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A Two-Parameter Poisson-Sujatha Distribution

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Abstract A two-parameter Poisson-Sujatha distribution which is a Poisson mixture of two-parameter Sujatha distribution, and includes Poisson-Sujatha distribution as particular case has been proposed. Its moments and moments based measures including coefficient of variation, skewness, kurtosis and index of dispersion have been obtained. Maximum likelihood estimation has been explained for estimating its parameters. Goodness of fit of the proposed distribution has been explained with two over-dispersed count datasets and the fit has been compared with one parameter Poisson-Lindley distribution and Poisson-Sujatha distribution and a generalization of Poisson-Sujatha distribution.

Keywords Sujatha distribution, Poisson-Sujatha distribution, Two-parameter Sujatha distribution, Moments based measures, Maximum likelihood estimation

1. Introduction

In statistics literature, Poisson distribution is the common distribution for modeling count data. However, the unique feature of equality of the mean and the variance (equi-dispersed) of Poisson distribution makes it unsuitable for modeling count data which are under-dispersed (mean greater than variance) or over-dispersed (mean less than variance). In recent years, several researchers have proposed Poisson mixture of lifetime distributions which are useful for over-dispersed or under-dispersed. The over-dispersed Poisson mixed distributions are Poisson-Lindley distribution (PLD), a Poisson mixture of Lindley distribution of Lindley (1958) proposed by Sankaran (1970), Poisson-Sujatha distribution (PSD), a Poisson mixture of Sujatha distribution of Shanker (2016 a) introduced by Shanker (2016 b), some among others.

The probability density function (pdf) of Sujatha distribution having scale parameter θ and introduced by Shanker (2016 a) is

$$f_1(x;\theta)=\frac{\theta^3}{\theta^2+\theta+2}\left(1+x+x^2\right)e^{-\theta x} \quad ;x>0,\ \theta>0 \quad (1.1)$$

Various properties including shapes of the density, moments and moments based measures, skewness, kurtosis, index of dispersion, hazard rate function, mean residual life function, stochastic ordering, mean deviation, stress-strength reliability, along with the estimation of parameter and applications for modeling lifetime data from biomedical science and engineering of Sujatha distribution are available in Shanker (2016 a). Kaliraja and Perarasan (2019) studied a stochastic model on the generalization of Sujatha distribution for the effects of two types of exercise on plasma growth hormone.

Shanker (2016 b) obtained Poisson-Sujatha distribution (PSD) by compounding Poisson distribution with Sujatha distribution. The PSD is defined by its probability mass function (pmf)

$$P_1(x;\theta)=\frac{\theta^3}{\theta^2+\theta+2}\frac{x^2+(\theta+4)x+\left(\theta^2+3\theta+4\right)}{(\theta+1)^{x+3}} \quad ; \quad (1.2)$$
$$x=0,1,2...,\ \theta>0$$

Statistical properties including shapes of pmf, moments and moments based measures, over-dispersion, unimodality and increasing hazard rate, estimation of parameter and applications to model over-dispersed data have been discussed by Shanker (2016 b). Wesley et al (2018) proposed a zero-modified Poisson-Sujatha distribution to model over-dispersed count data and discussed its several important properties and applications.

Shanker et al (2017) proposed a generalization Sujatha distribution (AGSD) using convex combination of exponential (θ), gamma $(2,\theta)$ and gamma $(3,\theta)$ distributions and defined by its pdf

$$f_2(x;\theta,\alpha)=\frac{\theta^3}{\theta^2+\theta+2\alpha}\left(1+x+\alpha x^2\right)e^{-\theta x} \quad ; \quad (1.3)$$
$$x>0,\ \theta>0,\alpha>0$$

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where θ is a scale parameter and α is a shape parameter. Lindley distribution and Sujatha distribution are the particular cases of AGSD for $\alpha = 0$ and $\alpha = 1$, respectively. Various statistical properties, estimation of parameters and applications of AGSD have been discussed by Shanker et al (2017).

Shanker and Shukla (2019) derived a generalization of Poisson-Sujatha distribution (AGPSD) by mixing Poisson distribution with AGSD. The AGPSD is defined by its pmf

$$\frac{\theta^3}{\theta^2+\theta+2\alpha}\frac{\alpha x^2+(\theta+3\alpha+1)x+\left(\theta^2+3\theta+2\alpha+2\right)}{(\theta+1)^{x+3}}; \quad (1.4)$$

$$x = 0.1.2....,\theta > 0,\alpha > 0$$

It can be easily shown that Poisson-Lindley distribution (PLD of Sankaran (1970) and PSD of Shanker (2016 b) are special cases of AGPSD for $\alpha = 0$ and $\alpha = 1$, respectively. Statistical properties based on moments, unimodality and increasing hazard rate, estimation of parameter and applications of AGPSD have been studied by Shanker and Shukla (2019).

Mussie and Shanker (2018) proposed a two-parameter Sujatha distribution (TPSD) defined by its pdf

$$f_2(x;\theta,\alpha) = \frac{\theta^3}{\alpha\theta^2+\theta+2}\left(\alpha+x+x^2\right)e^{-\theta x}; \quad (1.3)$$

$$x > 0,\theta > 0,\alpha \ge 0$$

where θ is a scale parameter and α is a shape parameter. It can be easily verified that (1.3) reduces to Sujatha distribution (1.1) and size-biased Lindley distribution (SBLD) for α = 1 and α = 0 respectively.

The main motivation for proposing a two-parameter Poisson-Sujatha distribution (TPPSD) are (i) Sujatha distribution is a better model than both exponential and Lindley distribution for modeling lifetime data, and PSD being a Poisson mixture of Sujatha distribution gives better fit than both Poisson and Poisson-Lindley distribution (PLD), and (ii) TPSD gives much better fit than exponential, Lindley and Sujatha distribution, it is expected that TPPSD being a Poisson mixture of TPSD would provide better fit over PLD, PSD and other discrete distributions.

Keeping these points in mind, a two-parameter Poisson-Sujatha distribution (TPPSD), a Poisson mixture of TPSD has been proposed and its moments and moments based measures have been obtained and their behaviors have been studied. Maximum likelihood estimation of TPPSD has been discussed for the estimation its parameters and its applications have been discussed with two examples of observed count datasets from ecology and demography.

2. A Two-Parameter Poisson-Sujatha Distribution

A random variable X is said to follow a two-parameter Poisson-Sujatha distribution (TPPSD) if $X \mid \lambda \sim P(\lambda)$ and $\lambda \mid \theta,\alpha \sim TPSD(\theta,\alpha)$. That is,

$$P(X=x \mid \lambda) = \frac{e^{-\lambda}\lambda^x}{x!}; x = 0,1,2,..., \lambda > 0, \text{ and}$$

$$f(\lambda \mid \theta,\alpha) = \frac{\theta^3}{\alpha\theta^2+\theta+2}\left(\alpha+\lambda+\lambda^2\right)e^{-\theta\lambda};$$

$$\lambda > 0,\theta > 0,\alpha \ge 0$$

The pmf of unconditional random variable X can be obtained as

$$P_2(x;\theta,\alpha) = P(X=x) = \int_0^\infty P(X=x \mid \lambda) f(\lambda \mid \theta,\alpha)\,d\lambda$$

$$= \int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\frac{\theta^3}{\alpha\theta^2+\theta+2}\left(\alpha+\lambda+\lambda^2\right)e^{-\theta\lambda}\,d\lambda \quad (2.1)$$

$$= \frac{\theta^3}{\left(\alpha\theta^2+\theta+2\right)x!}\int_0^\infty e^{-(\theta+1)\lambda}\left(\alpha\lambda^x+\lambda^{x+1}+\lambda^{x+2}\right)d\lambda$$

$$= \frac{\theta^3}{\left(\alpha\theta^2+\theta+2\right)x!}\left[\frac{\alpha\Gamma(x+1)}{(\theta+1)^{x+1}}+\frac{\Gamma(x+2)}{(\theta+1)^{x+1}}+\frac{\Gamma(x+3)}{(\theta+1)^{x+1}}\right]$$

$$= \frac{\theta^3}{\left(\alpha\theta^2+\theta+2\right)}\frac{x^2+(\theta+4)x+\left\{\alpha\theta^2+(2\alpha+1)\theta+(\alpha+3)\right\}}{(\theta+1)^{x+3}} \quad (2.2)$$

$$x = 0,1,2,....,\theta > 0,\alpha > 0$$

We would call this two-parameter Poisson-Sujatha distribution (TPPSD) because for $\alpha = 1$, it reduces to one parameter PSD given in (1.2).

It can be easily shown that TPPSD is unimodal and has increasing hazard rate. Since

$$\frac{P_2(x+1;\theta,\alpha)}{P_2(x;\theta,\alpha)} = \frac{1}{\theta+1}\left[1+\frac{2x+\theta+5}{x^2+(\theta+4)x+\left\{\alpha\theta^2+(2\alpha+1)\theta+(\alpha+3)\right\}}\right]$$

is decreasing function in x, $P_3(x;\theta,\alpha)$ is log-concave. Now using the results of relationship between log-concavity, unimodality and increasing hazard rate (IHR) of discrete distributions available in Grandell (1997), it can concluded that TPPSD has an increasing hazard rate and unimodal.

The behavior of the pmf of TPPSD for varying values of parameters θ and α are shown in figure 1.

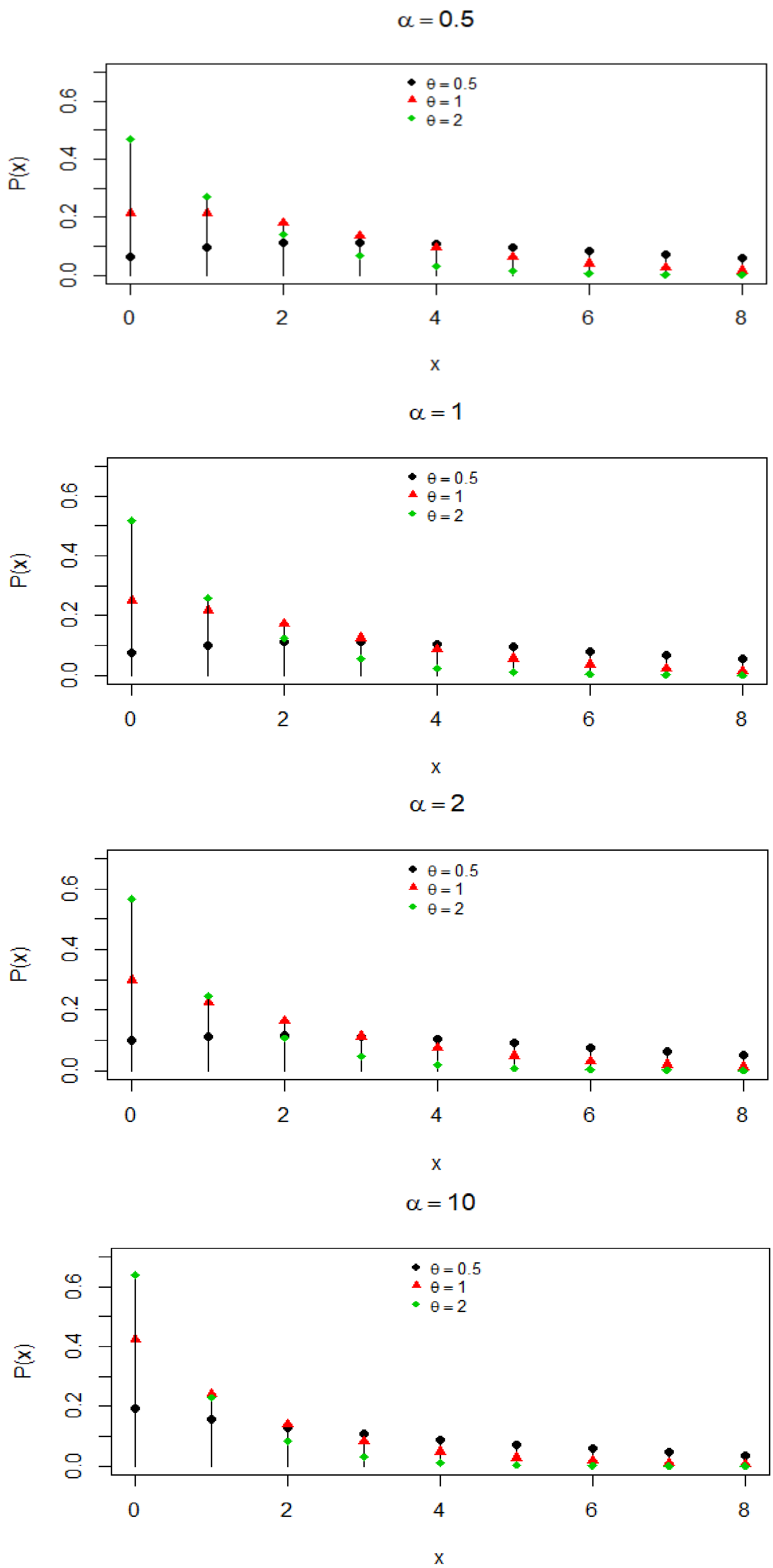

Figure 1. Behaviour of pmf of TPPSD for varying values of parameters θ and α

3. Moments Based Measures

The r th factorial moment about origin $\mu_{(r)}'$ of TPPSD (2.2) can be obtained as

$\mu_{(r)}' = E\left[E\left(X^{(r)} \mid \lambda\right)\right]$, where $X^{(r)} = X\left(X-1\right)\left(X-2\right)...\left(X-r+1\right)$. Using (2.1), the r th factorial moment about origin $\mu_{(r)}'$ of TPPSD (2.2) can be obtained as

$$\mu_{(r)}' = \frac{\theta^3}{\alpha\theta^2+\theta+2}\int_0^\infty \left[\sum_{x=0}^{\infty} x^{(r)}\frac{e^{-\lambda}\lambda^x}{x!}\right]\left(\alpha+\lambda+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

$$= \frac{\theta^3}{\alpha\theta^2+\theta+2}\int_0^\infty \left[\lambda^r\sum_{x=0}^{\infty}\frac{e^{-\lambda}\lambda^{x-r}}{(x-r)!}\right]\left(\alpha+\lambda+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

Taking $x-r=y$ within the bracket, we get

$$\mu_{(r)}' = \frac{\theta^3}{\alpha\theta^2+\theta+2}\int_0^\infty \left[\lambda^r\sum_{y=0}^{\infty}\frac{e^{-\lambda}\lambda^{y}}{y!}\right]\left(\alpha+\lambda+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

$$= \frac{\theta^3}{\alpha\theta^2+\theta+2}\int_0^\infty \lambda^r\left(\alpha+\lambda+\lambda^2\right)e^{-\theta\lambda}d\lambda$$

After some tedious algebraic simplification, a general expression for the r th factorial moment about origin $\mu_{(r)}'$ of TPPSD (2.2) can be expressed as

$$\mu_{(r)}' = \frac{r!\left\{\alpha\theta^2+(r+1)\theta+(r+1)(r+2)\right\}}{\theta^r\left(\alpha\theta^2+\theta+2\right)}\;;r=1,2,3,.... \qquad (3.1)$$

It can be easily verified that at $\alpha=1$, the expression (3.1) reduces to the corresponding expression of PSD. Substituting $r=1,2,3,\text{and } 4$ in (3.1), the first four factorial moments about origin of TPSD can be obtained as

$$\mu_{(1)}' = \frac{\alpha\theta^2+2\theta+6}{\theta\left(\alpha\theta^2+\theta+2\right)},\ \mu_{(2)}' = \frac{2\left(\alpha\theta^2+3\theta+12\right)}{\theta^2\left(\alpha\theta^2+\theta+2\right)},\ \mu_{(3)}' = \frac{6\left(\alpha\theta^2+4\theta+20\right)}{\theta^3\left(\alpha\theta^2+\theta+2\right)},\mu_{(3)}' = \frac{24\left(\alpha\theta^2+5\theta+30\right)}{\theta^4\left(\alpha\theta^2+\theta+2\right)}.$$

Now using the relationship between factorial moments about origin and moments about origin, the first four moment about origin of the TPPSD are obtained as

$$\mu_1' = \frac{\alpha\theta^2+2\theta+6}{\theta\left(\alpha\theta^2+\theta+2\right)}$$

$$\mu_2' = \frac{\alpha\theta^3+(2\alpha+2)\theta+12\theta+24}{\theta^2\left(\alpha\theta^2+\theta+2\right)}$$

$$\mu_3' = \frac{\alpha\theta^4+(6\alpha+2)\theta^3+(6\alpha+24)\theta^2+96\theta+120}{\theta^3\left(\alpha\theta^2+\theta+2\right)}$$

$$\mu_4' = \frac{\alpha\theta^5+(14\alpha+2)\theta^4+(36\alpha+48)\theta^3+(24\alpha+312)\theta^2+840\theta+720}{\theta^4\left(\alpha\theta^2+\theta+2\right)}$$

Using the relationship between moments about mean and the moments about origin, the moments about mean of TPPSD are obtained as

$$\mu_2=\frac{\alpha^2\theta^5+\left(\alpha^2+3\alpha\right)\theta^4+(12\alpha+2)\theta^3+(16\alpha+12)\theta^2+24\theta+12}{\theta^2\left(\alpha\theta^2+\theta+2\right)^2}$$

$$\mu_3=\frac{\left\{\begin{array}{l}\alpha^3\theta^8+\left(3\alpha^3+4\alpha^2\right)\theta^7+\left(2\alpha^3+25\alpha^2+5\alpha\right)\theta^6+\left(66\alpha^2+42\alpha+2\right)\theta^5\\+\left(60\alpha^2+148\alpha+20\right)\theta^4+(216\alpha+84)\theta^3+(72\alpha+168)\theta^2+144\theta+48\end{array}\right\}}{\theta^3\left(\alpha\theta^2+\theta+2\right)^3}$$

$$\mu_4=\frac{\left\{\begin{array}{l}\alpha^4\theta^{11}+\left(10\alpha^4+5\alpha^3\right)\theta^{10}+\left(18\alpha^4+72\alpha^3+9\alpha^2\right)\theta^9+\left(9\alpha^4+314\alpha^3+158\alpha^2+7\alpha\right)\theta^8\\+\left(600\alpha^3+940\alpha^2+140\alpha+2\right)\theta^7+\left(384\alpha^3+2576\alpha^2+1012\alpha+44\right)\theta^6\\+\left(3096\alpha^2+3544\alpha+368\right)\theta^5+\left(1224\alpha^2+6232\alpha+1560\right)\theta^4+(5184\alpha+3600)\theta^3\\+(1728\alpha+4512)\theta^2+2880\theta+720\end{array}\right\}}{\theta^4\left(\alpha\theta^2+\theta+2\right)^4}$$

The coefficient of variation $(C.V)$, coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis (β_2), and index of dispersion (γ) of TPPSD are thus given by

$$C.V=\frac{\sigma}{\mu_1'}=\frac{\sqrt{\alpha^2\theta^5+\left(\alpha^2+3\alpha\right)\theta^4+(12\alpha+2)\theta^3+(16\alpha+12)\theta^2+24\theta+12}}{\alpha\theta^2+2\theta+6}$$

$$\sqrt{\beta_1}=\frac{\mu_3}{\mu_2^{3/2}}=\frac{\left\{\begin{array}{l}\alpha^3\theta^8+\left(3\alpha^3+4\alpha^2\right)\theta^7+\left(2\alpha^3+25\alpha^2+5\alpha\right)\theta^6+\left(66\alpha^2+42\alpha+2\right)\theta^5\\+\left(60\alpha^2+148\alpha+20\right)\theta^4+(216\alpha+84)\theta^3+(72\alpha+168)\theta^2+144\theta+48\end{array}\right\}}{\left\{\alpha^2\theta^5+\left(\alpha^2+3\alpha\right)\theta^4+(12\alpha+2)\theta^3+(16\alpha+12)\theta^2+24\theta+12\right\}^{3/2}}$$

$$\beta_2=\frac{\mu_4}{\mu_2^{\,2}}=\frac{\left\{\begin{array}{l}\alpha^4\theta^{11}+\left(10\alpha^4+5\alpha^3\right)\theta^{10}+\left(18\alpha^4+72\alpha^3+9\alpha^2\right)\theta^9+\left(9\alpha^4+314\alpha^3+158\alpha^2+7\alpha\right)\theta^8\\+\left(600\alpha^3+940\alpha^2+140\alpha+2\right)\theta^7+\left(384\alpha^3+2576\alpha^2+1012\alpha+44\right)\theta^6\\+\left(3096\alpha^2+3544\alpha+368\right)\theta^5+\left(1224\alpha^2+6232\alpha+1560\right)\theta^4+(5184\alpha+3600)\theta^3\\+(1728\alpha+4512)\theta^2+2880\theta+720\end{array}\right\}}{\left\{\alpha^2\theta^5+\left(\alpha^2+3\alpha\right)\theta^4+(12\alpha+2)\theta^3+(16\alpha+12)\theta^2+24\theta+12\right\}^2}$$

$$\gamma=\frac{\sigma^2}{\mu_1'}=\frac{\alpha^2\theta^5+\left(\alpha^2+3\alpha\right)\theta^4+(12\alpha+2)\theta^3+(16\alpha+12)\theta^2+24\theta+12}{\theta\left(\alpha\theta^2+\theta+2\right)\left(\alpha\theta^2+2\theta+6\right)}.$$

It can be easily verified that at $\alpha=1$, expressions of these statistical constants of TPPSD reduce to the corresponding expressions for PSD.

The behaviors of coefficient of variation (C.V), coefficient of skewness (C.S), coefficient of kurtosis (C.K) and index of dispersion (I.D) of TPPSD for varying values of parameters θ and α have been explained through graphs and presented in figure 2.

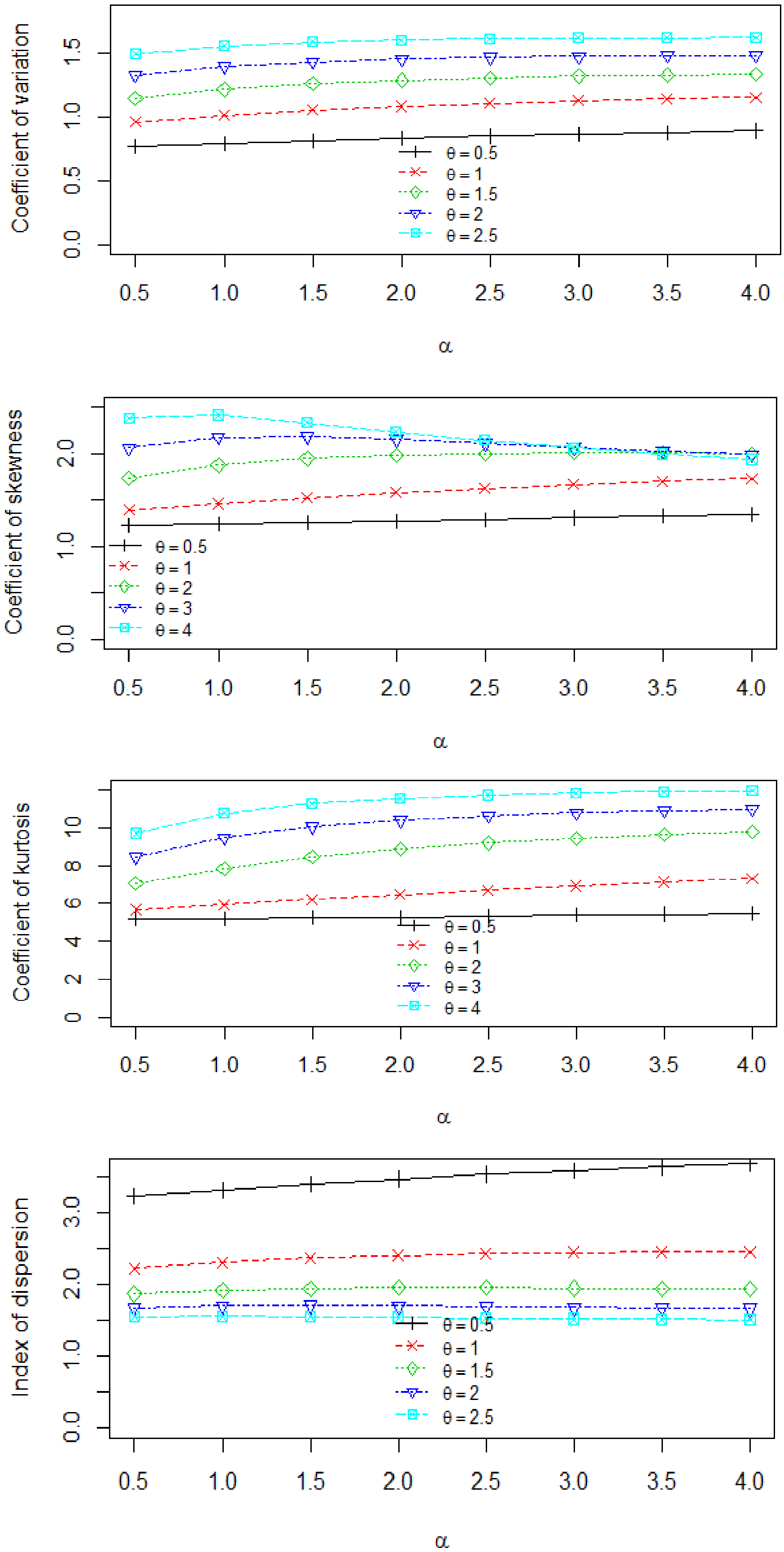

Figure 2. Behaviors of coefficient of variation (C.V), coefficient of skewness (C.S), coefficient of kurtosis (C.K) and index of dispersion (I.D) of TPPSD for varying values of parameters θ and α

4. Maximum Likelihood Estimation of Parameters

Suppose $(x_1, x_2, ..., x_n)$ be a random sample of size n from TPPSD and f_x be the observed frequency in the sample corresponding to $X = x \ (x = 1, 2, 3, ..., k)$ such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency. The likelihood function L of TPPSD is given by

$$L = \left(\frac{\theta^3}{\alpha\theta^2 + \theta + 2} \right)^n \frac{1}{(\theta+1)^{\sum_{x=1}^{k} (x+3) f_x}} \prod_{x=1}^{k} \left[x^2 + (\theta+4)x + \left\{ \alpha\theta^2 + (2\alpha+1)\theta + (\alpha+3) \right\} \right]^{f_x}$$

The log likelihood function is thus obtained as

$$\log L = n \log\left(\frac{\theta^3}{\alpha\theta^2 + \theta + 2} \right) - \sum_{x=1}^{k} (x+3) f_x \log(\theta+1)$$
$$+ \sum_{x=1}^{k} f_x \log\left[x^2 + (\theta+4)x + \left\{ \alpha\theta^2 + (2\alpha+1)\theta + (\alpha+3) \right\} \right]$$

The maximum likelihood estimates $(\hat{\theta}, \hat{\alpha})$ of (θ, α) of TPPSD is the solutions of the following log likelihood equations

$$\frac{\partial \log L}{\partial \theta} = \frac{3n}{\theta} - \frac{n(2\alpha\theta+1)}{\alpha\theta^2+\theta+2} - \frac{n(\bar{x}+3)}{\theta+1} + \sum_{x=1}^{k} \frac{(x+2\alpha\theta+2\alpha+1) f_x}{\left[x^2 + (\theta+4)x + \left\{ \alpha\theta^2 + (2\alpha+1)\theta + (\alpha+3) \right\} \right]} = 0$$

$$\frac{\partial \log L}{\partial \alpha} = -\frac{3n\theta^2}{\alpha\theta^2+\theta+2} + \sum_{x=1}^{k} \frac{(\theta^2+2\theta+1) f_x}{\left[x^2 + (\theta+4)x + \left\{ \alpha\theta^2 + (2\alpha+1)\theta + (\alpha+3) \right\} \right]} = 0,$$

where $\bar{x}$ is the sample mean. These two log likelihood equations do not seem to be solved directly because they do not have closed forms. Therefore, to find the maximum likelihood estimates of parameters an iterative method such as Fisher Scoring method, Bisection method, Regula Falsi method or Newton-Raphson method can be used. In this paper Newton-Raphson method has been used using R-software.

Table 1. Observed and Expected number of European red mites on Apple leaves, available in Bliss (1953)

Number of Red mites per leaf	Observed frequency	Expected frequency				
		PD	PLD	PSD	AGPSD	TPPSD
0	70	47.6	67.2	66.4	67.3	69.1
1	38	54.6	38.9	39.2	38.7	37.4
2	17	31.3	21.2	21.8	21.2	20.3
3	10	11.9	11.1	11.4	11.2	11.0
4	9	3.4	5.7	5.7	5.7	5.8
5	3	0.8	2.8	2.8	2.9	3.0
6	2	0.2	1.4	1.3	1.4	1.6
7	1	0.1	0.9	0.6	0.7	0.8
8	0	0.1	0.8	0.8	0.9	1
Total	150	150.0	150.0	150.0	150.0	150.0
ML estimates		$\hat{\theta} = 1.14666$	$\hat{\theta} = 1.26010$	$\hat{\theta} = 1.6533$	$\hat{\theta} = 1.4043$ $\hat{\alpha} = 0.2316$	$\hat{\theta} = 1.3640$ $\hat{\alpha} = 3.2989$
χ^2		26.50	2.49	3.41	2.99	2.43
d.f		2	4	4	3	3
p-value		0.0000	0.5595	0.4916	0.3931	0.4880
$-2\log L$		485.61	445.02	445.27	444.95	444.53
AIC		487.61	447.02	447.27	448.95	448.53

Table 2. Observed and Expected number of households according to the number of male migrants aged 15 years and above, available in Shukla and Yadav (2006)

Number of migrants	Observed frequency	Expected frequency				
		PD	PLD	PSD	AGPSD	TPPSD
0	242	208.9	240.1	239.9	239.9	240.8
1	97	136.7	98.8	98.7	98.8	97.8
2	35	44.7	39.0	39.3	39.3	39.0
3	19	9.7	15.0	15.1	15.1	15.1
4	6	1.6	5.6	5.6	5.6	5.7
5	3	0.2	2.0	2.0	2.0	2.1
6	0	0.0	0.7	0.7	0.7	0.7
7	0	0.0	0.3	0.2	0.2	0.3
8	0	0.2	0.4	0.5	0.4	0.5
Total	402	402.0	402.0	402.0	402.0	402.0
ML estimates		$\hat{\theta}=0.6542$	$\hat{\theta}=2.0329$	$\hat{\theta}=2.4665$	$\hat{\theta}=2.4835$ $\hat{\alpha}=1.0589$	$\hat{\theta}=2.3668$ $\hat{\alpha}=1.2916$
χ^2		41.58	1.46	1.52	1.53	1.43
d.f		2	3	3	2	2
p-value		0.0000	0.6915	0.6776	0.4653	0.4891
$-2\log L$		932.64	892.37	892.25	892.25	892.22
AIC		934.64	894.37	894.25	896.25	896.22

5. Applications

The applications and the goodness of fit of the TPPSD have been demonstrated with two real count datasets. The first dataset is from ecology regarding the observed number of European red mites on Apple leaves, available in Bliss (1953) and the second dataset is from demography regarding the observed number of households according to the number of male migrants aged 15 years and above, available in Shukla and Yadav (2006). These two datasets are over-dispersed data. Maximum likelihood estimation has been used to fit Poisson distribution (PD), Poisson-Lindley distribution (PLD), PSD, AGPSD and TPPSD. The AIC (Akaike information criterion) has been calculated using the formula $AIC=-2\log L+2k$, where k is the number of parameters involved in the distribution. The distribution having less value of chi-square and AIC is the better distribution. Based on the values of chi-square and AIC of the considered distribution, it is obvious that TPPSD is competing well with the considered one parameter and two-parameter discrete distributions and, therefore, TPPSD can be considered an important two-parameter discrete distribution for ecology and migration data.

6. Concluding Remarks

In this paper, a two-parameter Poisson Sujatha distribution (TPPSD) by compounding Poisson distribution with two-parameter Sujatha distribution, which includes Poisson-Sujatha distribution (PSD) as a special case, has been proposed. The unimodality and increasing hazard rate properties of the distribution has been explained. Its moments and moments based measures have been derived. The nature of coefficients of variation, skewness, kurtosis and index of dispersion has been discussed with varying values of parameters. The method of maximum likelihood estimation has been discussed. The applications of the proposed distribution has been explained through two examples of count data, one from ecology and one from demography and its goodness of fit has been found quite satisfactory over PD, PLD, PSD, and AGPSD.

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REFERENCES

[1] Bliss, C.I. (1953): Fitting negative binomial distribution to biological data, *Biometrics*, 9, 177 – 200.

[2] Grandell, J. (1997): Mixed Poisson Processes, Chapman& Hall, London.

[3] Kaliraja, M. and Perarasan, K. (2019): A new stochastic model on the generalization of Sujatha distribution for the effects of two types of exercise on plasma growth hormone, The International Journal of Analytical and Experimental Model Analysis, 11(9), 1164 – 1170.

[4] Lindley, D.V. (1958): Fiducial distributions and Bayes' theorem, *Journal of the Royal Statistical Society, Series B*, 20, 102- 107.

[5] Sankaran, M. (1970): The discrete Poisson-Lindley distribution, *Biometrics*, 26: 145-149.

[6] Shanker, R. (2016 a): Sujatha distribution and Its Applications, *Statistics in Transition new Series*, 17 (3), 1 – 20.

[7] Shanker, R. (2016 b): The discrete Poisson-Sujatha distribution, *International Journal of Probability and Statistics*, 5(1), 1- 9.

[8] Shanker, R., Shukla, K.K., and Hagos, F. (2017): A Generalization of Sujatha distribution and its Applications to Real lifetime data, *Journal of Institute of Science and Technology* (JIST), 22(1), 77 – 94.

[9] Shanker, R. and Shukla, K.K. (2019): A generalization of Poisson-Sujatha Distribution and Its Applications to Ecology, *International Journal of Biomathematics*, 12 (2), 1- 11.

[10] Mussie, T. and Shanker, R. (2018): A Two-Parameter Sujatha distribution, *Biometrics & Biostatistics International Journal*, 7(3), 188 – 197.

[11] Shukla, K.K. and Yadav, K.N.S. (2006): The distribution of number of migrants at household level, Journal of Population and Social Studies, 14(2), 153 – 166.

[12] Wesley, B., Agelica, M.T.R., Katiane, S.C., Marinaho, G.A., and Francisco, L.N. (2018): On Zero-modified Poisson-Sujatha distribution to model over-dispersed data, Austrian Journal of Statistics, 47(3), 1 – 19.