2π phase slip.

In the thermally activated regime [3], the vortex nucleation rate, at low temperature, is given by:

$$\Gamma = \Gamma_0 \exp\left((-2/3)(E_J/T)(1-\theta^2/\theta_{c0}^2)^{3/2}\right) . \qquad (3)$$

The attempt frequency $\Gamma_0 \sim 1.9\ 10^{10}\ Hz$, the junction energy $E_J \sim 50\ K$, and θ_{c0}, the critical phase difference at T=0, are fitted to the experimental data.

The probability of observing k phase slips *at most* in a time dt is then:

$$P(k) = \left(1+\Gamma dt/1!+\cdots+(\Gamma dt)^k/k!\right)\exp(-\Gamma dt) \, . \qquad (4)$$

In the simulation, the number of phase slips induced in each integration step is the least number k for which $P(k)$ is greater than some random number between 0 and 1. For each phase slip, θ is changed by 2π, κ_x by one quantum, and the membrane velocity is changed according to Eq.(2). When these changes are only applied at the beginning of the next integration step, dt plays the role of a delay for the completion of a phase slip.

Simulated data generated in this way are extremely

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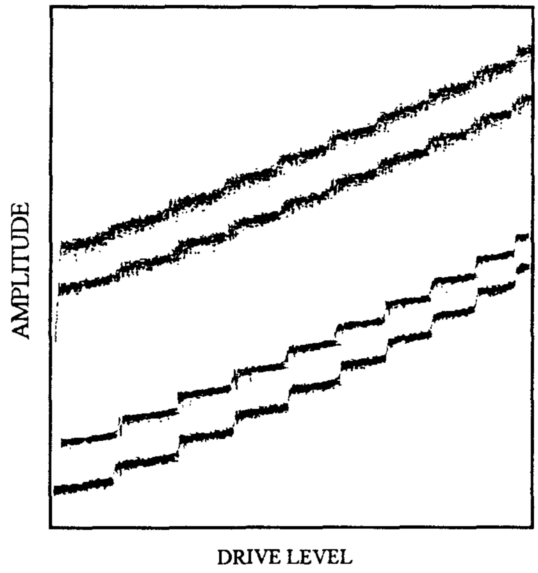

Figure 1. Real data (top and bottom traces), and simulated data (middle traces).

realistic, with the added advantage that the sequence of events leading to some peculiar behavior can be analyzed in great detail. Especially 'noisy' or 'pathological' traces recorded occasionally are seen to be due to the stochastic nature of the phase slip events, and should not be attributed, in most cases, to external disturbances.

This simulated data can be analyzed in the same way as real data [3], and the analysis method is strongly validated by the fact that the values extracted in the process, the fractional bias κ_x for instance, come out to be precisely the right ones.

As explained above, the simulation allows, through Eq.(4), for the occurrence of multiple slips due to the finite delay necessary for the completion of a phase slip. If the liquid in the orifice is being strongly accelerated when phase slippage is first triggered, the probability of nucleating vortices in salvoes before the liquid has time to slow down becomes very high, especially at low temperature where the nucleation rate, Eq.(3), is a very steep function of the velocity.

Indeed, the simulation shows qualitatively that multiple slips should become a real nuisance at operating frequencies in the *kHz* range, and that their magnitude and rate of occurrence can be very different in both directions, depending on the precise value of the fractional bias κ_x. But, even with very long and unrealistic delay times, the program mostly fails to reproduce the experimental observations regarding these multiple slips at low frequency [4,5]. They seem to be triggered at random for reasons which, very often, cannot be traced back along the preceding lines. Some other mechanism must be invoked for their explanation.

When the drive level is slowly swept up, staircase patterns are observed [1], similar to those seen in rf-SQUIDS. Two such patterns, obtained in the same cell at the same temperature but at 4 years interval, are represented in Fig.1, with the real data on top and bottom, and the simulation in the middle. When fed with the right parameters, the simulation program perfectly reproduces the experimental data.

The large qualitative difference between the two sets of data at 4 years interval is just a consequence of the time evolution of some important parameters governing the operation of the cell: the critical phase difference in the orifice went up from 30 (bottom) to 36 (top), the frequency ω_0 changed from 6.3 to 12.2 *Hz*, the *R* factor from 5.3 to 3.9, and multiple slips also became slightly more frequent.

The simulation allows us to quantify the influence of each individual parameter on the overall quality of the observed patterns. It shows that if these devices are to be used as gyrometers by observing the staircase patterns, the cell parameters must be carefully chosen, and extrapolations about their ultimate sensitivity must be done with great caution.

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