

ON THE DESIGN OF A (H)EV STEERABLE WARNING DEVICE USING ACOUSTIC BEAM FORMING AND ADVANCED NUMERICAL ACOUSTIC SIMULATION

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ABSTRACT – This paper describes the simulation-based design methodology used in the eVADER project for the development of targeted acoustic warning devices for increased detectability of Hybrid and Electric Vehicles (HEVs) while, at the same time, reducing urban noise pollution. A key component of this system is an external warning signal generator capable of projecting the warning signals to a contained area in front of the vehicle where potential at-risk situations are detected. Using acoustic beam forming principles a suitable warning strategy and an initial layout for realizing such a system is defined. Starting from this information, acoustic Finite and Boundary Element models of the transducer array allow assessing more realistically the performance impact of the system integration and of the most critical changes in the acoustic environment in which the signal generator needs to operate.

INTRODUCTION

This paper describes the development of targeted acoustic warning systems for increased detectability of Hybrid and Electric Vehicles (HEVs) in the context of the eVADER European project (1). It presents the numerical simulation based design process that has been used to develop the exterior warning generator, which is a key component of this system. After a general introduction and presentation of the scope of the eVADER project, a first section discusses acoustic beam forming principles and the different approaches to realize targeted acoustic fields based on a configuration of non-directional sources (such as e.g. loudspeakers). Using these principles a suitable warning strategy is defined and an initial overall array layout is proposed. Next, a second section presents the construction of numerical acoustic models of a transducer array installed on the bumper of the eVADER demonstrator vehicle. Such models allow assessing more realistically the impact of the system integration on the performance of the warning system. Using the acoustic responses obtained with these models, the strategy selected in the first section is verified. Next, a sensitivity study is presented to assess the impact of the most critical changes in the vehicle's acoustic environment: temperature changes, changes in the road surface properties and the influence of nearby scattering objects such as e.g. parked vehicles.

THE EVADER PROJECT & THE EXTERIOR WARNING SYSTEM DESIGN

This design study is part of the eVADER European project which aims at demonstrating acoustic warning systems for increased detectability of low noise vehicles at low speeds (below 35km/h) by Vulnerable Road Users (VRUs), while minimizing the impact on the

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environmental noise levels and signature. To this end, the eVADER system combines advanced VRU detection technology, carefully designed warning signals (presenting a trade-off between detectability and annoyance) and dedicated alerting strategies. To realize its dual goals, the eVADER project designs and builds a pedestrian warning system that will be integrated into a Nissan Leaf demonstrator vehicle. All aspects of the development of such a system are considered ranging from the psycho-acoustic perception of artificial warning sounds to automated pedestrian detection systems, risk estimation and interior and exterior warning system design. This paper focusses on the development of a crucial component of this system namely the exterior warning signal generator.

In order to optimally reduce the environmental noise impact, the design of this signal generator applies acoustic beam forming principles and follows the following steps:

- Starting from the functional specifications an initial beam forming assessment is performed based on simplified analytical source models. The outcome of this design step is a suitable beam forming algorithm and the selection of a limited subset of transducer configurations whose performance as installed on the bumper of the demonstrator vehicle is studied in more detail.
- To this end, a FEM-based numerical model of the bumper and the separate sources is constructed. Using the obtained acoustic transfer functions between each transducer and a number of target microphones, the optimal control strategy for each individual speaker is determined. To verify the system's performance, these speaker control parameters are used as inputs for an acoustic verification model. The obtained response predictions allow assessing the spatial distributions of the acoustic pressure in a much broader area than the microphones used in the control parameter identification.
- After this verification of the performance for the reference environmental conditions, a sensitivity study is performed to assess the system's robustness with respect to changes in the acoustical environment and to provide feedback to the design.
- Once the system has proven its robustness with respect to all relevant environmental changes, a well-founded view on the system's performance and possible critical implementation issues can be formulated.

This design study was executed as a collaboration between TNO and LMS, where TNO focussed their efforts on the study, selection and validation of the acoustic beam forming strategies and LMS looked into the numerical modelling aspects and overall process integration. This design study was also discussed by the authors in (2). In the subsequent sections of this paper the results of the different steps in this design study are presented for a transducer array of six small-sized loudspeakers (membrane radius 50mm) integrated in the front bumper of the eVADER demonstrator vehicle.

ACOUSTIC BEAMFORMING STRATEGIES

The eVADER approach to increase VRU safety and reduce noise pollution is based on the use of a sound beam conveying the warning signal in the direction of the VRU, while minimising acoustic output in other directions (Fig. 1). This section firstly describes the design of an array of acoustic sources based on simple source models. This design can be used as a starting point to generate more realistic transfer function models based on the geometry of a car or part of a car. This section starts with the definition of the initial geometry of the array, based on point source models and mirror point sources with respect to a perfectly reflecting ground plane. A comparison is given of two different beam forming algorithms. At the end of the section an

example is given for a more realistic configuration showing of the influence of the ground reflection and the influence of the beam former optimisation distance.

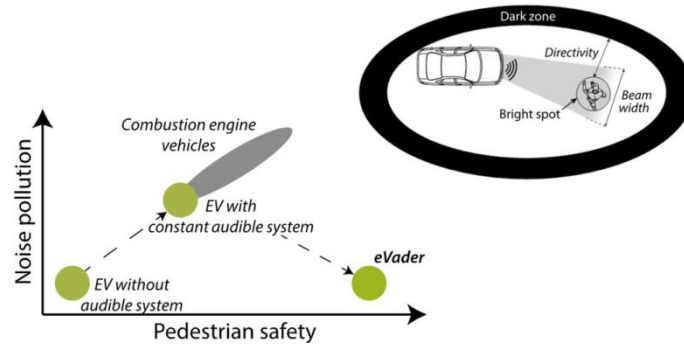


Figure 1: eVADER approach using a directional acoustic warning system.

SPECIFICATIONS

The array consists of a maximum of six acoustic sources. The frequency range to be covered is 300 Hz to 1.2 kHz, which is based on several prototype sounds developed within eVADER (3). Directivity is only required in the horizontal direction, not in the vertical one. The steering direction of the beam is between -60° and $+60^\circ$ using a single beaming direction; multiple beams at the same time are not required. The angular tracking speed is $300^\circ/\text{s}$.

SPATIAL ALIASING AND BEAM WIDTH

The limited number of sources in the array necessitates a trade-off between beam width, particularly at low frequencies, and the maximum frequency determined by spatial aliasing. For the often used Delay and Sum algorithm the beam width (BW) in radians is given by $BW=2\lambda/L$ with $\lambda=c/f$ the wavelength of sound with c the speed of sound [m/s], f the frequency [Hz], and L the size of the array [m]. Other algorithms may perform better resulting in a smaller BW for a given array length L and wavelength λ . However, practical high-resolution algorithms possess a similar relationship between BW, L and λ . Hence the size of the array has to be as large as possible for narrow beams. On the other hand the spacing between the array elements determines the upper frequency for which no 'aliased' beams are produced.

Spatial aliasing is determined by the spacing Δx between the sources and the spatial wavenumber k_x in the horizontal direction (4). Spatial aliasing does not occur if the $|k_x| \leq \pi / \Delta x$. The wavenumber in the horizontal direction is defined by $k_x = \omega \sin(\alpha) / c$, in which $\omega = 2\pi f$ is the angular frequency, and α is the angle between the propagation direction and the forward direction. For $\alpha = \pi/2$ $k_x = \omega/c = k$. The maximum frequency at which no aliasing occurs for a given spacing Δx is called the Nyquist frequency. The requirement for the spacing can be written as $\Delta x \leq \lambda / (2 \sin(\alpha))$. For a Nyquist frequency of 1.2 kHz and a maximum angle α of 60° we obtain $\Delta x \leq 0,165$ m. To introduce a certain margin and to further reduce the influence of spurious beams, a Nyquist frequency of 1.5 kHz was selected and a maximum angle α of 90° . This results in a spacing $\Delta x \leq 0,1143$ m.

For sensor arrays, many beam forming methods exist (5). For source arrays the number of methods that have been described in the literature is smaller. Some of the methods for source arrays are the Contrast Maximisation approach (6,7), the Acoustic Energy Difference method (8), and the Time Reversal approach (9). We can also define a so-called Least-Squares beam

former based on a description for sensor arrays. A method based on the reduction of the sound power with a constraint in the main steering direction is given in (10). This paper also compares five different beam formers. From these results it can be seen that the Acoustic Energy Difference method and the Sound Power Minimization method provide the sharpest beams. An advantage of the Sound Power Minimization approach and Delay and Sum approach is that a re-computation of the source coefficients when a new steering direction is required is computationally feasible within the timing constraints and with the computing resources of the embedded hardware suggested for eVADER. Although a re-computation is not necessary for a single target direction, it is more efficient when two or more, a-priori unknown, target directions are taken into account. The Acoustic Contrast Control method and the Acoustic Energy Difference method require computation of eigenvalues and eigenvectors, and are therefore less attractive for real-time modification of the steering direction and subsequent re-computation. A selection of the results for two methods will be presented in this paper: the delay and sum beam former and the sound power minimization beam former.

An example of the beam shape for a uniform array with constant spacing between the sources is given in the left side of Fig. 2. The Sound Power Minimization method gives more narrow beams than the Delay and Sum method, especially at low frequencies. A further reduction of the beam width is possible by using non-uniform array spacing, as can be seen in the right side of Fig. 2. The disadvantage of the high resolution methods such as the Sound Power Minimization method is that the required driving levels of individual sources can be higher (see Fig. 3). Regularisation is therefore needed, which is also beneficial for the robustness with respect to uncertainties. For other beaming directions similar results can be found, possibly with somewhat wider beams at large beaming angles.

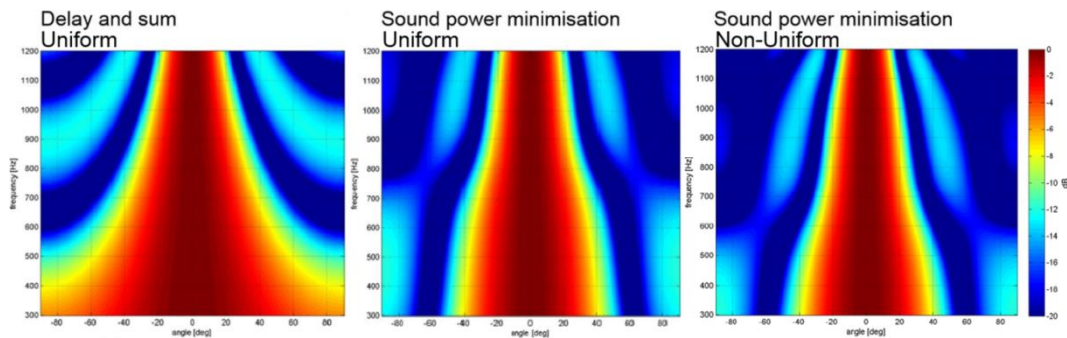


Figure 2: SPL in dB relative to the SPL in the main beam (steering at 0°) for an array of six point sources using uniform and non-uniform array geometries and delay and sum and sound power minimisation strategies.

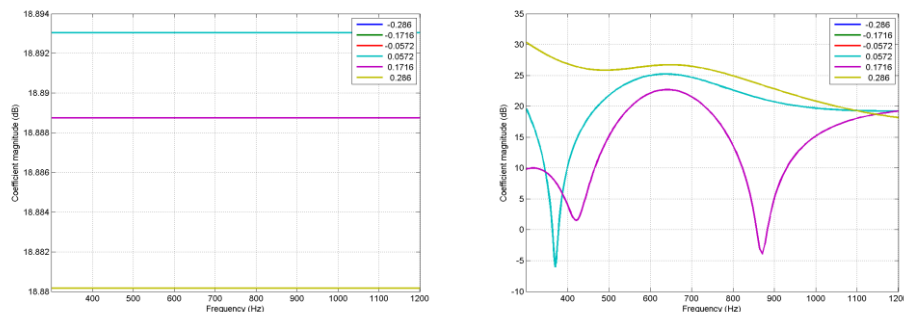


Figure 3: Driving level magnitude for delay and sum (left) and sound power minimisation beam former (right).

BEAMFORMING BASED ON MEASURED TRANSFER FUNCTIONS

The methods for driving an array of sources were selected such that measured transfer functions could be used, allowing calibration with experimental data. An advantage of such

an approach is that no assumptions need to be made regarding the source size and behaviour. Compactness is often assumed in derivations for sensor arrays but for source arrays such an approximation is unrealistic. The sound power is often critical and therefore the dimension of the sources is maximized and tends to be approximately equal to the array spacing. Furthermore, there is a certain coupling between the sources, due to acoustic, mechanical or electrical interaction. Since the present methods are based on measured transfer functions all such factors are automatically taken into account in the design of the beam forming algorithm, but the performance of the system of course depends on the actual geometry and conditions. An example of a typical beam forming control coefficient identification configuration in which 15 microphones are used at distance of 5m from the array is given in Fig. 4.

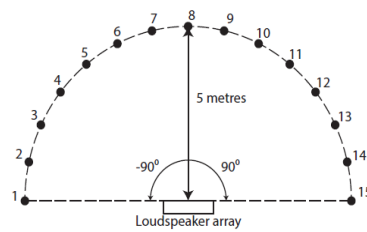


Figure 4: Example configuration used to define the beam former showing the loudspeaker array and a number of microphones on a circle, in this case using 15 microphones at a distance of 5m from the centre of the array.

For the initial assessment a simplified frequency domain point source model is used, based on the free-field Green's function. The reflecting ground surface is an important aspect of the present application. In a first approximation, each Green's function is augmented with a contribution due to an apparent mirror source. If the algorithm is used in such a way that the sound pressure in the main beaming direction at a specified distance is constrained then the ground reflection results in a degraded beam shape at frequencies with strong destructive interference. Several strategies exist to reduce the effect of such ground reflection interference. In Fig. 5, the influence of the optimisation distance is illustrated. An optimisation of the beam at a relatively large distance from the array, in this case 40m, leads to a smooth and narrow beam at the identification distance (left figure). However, if the beam is optimised at a relatively short distance from the array, in this case 5m, then the beam shape in the far field is degraded in the frequency range affected by the ground reflection (650-750Hz in the middle figure). Similar degradations are observed at other evaluation distances with other corresponding frequencies. Additionally, Fig. 6 shows that this degradation can be reduced by optimising the array for the far field. The effect of the ground reflection is still apparent at the shorter distances as an attenuation of the sound at specific frequencies, but it does not lead to widening of the beam. Optimisation of the beam former for the far field leads to a more natural behaviour, albeit with possible dips in the spectrum at certain receiver locations.

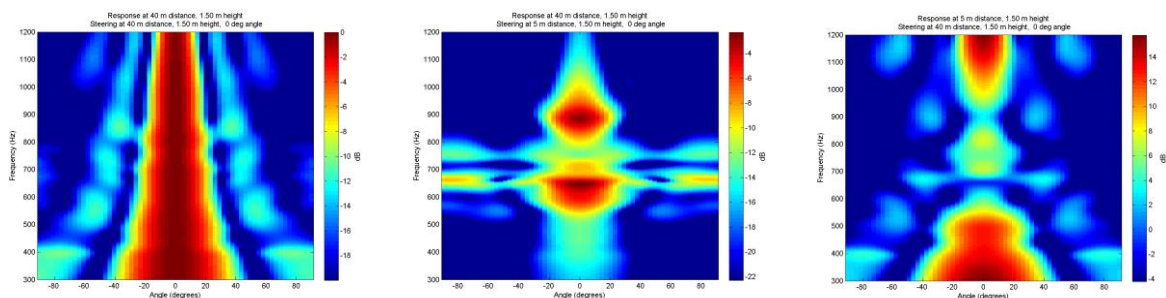


Figure 5: Beam shapes with reflecting ground surface; steering at an angle of 0° . Beam optimisation at 40m, beam evaluation at 40 m (left), beam optimisation at 5m, beam evaluation at 40m (middle), beam optimisation at 40m, beam evaluation at 5m (right).

ADVANCED ACOUSTIC CAE IN SUPPORT OF VRU WARNING SYSTEM DESIGN

In the scope of the eVADER exterior warning system design, the use of acoustic virtual prototypes presents very attractive perspectives. On the one hand realistic input data needed to assess the potential strengths and weaknesses of different acoustic beam forming algorithms can be quickly gathered. On the other hand the algorithms' and design robustness can be assessed by varying a wide range of environmental conditions. This section presents the construction of suitable acoustic CAE models of the warning system and their use in the design and verification of the signal generator. The versatility of such advanced CAE approaches in the context of (H)EV acoustic analysis has also been shown in (11,12).

PREDICTIVE CAE TOOLS FOR MODELLING VRU WARNING SYSTEMS

Whenever virtual prototypes are used for design support or performance evaluation, the selection of a suitable discretization approach and analysis type is key. When considering acoustic radiation problems, which involve solving an uncoupled exterior acoustic problem with imposed normal velocity boundary conditions in the frequency domain two families of numerical models are frequently used in the current engineering practice. The Boundary Element Method (BEM) is based on a boundary discretization and is hence very suited to model acoustic radiation. Moreover, significant advances such as the Fast Multipole BEM methods (13) and the Hierarchical Matrix (or H-Matrix) solvers (14) have been made in the BEM's computational efficiency. The Finite Element Method (FEM) has become the standard solution approach for solving interior acoustic and vibro-acoustic problems. Different strategies for coping with this limitation have been devised such as infinite elements and perfectly matched layer (PML) approaches (15) applied on fictitious outer FEM boundaries.

Based on the strengths and weaknesses of the different modelling approaches a combination of different strategies is proposed for this design study. For scenarios where only the bumper itself needs to be considered, the FEM-AML technique which is based on the PML approach is selected. In the settings where also the acoustic environment needs to operate (road surfaces etc.) are considered H-Matrix BEM present an attractive alternative.

REFERENCE CONFIGURATION FEM-AML MODEL

A detailed geometrical model of the front bumper of the eVADER demonstrator vehicle is incorporated in all the numerical models in this design study. A closed surface model to create the FEM model discretization is obtained by filling up any holes and surface mismatches. The front of the car is approximated by extruding the outer edge of the surface model in the length direction as is shown by the purple surface area in the left of Fig 6. Due to their small size, the six individual loudspeakers are incorporated as the six red circular vibrating pistons shown in the left side of Fig 6. To finalize the creation of the FEM-AML model, the pre-processing capabilities of the LMS Virtual.Lab Acoustics (16) suite of numerical tools are used to automatically generate a suitable convex wrapper mesh around the extended bumper geometry. This mesh is created in such a way that a minimal distance to the physical boundaries is respected and that the mesh runs up to and is perpendicular to a predefined perfectly reflecting plane. The right side of Fig 6 shows the final baseline FEM-AML model with an average mesh size such that it is at least valid up to 1.3kHz according to commonly used rules of thumb. The total computational cost for solving this baseline FEM model of 120.000 degrees of freedom is 9 seconds per frequency. In comparison, solving an equivalent H-matrix BEM model takes 210 seconds while providing similar acoustic predictions.

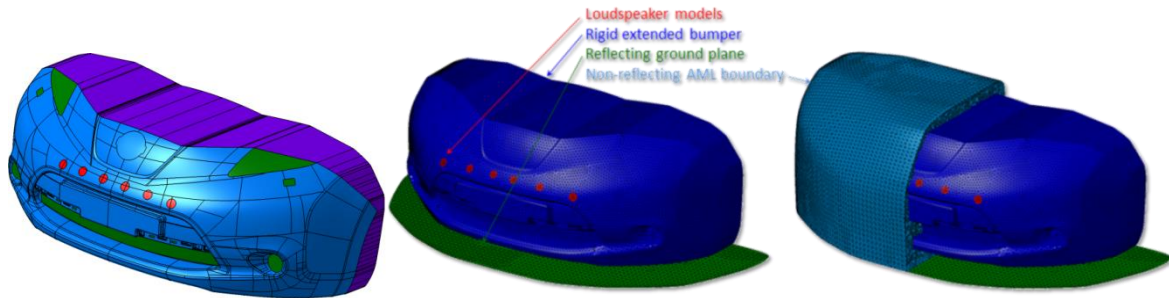


Figure 6. (left) Computational bumper geometry, (right) FEM-AML model of the warning signal generator

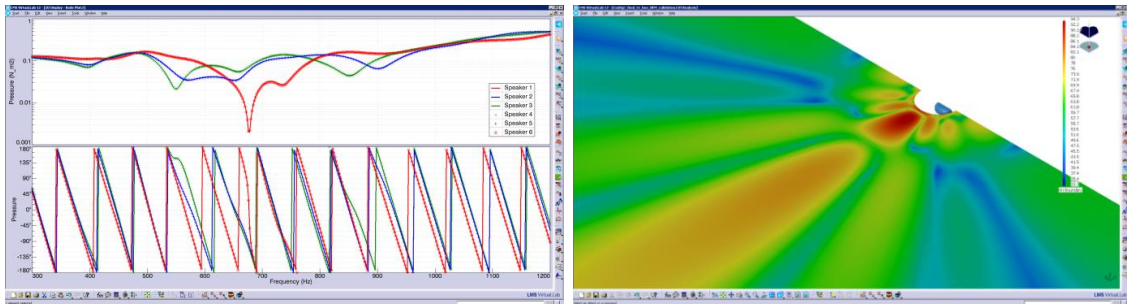


Figure 7. Acoustic pressure responses obtained with the baseline FEM-AML model: (left) individual acoustic transfer functions for the six speakers, (right) forward-facing acoustic beam at 900Hz

The FEM-AML model of the warning signal generator installed on the front bumper is used in two steps:

- Firstly to obtain the acoustic transfer functions between the surface velocity of each individual loudspeaker and the acoustic pressure at a number of semi-circular microphone arrays. These transfer functions are used as an input for the beam forming strategies discussed in section 2. An example of such transfer functions is shown in the left of Fig 7.
- Secondly, by introducing the optimal beam forming control parameters in the numerical model, the acoustic pressure generated by the warning generator can be reconstructed. This information is useful to assess the impact the warning generator has on the environment noise landscape. The right side of Fig 8 shows the acoustic pressure field at 900Hz in front of the bumper generated by an irregularly shaped beam forming array. The area considered in this picture extends up to 15m away of the bumper and illustrates the highly directive noise field. These results illustrate the suitability of the selected beam forming strategies to derive optimal control parameters.

DESIGN SENSITIVITY STUDY

Given the high level of exposure of the exterior warning system to a range of environmental conditions it is important that the dominant parameters are taken into account during the design of the system to ensure robustness. Based on a literature review the following parameters have been found to influence the propagation of acoustic waves in typical traffic environments: (i) Variations in the acoustic properties of the air that may be caused by the presence of temperature gradients or variations in relative air humidity, (ii) Aerodynamic disturbances of the propagating medium caused by wind gradients and air turbulence, (iii) Impact of second order reflections that may interfere with the direct field generated by the array such as the presence of reflecting objects and ground reflections and (iv) Interference with other noise sources that can be other sound sources in the environment or incoherent background noise

In the current study, the first and third class of influencing factors are considered. The fourth impact type is implicitly integrated into the total warning system that automatically selects a suitable absolute sound level to be generated.

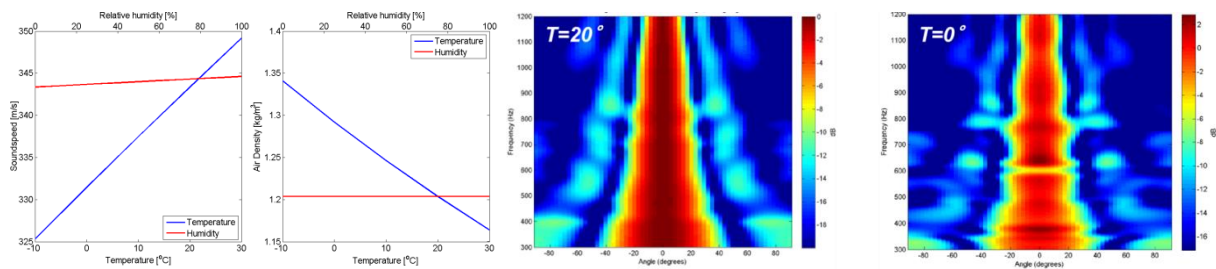


Figure 8. Impact of temperature and humidity on speed of sound and density (left) and on beam forming (right)

Both the ambient temperature and the air's relative humidity change the acoustic wave propagation properties and hence may significantly impact the warning system's performance. In order to identify the most influential parameter, Fig 8 shows the evolution of both density and sound speed for representative temperature (-10°C to +30°C, blue curves) and humidity (0 to 100%, red curves) ranges. The curves clearly show that the temperature has the most impact. Since temperature changes only impact the acoustic material properties of the ambient fluid, the selected FEM-AML technique remains valid. By combining these updated models with the control coefficients computed for the reference conditions, the robustness with respect to ambient fluid property changes is assessed. The right side of Fig 9 shows a representative example result of this study for a warning signal targeted at a 0° orientation at 20°C and at 0°C. A very good agreement between the obtained directivities can be observed.

A second parameter with a major impact on the warning system's performance is the acoustic absorption of the road surface. Apart from attenuating acoustic waves, the acoustic impedance of the road surface may also introduce asymmetry or even fully scatter the warning signal. In order to incorporate these effects a description of the frequency-dependent acoustic normal impedance is incorporated in a discretization of the road surface. In (17) a theoretical model was developed in which the acoustic surface impedance depends on the ambient fluid properties and also on the air flow resistivity and porosity of the asphalt, the so-called structural factor and the thickness of the asphalt layer. Typical values for these quantities were taken from literature. Since the exterior warning system is designed to warn at-risk VRUs up to large distances in front of the vehicle, the road surface should be appropriately modeled. Hence, the area where the road surface is located is modelled explicitly for a scenario where the vehicle is driving along a 20m long straight road which is two lanes wide, as is shown in the left of Fig 9. Additionally the impact that scattering objects in the vicinity of the warning signal generator is considered. To this end, the BEM model is extended by adding a model of two vehicles parked on the side of the road 5m in front of the approaching demonstrator vehicle. The vehicles themselves are considered to be perfectly reflecting and their additional impact on the spatial distribution and directivity of the acoustic warning signals will be assessed. The increased extent and complexity of the problem setting significantly impacts the choice of the modelling approach. Based on the model size (around 190.000 degrees of freedom for the road surface and 250.000 when including the parked cars) the H-matrix BEM was selected. Due to the increased computational cost (around 13-25 min per frequency of interest), the acoustic responses are only computed every 25Hz.

The right side of Fig 9 shows the spatial distribution of the acoustic pressure amplitude generated by the warning system at 600Hz for a steering angle of 0°. This case is considered

as the worst possible mode of operation since the central acoustic pressure amplitude lobe of the warning signal impinges directly on both stationary vehicles. As can be seen in the contour plots, the presence of the two vehicles shields the areas behind them as expected. The sound waves that are scattered by the vehicles do interfere with the central lobe of the warning signal but do not alter its shape nor do they generate areas of very low SPL.

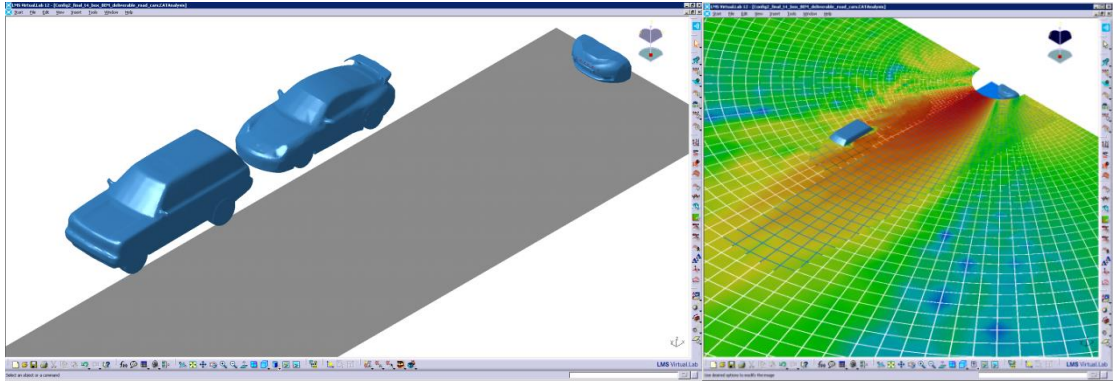


Figure 9. Impact of road surface impedance and nearby scattering objects on beam forming performance: (left) H-matrix BEM model discretisation, (right) spatial distribution of the acoustic pressure field at 600Hz

The conclusions above for the spatial pressure distributions at discrete frequencies are confirmed by the directivity analysis presented in Fig 10. On the left side the directivity obtained for the baseline problem setting is given. This is compared to the directivity for a 0° steering where only the road surface impedance (middle) and where both the road surface and the two parked vehicles (right) are included in the analysis. In both cases, the directivity pattern becomes asymmetric due to the asymmetric positioning of the vehicle with respect to the road (it occupies the right lane). Additionally the interference between the direct radiated acoustic field and the acoustic waves scattered by the parked vehicles is clearly visible in the image on the right but as was observed in Fig. 9 these interferences do not distort the overall directionality and hence they do not impact the system’s intended operation.

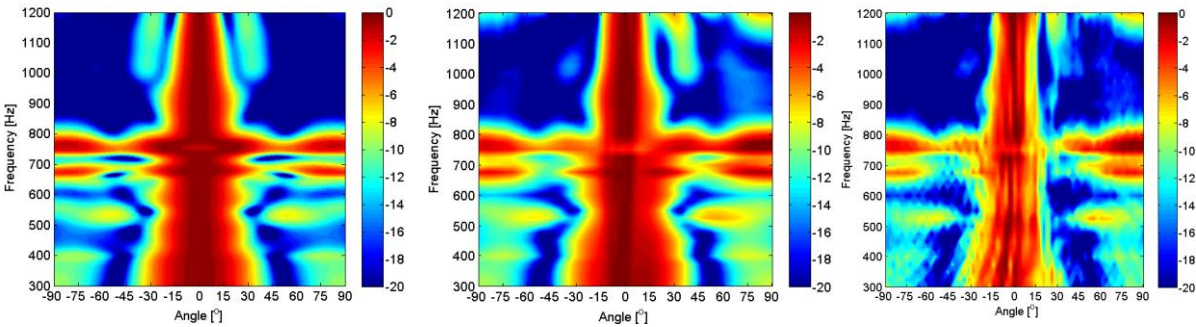


Figure 10. Impact of road surface impedance and nearby scattering objects on beam forming performance: (left) reference configuration, (middle) impact of 20m long asphalt road, (right) impact of asphalt road and parked cars

CONCLUSIONS

In this paper, a CAE-based analysis and evaluation is proposed to support the design of the eVADER acoustic exterior warning system. In this process, advanced acoustic beam forming algorithms are linked to high-fidelity numerical models for the assessment of the acoustic radiation of the different transducers as installed on the vehicle. In this way, a thorough assessment of different configurations can be executed in a flexible and efficient manner. The

choice of modelling technique is based on the problem setting and the expected computation times for different CAE models.

Based on the control coefficients for a localized warning signal targeted at different angles with respect to the forward direction of travel, a verification study of the spatial distribution of the acoustic pressure was performed. In a next step, the use of detailed numerical models allows to incorporate various parameter changes in the problem setting to assess the robustness of the proposed design. In this sensitivity study, three key types of changes in the acoustic environment of the warning system are considered. Based on these results the proposed speaker configuration was proven to be robust with respect to environmental changes and the design is deemed ready for integration into the eVADER vehicle.

ACKNOWLEDGEMENTS

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