A new approach to model tyre/road contact.

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Abstract
In the Structural Dynamics and Acoustics group at the University of Twente, we aim to develop a quantitative
tyre/road noise model. An essential part of this model is an accurate contact algorithm which is fast enough
to simulate tyre vibrations up to the acoustic frequencies. In this paper we present a contact algorithm,
describing the contact between a tyre and a road surface, which has the potential to be made very fast using
the multigrid techniques developed in the field of elasto-hydrodynamic lubrication. For the development of
the algorithm a flexible ring model is used to describe the tyre. The friction model is based on Coulomb’s
friction. We present (quasi-)static results obtained from the algorithm for various friction coefficients, as
well as frictionless results for a rotating tyre. The vibrations of the tyre obtained by this model have been
used to calculate the radiated sound field by means of a boundary element program (BEMSYS).

1 Introduction

For vehicles driving under normal conditions the most important noise source is the noise generated by the
interaction between the rotating tyre and road surface. To understand the complex interaction between tyre
and road and the noise that is radiated due to this interaction, researchers have developed models describing
the dynamical behavior of the tyre, developed contact models and models to predict the acoustic radiation
from a wide variety of noise generating mechanisms (radial- and tangential vibration of the tyre, stick-
slip, stick-snap, cavity resonance, air pumping, air resonance and the horn effect). For an overview and
explanation of all noise generating mechanisms, the reader is referred to [6].

Essential to a quantitative prediction of noise, is an excellent description of the tyre vibrations for the tyre
being in contact with the road surface. For the dynamical behavior of the tyre, being a very complex structure
with respect to both the geometry as well as the rubber material behavior, sophisticated models are available
(even in commercial FEM packages like ABAQUS). Unfortunately, the capability of the models to describe
vibrations up to the kHz frequencies is limited; the accuracy is less and the computation times are large.

With respect to contact algorithms, the ones being used today in tyre/road noise studies, are not as fast as
the models used in the field of elasto-hydrodynamic lubrication, see [5], [8], [10] and [9]. The multigrid and
multilevel integration techniques used in the latter field, speed up calculation times to such an extent that
high accuracy solutions can be obtained within a minimum of computation times. As a result, all kind of
transient effects influencing the lubricant film, like the over-rolling of texture and vibrations of the rolling
elements, can be simulated well within engineering accuracy.

Our intention is to bridge the gap between the fast numerical methods used in elasto-hydrodynamic lubrica-
tion and the contact algorithms used in tyre/road noise studies. For this, we need a (general) contact algorithm
that allows for incorporation of the fast numerical techniques. Therefore, we present our first attempt for a
new approach to model tyre-road contact, which, albeit in a rudimentary from, resembles the approach used
in elasto-hydrodynamic lubrication. The results reported here have been generated in the MSc projects of
M.T. van Zoelen [16] and A. Berendsen [2].
2 Tyre/road noise model

The model requires essentially three separate numerical models; a model for the tyre, a contact model (which includes a contact condition and a friction model) and a radiation model. This approach allows for the development of the individual models independently from the other two, without affecting the entire model structure.

The tyre is modeled by a 2D flexible ring model, as described in [4], [14] and [15]. A contact condition is used, which states that no part of the tyre may penetrate the road surface (the condition is actually an inequality or constraint equation). The friction model is based on Coulomb’s friction. The calculated structural vibrations of the tyre are used as input for a 3D boundary element code (BEMSYS), see [13], to calculate the radiated sound.

2.1 Tyre model

A 2D flexible ring model is used to explain the suggested approach. The model is shown in figure (1). The ring is assumed to be a thin elastic body of thickness \( h \) and radius \( R \), the thickness being small compared to the radius and displacements of the ring. Stress stiffening due to the pressurized air in the tyre is accounted for. The radial displacement, \( u_r(\theta) \), is assumed to be constant over the thickness, whereas the tangential displacement \( u_\theta(\theta) \) varies linearly over the thickness. The material is assumed to be isotropic and Hooke’s law is used to relate strains to stresses (Young’s modulus \( E \)). Rotary inertia is neglected. The ring is supported by an elastic foundation (a distributed radial and tangential stiffness \( k_r \) and \( k_\theta \), respectively), representing the side-wall stiffness.

![Ring model](image)

Figure 1: Ring model (taken from [14]) (a). Degrees of freedom \( u_r \) and \( u_\theta \). Note that the definition of \( \theta \) used in the current paper is according to figure (b).

The equations of motion for the flexible ring are, see [4], [14] and [15]:

\[
\rho \ddot{u}_r + \lambda \dot{u}_r + \frac{EI}{R^2} \left( \frac{\partial^3 u_r}{\partial \theta^3} - \frac{\partial^3 u_\theta}{\partial \theta^3} \right) + \frac{Eh}{R^2} \left( u_r + \frac{\partial u_\theta}{\partial \theta} \right) + \frac{p_0}{R} \left( u_r - \frac{\partial^2 u_r}{\partial \theta^2} + 2 \frac{\partial u_\theta}{\partial \theta} \right) + k_r u_r = f_r \quad (1)
\]

\[
\rho \ddot{u}_\theta + \lambda \dot{u}_\theta - \frac{Eh}{R^2} \left( \frac{\partial^2 u_r}{\partial \theta^2} + \frac{\partial u_r}{\partial \theta} \right) + \frac{EI}{R^3} \left( \frac{\partial^3 u_r}{\partial \theta^3} - \frac{\partial^2 u_\theta}{\partial \theta^2} \right) + p_0 \left( u_\theta - \frac{\partial^2 u_\theta}{\partial \theta^2} - 2 \frac{\partial u_r}{\partial \theta} \right) + k_\theta u_\theta = f_\theta, \quad (2)
\]

where \( \rho \) is the density, \( \lambda \) the damping coefficient, \( I = h^3/12 \) denotes the area moment of inertia per unit width, \( p_0 \) is the air pressure and \( f_r \) and \( f_\theta \) are the distributed forces per unit width in the, respectively, radial and tangential direction.

For convenience, the differential equations (1) and (2) are rewritten as:
and have been discretized using a finite difference scheme, given in Appendix A. Note that the method is not restricted to finite difference discretization; a finite element discretization can also be used. This requires an iterative solver on the element matrix level and not on the (global) system matrix level.

Essential in the development of a fast numerical solution procedure is that the tyre model equations can be used to solve the unknown displacements \( u_{r,\theta} \) if the forces \( f_{r,\theta} \) are known, but also that they can be used to calculate the unknown forces \( f_{r,\theta} \) if the displacements \( u_{r,\theta} \) are given. This is the case for the position of points on the tyre which are in the footprint, as these positions are equal to the position of the road surface (or, if friction is included, can be calculated from the position of the road and the friction model used).

For simplicity, we use a coordinate system which is fixed to the tyre. This implies that, in the transient calculations, we let the road surface rotate around the tyre. This is however not essential to the method.

### 2.2 Contact model

As the tyre comes in contact with the road surface, the tyre will deform. The actual position of all points of the tyre in the footprint obviously depends on the position of the road. However, it also depends on the friction forces between the tyre and the road surface, as well as the tyre model (note that the displacements \( u_{r,\theta} \) should also satisfy the differential equations in the footprint!). The normal and friction force will cause the tyre to adhere to the road surface and, as already indicated, the position of the tyre in the footprint equals the position of the road surface. If the friction force exceeds a maximum friction force at some points in the footprint (as in Coulomb’s friction) these points will slip and their position, i.e. the displacements \( u_{r,\theta} \), is determined by the geometry of the road surface, the maximum friction force and the tyre model.

#### 2.2.1 Contact condition

The contact condition is simply a constraint equation, specifying that the tyre can not penetrate the road surface. For simplicity, we discuss a road surface with no texture, but we have extended the algorithm to various types of texture. For the ring model given above and a smooth road surface, this implies that the radial and tangential displacements of the tyre should satisfy:

\[
g(u_r, u_\theta) = (R + u_r) \cos(\theta) - u_\theta \sin(\theta) + R - \delta \geq 0,
\]

where \( \delta \) denotes the mutual approach of the rim to the road surface, see figure (1). Thus, if \( g(u_r, u_\theta) < 0 \) the tyre penetrates the road surface and the contact condition is violated. On the other hand, in the contact region or footprint the displacements of the tyre satisfy the equality:

\[
g(u_r, u_\theta) = 0,
\]

which, as is discussed below, is one of the equations for the two unknown displacements \( u_{r,\theta} \) in the footprint.

#### 2.2.2 Friction model

The contact forces \( f_{r,\theta} \) and displacements \( u_{r,\theta} \) are, besides the contact condition, determined by the friction model. For given displacements \( u_{r,\theta} \) of any point of the tyre, one can calculate the actual forces needed to
keep that point at that position. Hence, one can also calculate the (normal) normal force $p_n$ and shear stress $\tau$. For a non-textured road surface, $p_n$ and $\tau$ follow from:

$$
p_n = f_r \cos(\theta) - f_\theta \sin(\theta)
\tag{7}
$$

$$
\tau = f_r \sin(\theta) + f_\theta \cos(\theta)
\tag{8}
$$

As the displacements $u_{r,\theta}$ are known, we can use equations 3 and 4 to calculate, respectively, the forces $f_r$ and $f_\theta$. If the contact is assumed to be frictionless, i.e. if $\tau = 0$, this implies

$$
L_r(u_\theta, u_r) \sin(\theta) + L_\theta(u_\theta, u_r) \cos(\theta) = 0
\tag{9}
$$

and together with $g(u_r, u_\theta) = 0$, one can solve for $u_r$ and $u_\theta$.

If friction is accounted for $|\tau| \leq \mu |p_n|$ when there is no slip and $u_{r,\theta}$ can, initially, be determined from the position of the road surface. If $|\tau| > \mu |p_n|$ slip occurs and, besides the contact condition, $\tau = \mu p_n$. Substitution of equations 7 and 8 yields:

$$
L_r(u_\theta, u_r) (\sin(\theta) \mp \mu \cos(\theta)) + L_\theta(u_\theta, u_r) (\cos(\theta) \pm \mu \sin(\theta)) = 0,
\tag{10}
$$

where the sign ($\pm$) depends on the sign of $\tau$.

### 2.3 Radiation model

The vibrations of the tyre, calculated by the tyre/contact model, are used as input for the 3D boundary element code (BEMSYS), see [13]. Hence, phenomena like the horn-effect are accounted for. Noise absorption of the road surface can be included as well. With respect to the current 2D tyre model, we assume all points to vibrate along the entire width of the tyre.

![Figure 2: Boundary element source mesh (a) and source and field mesh (b).](image)

As in the BEM code the source mesh is fixed, we need to transform the vibrations of the rotating tyre to normal vibrations on a fixed source mesh. The average shape of the tyre, as calculated by the tyre/contact model, is used as the 3D source mesh, see figure 2. The vibrations are subsequently interpolated to this fixed source mesh. Furthermore, the vibrations of the (now stationary) source mesh are assumed to be harmonic in the boundary element code. Transforming the time signal to the fourier domain seems trivial. However, for some points points on the source mesh close to the contact region, the actual displacement can be such that they come in and out of contact. Once these points are in contact, they should not be able to radiate sound
at all, as the position of that point in the contact is assumed to be fixed. Therefore, near the contact region, we keep track of the volume (between the tyre and the road surface) in the very small region between the footprint and those points which are never in contact. The change of the volume in time is then converted to a harmonic velocity and this value is set to the velocity of an element radiating from the contact region into the ’horn’. Note that a similar approach can be used to simulate air-pumping.

2.4 Solution procedure

The deformation of the tyre outside the footprint should satisfy the differential equation (or element matrix equations). For all points in the footprint, the position is given and one can calculate the contact forces needed to put the tyre at that position. We use this procedure to iterate to the solution.

Using the finite difference equations, the discrete analogue of the contact condition and the friction model, a Newton-Raphson update is, consecutively, calculated for each of the discrete points of the tyre (we use Gauss-Seidel relaxation). Before proceeding with the next point, we check whether the contact condition is violated. If so, the position of that discrete point is fixed to the road surface. The forces required to keep that point at that position is calculated (the normal and shear forces \( p_n \) and \( \tau \)). It is then checked whether \(|\tau|\) is smaller than \( \mu |p_n| \). If so, the forces are correct and the algorithm can proceed with the next point. If not, the position of that point of the tyre is calculated from solving the displacements from the equations \( g(u_r, u_\theta) = 0 \) and equation 10. For points already fixed to the road surface, one does need to check whether the normal force becomes negative. If so, contact is lost and the Newton-Raphson update is calculated based on the finite difference equations again.

3 Results

In this section, some initial results are presented, based on the current approach. The values used in the simulations are; \( R = 0.32 \, [m] \), \( E = 4.8 \times 10^8 \, [N/m^2] \), \( h = 0.008 \, [m] \), \( \rho_0 = 2.0 \times 10^5 \, [N/m^2] \), \( k_\theta = 1.10 \times 10^6 \, [N/m^3] \), \( k_r = 1.23 \times 10^7 \, [N/m^3] \) and \( \delta = 0.04 \, [m] \). The number of points on the tread is \( N = 256 \). The rotational speed is \( \Omega = 105 \, [rad/s] \), density \( \rho = 1200 \, [kg/m^3] \) and damping coefficient \( \lambda = 0.05 \, [Ns/m^3] \). The width of the tyre is assumed to be \( 0.20 \, [m] \).

3.1 (Quasi-) static results

![Figure 3: Contact forces (green arrows) and deformation of the ring (red), frictionless (a), incl. friction \( \mu = 0.2 \) (b), incl. friction \( \mu = 0.7 \) (c).](image)

In figure 3, the steady state solutions of the ring in contact with the road surface are shown. If a frictionless contact is assumed, the solution can readily be obtained. If a non-zero friction coefficient is used the solution becomes path dependent. Hence, we have solved a quasi-static solution (no inertia and damping forces),
when the road surface is pressed into the tyre, having a vertical displacement only. In this simulation, the final indentation of the tyre $\delta$ has been set to 0.04. As can be seen, the shear forces increase with increasing friction coefficient $\mu$.

### 3.2 Transient results

From the static solution, the tyre is set in motion and inertia and damping are accounted for. The displacements (an example is given in figure 4) are obtained and interpolated to the fixed source mesh. In figure 5, the resulting frequency spectrum is shown for a point on the source mesh which is near to the contact (at the front side of the tyre).

![Figure 4: Transient response (normal displacement) of one of the points on the tyre v. time.](image)

The spectrum clearly shows resonance peaks associated with the eigenfrequencies of the tyre. Additional calculations have confirmed the observation in [3] that due to the change in propagation speed of waves traveling in and opposite to the direction of rotation, the resonance frequencies of the stationary ring $f_n$ is split into two resonance frequencies $f_n \pm n\Omega/(2\pi)$, where $n$ is the mode number and $\Omega$ the rotational frequency. At higher frequencies the signal is less clear.

### 3.3 Radiation results

The dynamic analysis has, as indicated above, been used as input for the boundary element calculation. The calculated pressures in the field mesh, at 1050 Hz. and 1400 Hz, are shown in figure 6. No absorption of the road surface has been used in this calculation.

![Figure 5: Frequency spectrum for a point near the contact region](image)
Especially at 1400 Hz, one can observe the directivity of the sound field. Sound radiation to the sides is much less than the radiation to the front and rear. One also observes the horn effect near 1050 Hz, amplifying the sound generated near the contact region.

As a final example, the difference in sound pressure level between the sound generated on a textured profile (a single sin-wave) and a smooth road is shown in figure 7. Clearly more sound is being radiated. However, also the directivity is changed; more sound is radiated to the sides.

4 Conclusion

To quantitatively predict tyre/road noise, it is essential to accurately describe the high frequency vibrations of the tyre. In addition, it is necessary to have contact algorithms which calculate these vibrations within reasonable computation times. The contact algorithms used in the field of elasto-hydrodynamics are, essentially, fast enough to accomplish this task. The contact algorithms described in this paper is a first stage in the development of such a fast algorithm. It is shown that the algorithm (contact condition and friction model), together with a tyre model and a (3D boundary element) radiation model can be used to predict sound radiated from the tyre.

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A: Discretization

The equations of motion for the tyre (1 and 2) are discretized using finite differences. Using first order discretization in time and second order in θ, the following finite difference equations are obtained:
\[ C_1 \frac{u_{\theta i,k} - 2u_{\theta i,k-1} + u_{\theta i,k-2}}{h_t^2} + C_2 \frac{u_{\theta i,k} - u_{\theta i,k-1}}{h_t} + C_3 \frac{-u_{r i-2,k} + 2u_{r i-1,k} - 2u_{r i+1,k} + u_{r i+2,k}}{2h_{\theta}^3} \]
\[ + \ C_4 \frac{u_{\theta i-1,k} - 2u_{\theta i,k} + u_{\theta i+1,k}}{h_{\theta}^2} + C_5 \frac{u_{r i+1,k} - u_{r i-1,k}}{2h_{\theta}} + C_6 u_\theta = f_\theta \]  

(11)

\[ + \ C_7 \frac{u_{r i-2,k} - 4u_{r i-1,k} + 6u_{r i,k} - 4u_{r i+1,k} + u_{r i+2,k}}{h_{\theta}^4} + C_8 \frac{-u_{\theta i-2,k} + 2u_{\theta i-1,k} - 2u_{\theta i+1,k} + u_{\theta i+2,k}}{2h_{\theta}^3} \]
\[ + \ C_9 \frac{u_{r i-1,k} - 2u_{r i,k} + u_{r i+1,k}}{h_{\theta}^2} + C_{10} \frac{u_{\theta i+1,k} - u_{\theta i-1,k}}{2h_{\theta}} + C_{11} u_r = f_r, \]  

(12)

where subscript \( i \) is the spatial index, \( k \) is the time index, \( h_\theta = 2\pi/N \) denotes the step size and \( h_t \) the time step. The constants \( C_1 \) to \( C_9 \) are defined according to:

\[ C_1 = \rho h \quad C_2 = \lambda \quad C_3 = \frac{EI}{R^3} \quad C_4 = -\frac{Eh}{R^2} - \frac{EI}{R^4} - \frac{p_0}{R} \quad C_5 = -\frac{Eh}{R^2} - \frac{2p_0}{R} \]
\[ C_6 = \frac{p_0}{R} + k_\theta \quad C_7 = C_3 \quad C_8 = -C_3 \quad C_9 = \frac{p_0}{R} \quad C_{10} = -C_5 \quad C_{11} = \frac{Eh}{R^2} + \frac{p_0}{R} + k_r \]