# An Iterative Quality-based Localization Algorithm for Ad Hoc Networks

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**Abstract.** In this paper an iterative quality-based algorithm for location discovery is presented which can be used in wireless ad hoc sensor networks. The algorithm will take the reliability of measurements into account and will produce a reliability index for every estimated location using a statistical approach. The algorithm can also work in a hybrid network with different kinds of distance measuring techniques. It will use the reliability of each of these methods in the final result. Satisfactory results can be achieved with this approach.

## 1 Introduction

The vision of ubiquitous computing requires the development of devices and technologies, which can be pervasive without being intrusive. Recently, we have seen major progress in developing the off-the-desktop computing paradigm that moves towards the notion of a pervasive, wearable, unobtrusive, disappearing, or invisible computer. Intuitively it can be defined as making computer systems pervasive, enhancing user interaction with the environment, as seamlessly as possible. Obviously context awareness and wireless communication plays a major role in this area. Therefore, the success of pervasive

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computing will be closely related to their capability to locate themselves in the contexts of its environment.

## 2 Related Work

Researchers have developed many localization techniques. We will discuss some techniques briefly. *GPS* [HOF97] is probably the most common used localization technique used for outdoor localization. The *Active Badge* system [WAN92] was one of the first badge systems for indoor use. The *Active Bat* system [HAR99] makes use of the combination of two techniques to pinpoint an object's location: the time difference of arrival (TDOA) of ultrasound waves and radio waves. The *RADAR* system [BAH00] uses the received radio signal strength (RSSI) to determine an object's location. Estrin et al. did some experiments with ad-hoc outdoor localization techniques; one of them is described in [EST00]. Savarese [SAV01] proposed an algorithm to calculate relative positions of nodes based on range measurements between the nodes. It will be discussed more in section three.

## 3 The IQL Algorithm

This section gives a brief description of the IQL algorithm. For it to work, a few assumptions are made. The first is that the devices have certain processing capabilities and identical communicative properties. The second is that the error-rate in the range measurement method is gauss-distributed. The third is symmetric (wireless) communication capabilities between nodes that can sense their mutual distance. Nodes that can sense each other and can have direct communication will be called *neighbours*.

All measurements are more or less reliable; they have a certain amount of error. As is usual for errors of measurements, the distribution of those error values will follow the Gaussian distribution, with the mean around its true value, and some standard deviation. For range measurements, the standard deviation typically depends on the true distance in a linear way, which means that the range measurements can be modelled to be drawn from a Gaussian statistical distribution, where the mean is the true range $R$, and the standard deviation is calculated as $R \cdot S$, where $S$ is the measurement precision constant, between 0 and 1, which depends on the method of measurement being used. In the case of RSSI range measurements, [BAH00] presents a set of test results, which suggest this assumption to be valid. Incorporating precision into the localization calculations has the benefit of having precision information next to a location as a result. An additional benefit is that different range sensing techniques can be used in order to estimate the positions of nodes in an ad hoc network, depending on what hardware is available in each node. The IQL algorithm first uses the ABC-algorithm to determine an initial place, after which the WLS-algorithm is used iteratively to refine the position. In the WLS-algorithm the Gauss-distribution is used to determine the reliability of measurements.

### Weighed Least Squares Algorithm

The calculation of a location from a set of range measurements between the node to be calculated and a set of nodes with known or estimated positions can be performed in a way similar to that of GPS (described in [MIS99]) and other systems. Starting from an initial estimation, an improvement vector is calculated iteratively and added to the previous estimation until the improvement vector is smaller than a certain value. This improvement vector is calculated with a weighed linear least squares formula, $wA\,\mathbf{x} = w\mathbf{b}$, with $A$ a matrix, and $\mathbf{x}$ and $\mathbf{b}$ column vectors. Denoting the true position of the node to be calculated as $\mathbf{x} = (x, y)$ and the initial estimation of the position as $\mathbf{x}^{est}$ and the positions of the $n$ neighbour nodes as $\mathbf{x}_i$, $i = 1..n$, the measured distances to those nodes can be denoted as $r_i = \| \mathbf{x}_i - \mathbf{x} \| + e_{\mathrm{i}}$, $i = 1..n$, and the nodes true position as $\mathbf{x} = \mathbf{x}^{est} + d\mathbf{x}$.

In the same way, $r^{est}_i = \| \mathbf{x}_i - \mathbf{x}^{est} \|$, and

$$dr_i = r_i - r^{est}_i = \| \mathbf{x}_i - \mathbf{x}^{est} - d\mathbf{x} \| - \| \mathbf{x}_i - \mathbf{x}^{est} \| + e_{\mathrm{i}},\ i = 1..n$$

$$\approx -\frac{(\mathbf{x}_i - \mathbf{x}^{est})}{\| x_i - \mathbf{x}^{est} \|} \cdot d\mathbf{x} + e_i$$

$$= dir_i \times d\mathbf{x} + e_I,$$

with $dir_i$ the direction vector of length 1 from the node's estimated position to node *i*.

For each range measurement $r_i$ a weight is calculated according to formula 1, where $s^{edge}{}_{0,i}^{2}$ is the variance of the range measurement to node *i*, and $s^{node}{}_{i}^{2}$ is the 1-dimensional variance of the precision value of node *i*.

$$w_i = \sqrt[-2]{s^{\,edge}{}_{0,i}^{\;2} + s^{\,node}{}_{0,i}^{\;2}} \qquad \textbf{(1)}$$

Matrix *A* is then constructed by filling the *i*th row with the weighed direction vector of range measurement *i*, $dir_i \cdot w_i$, where $w_i$ is calculated as in equation 1, and vector ***b*** is constructed as a column vector filled with the values $dr_i \cdot w_i$. The least squares solution of the equation $w\boldsymbol{Ax} = w\boldsymbol{b}$ will then look like this:

$$\begin{bmatrix} dir_{x,1}/w_1 & dir_{y,1}/w_1 \\ \vdots & \vdots \\ dir_{x,n}/w_n & dir_{y,n}/w_n \end{bmatrix} \begin{bmatrix} dx & dy \end{bmatrix} = \begin{bmatrix} dr_1/w_1 \\ \vdots \\ dr_n/w_n \end{bmatrix} \qquad \textbf{(2)}$$

This way the improvement vector ***dx*** is calculated. From this calculation the node's precision can be calculated as well. $Cov = (A^T A)^{-1}$ is the covariance matrix of all the measurements and assuming that the x- and y-values involved in the calculation are independent. The node's two-dimensional standard deviation is calculated as $\sqrt{Cov_{1,1} + Cov_{2,2}}$ . Using a calculation like this, a higher number of neighbouring nodes will result in a lower standard deviation. When the estimated position of the nodes is calculated like this, the optimal location will be calculated, based on the given ranges to and positions of the neighbour nodes.

## ABC Algorithm

Having a way to calculate a node's position based on the position of neighbouring nodes, it is still necessary to be able to make an estimation of those neighbouring nodes first. This can be done using the Assumption Based Coordinates algorithm described in [SAV01]. In a distributed environment, a node has no knowledge of the positions of nearby nodes at the start of the localization process. A local map has to be made by each node from the available range measurements. The

algorithm is based on assumptions about the locations of neighbouring nodes. These assumptions have to be made, because an exact position can't be determined without having enough (three or more) range measurements. Each node (numbered 0) starts by setting its own (relative) coordinates at $\boldsymbol{x}_0 = (0,0)$, and its standard deviation $s^{node}{}_0$ at a value of nearly 0 (just to ensure that division will not produce infinity). For two neighbouring nodes $n_i$, $n_j$ to which the ranges are known, as well as between each other, it will then set their locations at
$\boldsymbol{x}_i = (r_i,0)$, and $\boldsymbol{x}_j = (r_{0,\mathrm{i}}{}^2 + r_{0,\mathrm{j}}{}^2 - r_{i,\mathrm{j}}{}^2 / 2\, r_{0,\mathrm{i}}, \sqrt{r_{0,\mathrm{j}}{}^2 - x_j{}^2}\,)$,
and their standard deviations at
$s^{node}{}_i = s^{edge}{}_{0,i} / 2$, $s^{node}{}_j = \sqrt{s^{edge}{}_{0,\mathrm{j}}{}^2 + s^{edge}{}_{i,\mathrm{j}}{}^2} / 2$..

Now the weighed least squares algorithm (or WLS) can be used to calculate better estimations for $\boldsymbol{x}_i$, $\boldsymbol{x}_j$, $s^{node}{}_i$ and $s^{node}{}_j$.

In here the notation $r_{i,j}$ is used to denote the measured range between node $n_i$ and $n_j$, $s^{edge}{}_{i,j}$ for the standard deviation, and $s^{node}{}_i$ for node $n_\mathrm{i}$'s two-dimensional standard deviation.

For all subsequent nodes in the network for which position estimation is required, the ABC algorithm continues in the following manner: To calculate position $\boldsymbol{x}_l$ of node $n_l$, find three nodes $n_i$, $n_j$, $n_k$ whose positions are known, as well as the ranges between them and the ranges from these three nodes to node $n_l$. Triangulation determines the estimated position of node $n_l$, and the subsequent application of the WLS algorithm optimises these values for the ranges to its neighbours with known location, and assigns a value to $s^{node}{}_l$.

If somewhere during this process at least three nodes are located that have a known absolute position and precision, then a coordinate transformation can be performed to calculate the absolute coordinates of each node located so far, and an additional WLS calculation to calculate their precision values relative to those fixed nodes, instead of to node $n_0$'s position. Between anchor nodes and other nodes whose positions are known range measurement values are artificially created, with a high precision associated to it, to enforce the correct placement of these nodes.

## 4 Simulation Results

This research has only focused on the performance that can be achieved by using the calculations presented, and has not gone into the details of message passing in the network. Only a few cases will be presented describing the results of some experiments assuming a certain network connectivity and availability of information in the network. Another assumption in this paper is that all nodes are static, so their locations will not change over time, during the execution of the algorithm, thereby abstracting from the timing parameters subject to the network communication and calculation.

To investigate the performance of the IQL-algorithm, a series of simulations are performed which are described below. This algorithm has been simulated on networks of varying connectivity where all nodes have all location and range information available in the network. The networks contain a small amount of anchor nodes with known position and very high precision ($s \approx 0$). All other nodes have an initially unknown position and use the same distance measurement method. Since all nodes have the same information, each node can calculate its position as well as the position of all other nodes in the network, so they produce the same results. This means that the calculation will have to be performed for only one node, in order to test the performance of the algorithm. All calculations will be performed in a 2-dimensional space, which is chosen as the initial implementation because it is simpler, and many application areas will only have to work with 2-dimensional location information.

### Results of the Simulations

If nodes in the network are assigned a wrong location initially, they will disrupt the further correct placement of other nodes in the network. Figure 1 and 2 show typical networks of 20 nodes, with average connectivity of 9 neighbour nodes per node, and measurement precision $S = 0.2$ respectively $S = 0.05$. The true locations of the nodes, the small dots, are connected by lines with the estimated locations, the circles. The sizes of the circles indicate the node's precision. The lines between nodes indicate presence of a range measurement between these nodes.

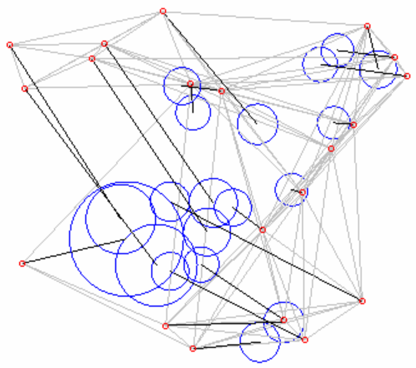

Fig. 1. Solution for anchorless network of 20 nodes with range precision of $\sigma$=0.2.

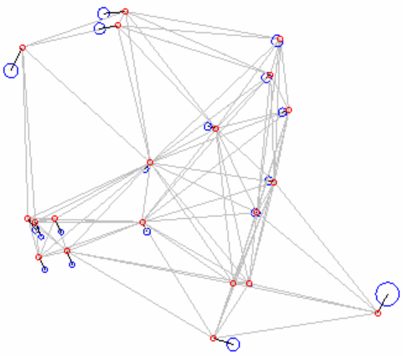

Fig. 2. Solution for anchorless network of 20 nodes with range precision of $\sigma$=0.05.

The large error especially occurs for nodes at the edges of the network, when all neighbour nodes are located only on one side of the node. During triangulation it is not easy to determine at which side of its neighbour nodes the next node to be placed should be located. Due to the higher error in the range measurements between the nodes in figure 1, this effect will occur more often.

It is clear that the more precise the measurements get, the less frequent this kind of behaviour occurs. Besides that, more densely connected networks perform slightly better as well. But for most location sensing systems, both solutions are not practical, or even not possible, due to the higher cost of hardware and energy consumption required to achieve more precise measurements, or longer ranging distances. Besides that, only having relative locations will not suffice in many application areas, therefore adding a few anchors to the network would be an acceptable solution. Adding four anchor nodes to the network greatly increases the performance of the algorithm, even by so much that most location errors lay well within the expected range as indicated by the calculated precision for each node. Figure 3 shows that for a network of size 40, and $S = 0.2$, about 67% of the relative location errors are below 1,

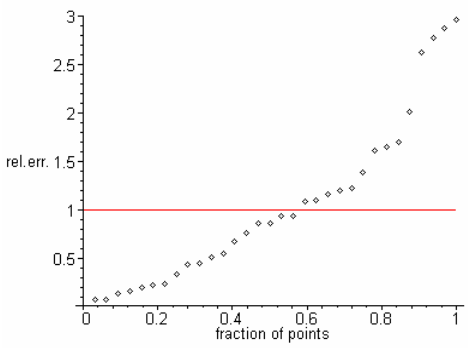

Fig. 3. Relative error of the nodes in a 40-node network with $S = 0.2$ of which 8 are anchor nodes.

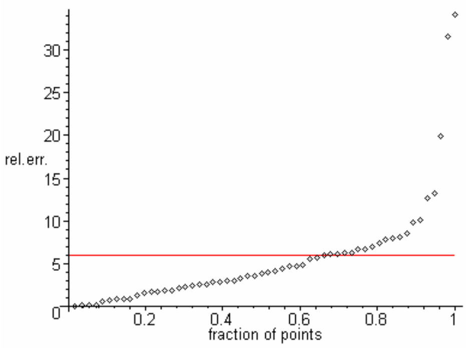

Fig. 4. Relative error of the nodes in a 60-node network with $S = 0.05$ of which 4 are anchor nodes.

which is the expected value for errors in a Gaussian distributed stochastic variable. This means that the calculated precision can truly be considered to be the real precision of the nodes in the network. For more precise measurements, the network sizes are allowed to get larger, while the calculated precision still remains a valid description of the true position error. Figure 4 shows that for about 68% of the nodes the relative error remains below a value of about 6. The calculated precision is not proportional to the relative error. A formula can be devised, though, which will generate correct precision values. In this research however, no effort has been put into constructing this formula. We just assume the existence of it.

## Implementation and Future Work

At the moment the algorithm is centralized. When distributing the algorithm, a device would just take a few nodes around it into its calculation, while the positions of other nodes will be communicated to it across the network. This way it is even possible for each node to only calculate its own position, and broadcast it over the network, for other nodes to use it in their calculations.

These different scenarios will of course have an impact on the computation and network communication needs, and depending on the hardware limitations of the network nodes, different approaches can be taken. The precision that is needed can as well be a factor on which to decide the amount of neighbour nodes to use in a nodes position calculation. Even though all these factors present a lot of new questions towards the performance of a location discovery system using the algorithms presented in this paper, it would reach too far to incorporate all of them in this study. Besides the two-dimensional space this algorithm is currently developed for, it could be very useful to have it work in full three-dimensional space to many application areas. Some more changes could also be made to the algorithm. In the current state, it can only locate nodes that have at least three neighbouring nodes in order to determine the location. For nodes that have only two neighbours within measuring distance, it should be allowed, and possible to also localize them based on the information in the network. This has not had the focus in our research so far, but could be added in future versions. Another future work would be adapting the algorithm in a way that it can be used for a dynamic environment.

## 5 Conclusions

In this paper an algorithm for localization, which takes into account the precision of measurements, to produce a location with precision indication. This algorithm can be used for different range measurements techniques. Despite the great amount of further research that can to be done on the use of this algorithm, it seems quite promising to us for use in many different application areas. As is shown in section five, the reliability of the position becomes quite acceptable when the gauss-distribution is very low. For a somewhat greater gauss-distribution, the precision becomes acceptable if a few anchors are added into the network. When compared to the ABC-algorithm and the WLS-algorithm, the IQL-algorithm produces a better result.

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