Limitations of Robust Adaptive Pole Placement Control

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***Abstract*—In this paper we investigate the limitations of adaptive pole placement control of AR systems when an additive disturbance is present. The magnitude of the disturbance is dependent on past input and output values. It is shown that the control objective of robust stabilization restricts the set of systems one can deal with. We subsequently propose and analyze an adaptive algorithm which can be applied to convex, not necessarily bounded, subsets of the aforesaid set. In particular, it is shown that all signals in the control loop remain bounded.**

I. Introduction

A. System Description

We consider the problem of robustly and adaptively stabilizing a linear time-invariant, discrete time, single-input/single-output (SISO), finite dimensional system. The system is represented by

$$A_0(q^{-1})y_t = b_0 u_{t-d} + \delta_t \qquad t \geq 0 \tag{1}$$

where

$$A_0(q^{-1}) = 1 + a_{01}q^{-1} + a_{02}q^{-2} + \cdots + a_{0n}q^{-n} \tag{2}$$

and u_t is the input. y_t is the measured output, and δ_t represents the deviation from nominal behavior. q^{-1} is the backward shift operator. The description of the deviation is given by.

$$|\delta_t| \leq \epsilon_0 + \epsilon_1 \sup_{t-k<l<t} (|u_l|, |y_l|) \qquad t \geq 0 \tag{3}$$

for some $\epsilon_0 \geq 0$ and $\epsilon_1 \geq 0$ where $(|u_l|, |y_l|)$ denotes the maximum of $|u_l|$ and $|y_l|$, and k is finite. This deviation description is similar to the one used in [9]. In the rest of the paper we will refer to δ_t as noise, but this does not necessarily mean that δ_t has a stochastic nature. δ_t is simply the mismatch between the model and the actual plant.

B. Robust and Adaptive Stabilization

With robust stabilization we mean that a feedback control designed on the basis of the "nominal" model

$$A_0(q^{-1})y_t = b_0 u_{t-d} \tag{4}$$

actually stabilizes the system (1) as well. (The difference between the model and the system is the disturbance term δ_t.) We say that a system is stabilized if all signals in the control loop remain bounded by $K\epsilon_0$ where K is a constant independent of the realization of δ.

Adaptive refers to the fact that the actual parameters, i.e., b_0 and the coefficients of the $A_0(q^{-1})$ polynomial, are not known.

Manuscript received June 15, 1992; revised July 16, 1993.

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IEEE Log Number 9401693

Robust and adaptive stabilization requires recovery of the robust stability (as described above) in the limit as time tends to infinity, on the basis of input/output measurement and our prior information.

Our prior knowledge includes the noise parameters ϵ_0 and ϵ_1, the order n, and the time delay d, of the nominal system, as well as $k-1$, the size of the noise window in (3).

Furthermore, we need a measure of the controlled system's robustness. A realization of δ_t can be written as

$$\delta_t = \epsilon_{0t} + \epsilon_{1t} \sup_{t-k<l<t} (|u_l|, |y_l|)$$

where $-\epsilon_0 \leq \epsilon_{0t} \leq \epsilon_0$, and $-\epsilon_1 \leq \epsilon_{1t} \leq \epsilon_1$, and in the sequel we will often talk about a realization of $\{\epsilon_{0,t}, \epsilon_{1,t}\}$ rather than a realization of $\{\delta_t\}$. $\{\epsilon_{0,t}, \epsilon_{1,t}\}$ will be referred to as a sequence of disturbance gains. In closed loop, y_t and u_t can be written as $y_t = [G_y\epsilon_0]_t$ and $u_t = [G_u\epsilon_0]_t$. The robustness measure will be linked to the gain ($= l_\infty$ induced operator norm) of the operators G_u and G_y. In addition to our prior knowledge concerning the system, we also assume that we are given a robustness margin ϵ_1'. This point will be clarified later on in Section II.

C. The Controller

Assume that the control designer believes that the control objective of robust and adaptive stabilization can be achieved using a pole-placement controller. In particular, the designer thinks that the closed-loop poles may be placed at the predetermined zeros of the polynomial

$$A^*(q^{-1}) = 1 + a_1^*q^{-1} + a_2^*q^{-2} + \cdots + a_{2n-1}^*q^{-2n+1}. \tag{5}$$

This pole-placement controller takes the form

$$C_0(q^{-1})u_t = D_0(q^{-1})y_t \tag{6}$$

where

$$C_0(q^{-1}) = 1 + c_{01}q^{-1} + c_{02}q^{-2} + \cdots + C_{0n-1}q^{-n+1} \tag{7}$$
$$D_0(q^{-1}) = d_{00} + d_{01}q^{-1} + d_{02}q^{-2} + \cdots + d_{0n-1}q^{-n+1}. \tag{8}$$

$C_0(q^{-1})$ and $D_0(q^{-1})$ are determined via the Diophantine equation

$$A_0(q^{-1})C_0(q^{-1}) - b_0q^{-d}D_0(q^{-1}) = A^*(q^{-1}). \tag{9}$$

In this paper we will assume that $A^*(q^{-1})$ can be any fixed polynomial belonging to a given set $\mathcal{A}$. (Every $A^*(q^{-1}) \in \mathcal{A}$ is supposed to be a Schur polynomial.) Typically we will restrict the locations of the zeros of $A^*(q^{-1})$, and this requirement will determine the set $\mathcal{A}$.

D. Problem Formulation

The "belief" of the control designer that the control objective can be achieved by a pole-placement controller actually imposes restrictions on the system (1). Indeed, as we will see, $(A_0(q^{-1}), b_0)$ cannot be arbitrary in view of the proposed control strategy (5)–(9). The prior knowledge implied by this "belief" of the control designer will be made explicit in the sequel.

The problem addressed in this paper can be formulated as follows: We have a set of models (AR-models as given by (4)), a control design method (pole placement with specified closed-loop polynomial) and a control objective (stabilization of the actual plant with a certain robustness margin). The controller designed on the basis of the nominal model (4) is applied to the actual plant as given by (1). We now ask the question: What kind of restrictions must be imposed on the set of models and plants to make the control objective achievable? The first part of this paper deals with this problem. In the second part we propose an adaptive algorithm exploiting our prior knowledge.

In this paper we investigate what type of control problem one can deal with in the adaptive context, especially the indirect philosophy of "identify" then "control" using the certainty equivalence principle [1]. The approach we take is to view the control problem from an identification perspective as introduced in the ideal case by Polderman [7] and continued in [8]. As it turns out, the type of control problem that seemingly can be dealt with is of a larger generality than that which is currently covered by the more classical robust adaptive control literature alluded to below. This raises the question: "Does there exist an adaptive algorithm that can deal with it?" We provide an affirmative answer by constructing an adaptive algorithm, based on previous work by Middleton *et al.* [4].

In classical "robust" adaptive control (e.g., [2], [4], [6], [7], [10], [12]), "robust" refers to the fact that boundedness of all signals is guaranteed in the presence of some unmodeled dynamics, as, e.g., given by (3). The size (ϵ_0, ϵ_1) of the unmodeled dynamics that can be tolerated is determined by the particular control algorithm. It is typically not the philosophy of these papers to guarantee a certain predetermined robustness measure. (Although one can try to unravel the functional dependence of the tolerated ϵ_0, ϵ_1 on the algorithm's parameters.) Here we pose a problem more akin to traditional robust control, but in an adaptive context.

The paper is organized as follows. In Section II we examine AR systems. The adaptive algorithm is presented and analyzed in Section III, and conclusions are given in Section IV.

II. AR System

In this section we examine an AR nominal system. First, using small gain arguments, we analyze the restrictions imposed on the system by the control objective. Then we investigate the achievable performance from an identification perspective. Finally we derive convex subsets of the set of systems which satisfy the control objective.

A. Robustness Considerations

The system is given by (1)–(3). Suppose that the system can be stabilized by (static) state feedback. The pole placement controller is given by

$$C_0(q^{-1})u_t = D_0(q^{-1})y_t \tag{10}$$

where

$$A_0(q^{-1})C_0(q^{-1}) - b_0 q^{-d} D_0(q^{-1}) = A^*(q^{-1}). \tag{11}$$

$A^*(q^{-1})$ is the desired closed-loop polynomial. Assume further that we want every closed-loop polynomial $A^*(q^{-1})$ in a set $\mathcal{A}$ to be achievable. By "achievable" we mean that if we pick an arbitrary $A^*(q^{-1}) \in \mathcal{A}$ the corresponding pole placement controller will stabilize the system. For robustness reasons we have required that all closed-loop polynomials in a set and not only a single closed-loop polynomial should be achievable. This requirement excludes singular cases as $A^*(q^{-1}) = A_0(q^{-1}), b_0 = 0$. In that case a slight change in either $A^*(q^{-1})$ or $A_0(q^{-1})$ would result in infinite input, a highly undesirable feature.

With the control design based on the nominal model (4) the equations for y_t and u_t can be rewritten as

$$A^*(q^{-1})y_t = C_0(q^{-1})\delta_t \tag{12}$$

$$A^*(q^{-1})u_t = D_0(q^{-1})\delta_t. \tag{13}$$

From (12) and (13) we can derive upper bounds for $\|y\|_\infty$ and $\|u\|_\infty$. For the sake of simplicity, we assume that the initial conditions are zero. (12)–(13) give

$$\|y\|_\infty \le \left\| \frac{C_0(q^{-1})}{A^*(q^{-1})} \right\|_1 \|\delta\|_\infty$$

$$\|u\|_\infty \le \left\| \frac{D_0(q^{-1})}{A^*(q^{-1})} \right\|_1 \|\delta\|_\infty$$

$\|\cdot\|_1$ denotes the one-norm of the impulse response. We substitute for δ and obtain

$$\|y\|_\infty \le \left\| \frac{C_0(q^{-1})}{A^*(q^{-1})} \right\|_1 (\epsilon_0 + \epsilon_1(\|y\|_\infty, \|u\|_\infty))$$

$$\|u\|_\infty \le \left\| \frac{D_0(q^{-1})}{A^*(q^{-1})} \right\|_1 (\epsilon_0 + \epsilon_1(\|y\|_\infty, \|u\|_\infty)).$$

After some algebra we find that

$$\|y\|_\infty \le \frac{\left\| \frac{C_0(q^{-1})}{A^*(q^{-1})} \right\|_1}{1 - \epsilon_1\left(\left\| \frac{C_0(q^{-1})}{A^*(q^{-1})} \right\|_1, \left\| \frac{D_0(q^{-1})}{A^*(q^{-1})} \right\|_1\right)} \epsilon_0 \tag{14}$$

$$\|u\|_\infty \le \frac{\left\| \frac{D_0(q^{-1})}{A^*(q^{-1})} \right\|_1}{1 - \epsilon_1\left(\left\| \frac{C_0(q^{-1})}{A^*(q^{-1})} \right\|_1, \left\| \frac{D_0(q^{-1})}{A^*(q^{-1})} \right\|_1\right)} \epsilon_0. \tag{15}$$

From (14) and (15), we find that we must impose bounds on $\epsilon_1(\|(C_0(q^{-1})/A^*(q^{-1}))\|_1, \|(D_0(q^{-1})/A^*(q^{-1}))\|_1)$. A natural robustness requirement is

$$\epsilon_1\left(\left\| \frac{C_0(q^{-1})}{A^*(q^{-1})} \right\|_1, \left\| \frac{D_0(q^{-1})}{A^*(q^{-1})} \right\|_1\right) \le 1 - \epsilon_1' \tag{16}$$

for some $0 < \epsilon_1' \le 1$. By assumption we are given the robustness margin ϵ_1'. It is clear that the possible range of systems $(A_0(q^{-1}), b_0)$ is limited with respect to ϵ_1, ϵ_1' and $\mathcal{A}$.

Incorporating the requirement (16) in (14) and (15) gives

$$(\|y\|_\infty, \|u\|_\infty) \le \frac{1 - \epsilon_1'}{\epsilon_1 \epsilon_1'} \epsilon_0. \tag{17}$$

This limit is solely determined by our prior knowledge and expectations. It also provides bounds on the gain (l_∞ induced norm) of the operators G_y and G_u where

$$y_t = [G_y(\epsilon_1')\epsilon_0]_t \qquad u_t = [G_u(\epsilon_1')\epsilon_0]_t.$$

Due to the dependence of δ_t on old values of y_t and u_t the nominal gains $\|(C_0(q^{-1})/A^*(q^{-1}))\|_1$ and $\|(D_0(q^{-1})/A^*(q^{-1}))\|_1$ increase with a factor $(1/1 - \epsilon_1(\|(C_0(q^{-1})/A^*(q^{-1}))\|_1, \|(D_0(q^{-1})/A^*(q^{-1}))\|_1))$. The robustness margin ensures that this factor is less than $1/\epsilon_1'$, and hence it also ensures closed-loop stability. Denote the set of allowable models by $P(\epsilon_1, \mathcal{A}, \epsilon_1')$. It is characterized by the control objective

$$P(\epsilon_1, \mathcal{A}, \epsilon_1') = \Bigg\{(A_0(q^{-1}), b_0) | \epsilon_1\left(\left\| \frac{C_0(q^{-1})}{A^*(q^{-1})} \right\|_1, \left\| \frac{D_0(q^{-1})}{A^*(q^{-1})} \right\|_1\right) \le 1 - \epsilon_1' \quad \forall A^*(q^{-1}) \in \mathcal{A}, \; A_0(q^{-1})C_0(q^{-1}) - b_0 q^{-d} D_0(q^{-1}) = A^*(q^{-1}) \Bigg\} \tag{18}$$

This set has nothing to do with adaptation in the sense that even if $(A_0(q^{-1}), b_0)$ is exactly known, they have to belong to $P(\epsilon_1, \mathcal{A}, \epsilon_1')$ to make the control objective achievable. Hence membership of $P(\epsilon_1, \mathcal{A}, \epsilon_1')$ is a minimum requirement to achieve the control objective in an adaptive context.

Example—First Order System: A first-order system is given by

$$y_t + a_0 y_{t-1} = b_0 u_{t-1} + \delta_t.$$

The pole-placement controller takes the form

$$u_t = \frac{a_0 - a^*}{b_0} y_t$$

where $-a^*$ is the closed-loop pole. Assume that we want every a^* in an interval (a_1^*, a_2^*) to be achievable, i.e., $\mathcal{A} = \{1 + a^* q^{-1} | a^* \in (a_1^*, a_2^*)\}$. The set P is now given by

$$P(\epsilon_1, \mathcal{A}, \epsilon_1') = \left\{(1 + a_0 q^{-1}, b_0) | \epsilon_1 \left(\frac{1}{1 - |a^*|}, \frac{|a_0 - a^*|}{|b_0|(1 - |a^*|)}\right) \le 1 - \epsilon_1' \forall a^* \in (a_1^*, a_2^*)\right\}. \quad (19)$$

Notice that P is a union of two disjoint convex sets and in particular that b_0 has to be bounded away from zero since the inequality in (19) must be valid for all $a^* \in (a_1^*, a_2^*)$. ■

B. Unfalsified Models

The models compatible with our prior knowledge can be written as

$$A(q^{-1}) y_t = b u_{t-d} (A(q^{-1}), b) \in P(\epsilon_1, \mathcal{A}, \epsilon_1'). \quad (20)$$

We use the notation $(A(q^{-1}), b)$ to distinguish an arbitrary model from the "true" model $(A_0(q^{-1}), b_0)$. Similarly for the controller where we use $C(q^{-1})$ and $D(q^{-1})$ instead of $C_0(q^{-1})$ and $D_0(q^{-1})$. As we observe the input and output data some of these models (20), become falsified. We say that a model is falsified if the prediction error $\epsilon_t = A(q^{-1}) y_t - b u_{t-d}$ is greater than the largest possible disturbance δ_t, i.e., a model is falsified if

$$|\epsilon_t| = |A(q^{-1}) y_t - b u_{t-d}| > \epsilon_0 + \epsilon_1 \sup_{t-k<l<t} (|y_l|, |u_l|)$$

for some t.

This condition implies that b and the coefficients of $A(q^{-1})$ cannot be the true parameters. From a pure open-loop identification point of view, we introduce the set of unfalsified models, compatible with a single realization of the data and our prior knowledge

$$F_{y_{\text{init}}, \{\epsilon_{0,t}, \epsilon_{1,t}\}, \{u_t\}} = \Big\{(A(q^{-1}, b) | (A(q^{-1}), b) \in P(\epsilon_1, \mathcal{A}, \epsilon_1'), \; |A(q^{-1}) y_t - b u_{t-d}| \le \epsilon_0 + \epsilon_1 \sup_{t-k<l<t} (|y_l|, |u_l|) \forall t \ge 1\Big\}. \quad (21)$$

$\{\epsilon_{0,t}, \epsilon_{1,t}\}$ is a sequence of disturbance gains defining a disturbance sequence which satisfies the presumed constraint, y_{init} are the initial conditions of the system, and $\{u_t\}$ is the applied input sequence. As we are only observing a single realization, it is obvious that our ability to falsify a model will depend on the initial conditions, the actual disturbance sequence and the applied input, hence $F_{y_{\text{init}}}$, $\{\epsilon_{0,t}, \epsilon_{1,t}\}$, $\{u_t\}$. If $(A(q^{-1}), b) \in F_{y_{\text{init}}}$, $\{\epsilon_{0,t}, \epsilon_{1,t}\}$, $\{u_t\}$ then the observed data do not negate the possibility that $(A(q^{-1}), b)$ is the true model.

Our input signal u_t is not, however, completely free, but generated by the feedback law $C(q^{-1}) u_t = D(q^{-1}) y_t$ where $A(q^{-1}) C(q^{-1}) b q^{-d} D(q^{-1}) = A^*(q^{-1})$. The models must therefore be considered from a closed-loop identification point of view. We, therefore, consider the set of closed-loop regulated unfalsified models

$$\begin{aligned} &G_{y_{\text{init}}, \{\epsilon_{0,t}, \epsilon_{1,t}\}, A^*(q^{-1})} \\ &= \{(A(q^{-1}), b) | (A(q^{-1}), b) \in P(\epsilon_1, \mathcal{A}, \epsilon_1') \\ &\quad |A(q^{-1}) y_t - b u_{t-d}| \le \epsilon_0 + \epsilon_1 \sup_{t-k<l<t} (|y_l|, |u_l|) \\ &\quad C(q^{-1}) u_t = D(q^{-1}) y_t \\ &\quad A(q^{-1}) C(q^{-1}) - b q^{-d} D(q^{-1}) = A^*(q^{-1})\} \end{aligned} \quad (22)$$

$A^*(q^{-1}) \in \mathcal{A}$ fixed.

Interpretation: We are given a single realization of the disturbance gains, the initial conditions, and the desired closed-loop polynomial. The set $G_{y_{\text{init}}}$, $\{\epsilon_{0,t}, \epsilon_{1,t}\}$, $A^*(q^{-1})$ contains those models $(A(q^{-1}), b)$ that will not be falsified if we apply the input sequence $C(q^{-1}) u_t = D(q^{-1}) y_t$. Hence in this case the input sequence is not free, but dependent on the particular model under consideration. The sets $F_{y_{\text{init}}}$, $\{\epsilon_{0,t}, \epsilon_{1,t}\}$, $\{u_t\}$ and $G_{y_{\text{init}}}$, $\{\epsilon_{0,t}, \epsilon_{1,t}\}$, $A^*(q^{-1})$ signify the important difference between open-loop and closed-loop identification. ■

G is important in understanding the asymptotic behavior of the adaptive idea. If on G the behavior is not acceptable, adaptation has no hope of being successful. For any model in $G_{y_{\text{init}}}$, $\{\epsilon_{0,t}, \epsilon_{1,t}\}$, $A^*(q^{-1})$ we have that

$$A^*(q^{-1}) y_t = C(q^{-1}) \epsilon_t$$

where ϵ_t is the prediction error $A(q^{-1}) y_t - b u_{t-d} = \epsilon_t$, and it satisfies the assumed noise constraint. Hence the system behaves as if it were regulated with correct pole placement and driven by a noise sequence (the prediction error) satisfying the *a priori* assumptions.

Remark 1: This is an important observation since it implies that we have a model at hand that fits the data and the uncertainty, and from an identification point of view we cannot do better. Hence, for every $(A(q^{-1}), b)$ in $G_{y_{\text{init}}}$, $\{\epsilon_{0,t}, \epsilon_{1,t}\}$, $A^*(q^{-1})$ the system behaves as if it were regulated with correct pole placement. It is not necessary to identify the true parameters to achieve this behavior since G (22) will, in general, contain more than just the true model. ■

Remark 2: Here we have used the term unfalsified models. This does not mean that the model is unfalsifiable. If we carry out enough experiments, we will most probably find out that the model is incorrect, and hence we have falsified the model. In our case we have restricted our experiments to closed-loop identification, and this is the reason why not all models which, in principle, are falsifiable will be falsified. ■

Remark 3: The analysis conducted here is in the field of set membership identification coupled with robust control (see, e.g., [5] or [11]). In the adaptive algorithm proposed in Section III we use a more classical identification algorithm. ■

C. Convex Sets of Allowable Models

Since the robustness margin is given by (16), we are interested in the set of $(A(q^{-1}), b)$ which gives bounded $C(q^{-1})$ and $D(q^{-1})$ as a function of the (unknown) parameters. When analyzing the adaptive algorithm, we will have to modify the robustness measure slightly, but the boundedness of $C(q^{-1})$ and $D(q^{-1})$ remains crucial.

We have shown that for a first-order system $P(\epsilon_1, \mathcal{A}, \epsilon_1')$ consists of two disjoint convex sets, but this does not generalize to higher order systems. Neither does the set

$$\{(A(q^{-1}), b) \mid \|C(q^{-1})\|_1 < K_1, \|D(q^{-1})\|_1 < K_2, A(q^{-1})C(q^{-1}) - bq^{-d}D(q^{-1}) = A^*(q^{-1})\}$$

consist of convex subsets. The reason why convex sets of parameters are desirable is that during the adaptation of the parameters, the estimate might be outside the allowable region. If we project the estimate onto a convex set containing the true parameters, we are guaranteed that the projected estimate is closer to the true parameters than the original estimate.

We will now try to find some convex sets, S_i, such that if $(A(q^{-1}), b) \in S_i$, then the norms of $C(q^{-1})$ and $D(q^{-1})$ are bounded. We start with deriving some sufficient conditions on $(A(q^{-1}), b)$ which guarantee boundedness of $C(q^{-1})$ and $D(q^{-1})$. $C(q^{-1})$ and $D(q^{-1})$ are solutions of the Diophantine equation

$$A(q^{-1})C(q^{-1}) - bq^{-d}D(q^{-1}) = A^*(q^{-1}). \tag{23}$$

The equations for the $d-1$ first coefficients in (23) are

$$\begin{bmatrix} 1 & & & & \\ a_1 & 1 & & & \\ a_2 & a_1 & 1 & & \\ \vdots & & & \ddots & \\ a_{d-1} & a_{d-2} & \cdots & a_1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ c_1 \\ c_2 \\ \vdots \\ c_{d-1} \end{bmatrix} = \begin{bmatrix} 1 \\ a_1^* \\ a_2^* \\ \vdots \\ a_{d-1}^* \end{bmatrix}. \tag{24}$$

These equations determine $c_1, \cdots, c_{d-1}$ uniquely, and it follows from (24) that a sufficient condition for boundedness of $c_1, \cdots, c_{d-1}$ is boundedness of $a_1, \cdots, a_{d-1}$.

The equations for the last $n-d$ coefficients are given by

$$\begin{bmatrix} a_n & \cdots & a_{d+2} & a_{d+1} \\ & \ddots & & \vdots \\ & & a_n & a_{n-1} \\ & & & a_n \end{bmatrix} \begin{bmatrix} c_d \\ \vdots \\ c_{n-2} \\ c_{n-1} \end{bmatrix} = \begin{bmatrix} a_{n+d}^* \\ \vdots \\ a_{2n-2}^* \\ a_{2n-1}^* \end{bmatrix}. \tag{25}$$

These equations uniquely determine the coefficients $c_d, \cdots, c_{n-1}$, provided a_n is bounded away from 0 and $a_i/a_n, i = d+1, \cdots, n-1$ are bounded, $c_d, \cdots, c_{n-1}$ are bounded. This requirement is obtained by requiring all elements of the inverse of the matrix in (25) to be bounded.

The $C(q^{-1})$ polynomial is uniquely specified by the above equations.

The $D(q^{-1})$ polynomial is given by the remaining equations which can be written as

$$\begin{bmatrix} a_d & \cdots & 1 & 0 & \cdots & 0 \\ a_{d+1} & \cdots & a_1 & 1 & \cdots & 0 \\ \vdots & & & & \ddots & \\ a_n & \cdots & & & \cdots & 1 \\ & \ddots & & & & \\ & & a_n & a_{n-1} & \cdots & a_d \end{bmatrix} \begin{bmatrix} 1 \\ c_0 \\ \vdots \\ c_{n-1} \end{bmatrix} - b \begin{bmatrix} d_0 \\ \vdots \\ d_{n-1} \end{bmatrix} = \begin{bmatrix} a_d^* \\ a_{d+1}^* \\ \vdots \\ a_{n+d-1}^* \end{bmatrix}. \tag{26}$$

It follows that if b is bounded away from 0 and $a_i/b, i = d, \cdots, n$ are bounded, then the coefficients $d_0, \cdots, d_{n-1}$ are bounded. This can be seen by dividing the above equations by b.

To summarize the sufficient conditions, we have the following:

C1) b bounded away from zero, i.e., $|b| \geq K_1$.
C2) $a_i, i = 1, \cdots, d-1$ bounded, i.e., $|a_i| \leq K_2$.
C3) $a_i/b, i = d, \cdots, n$ bounded, i.e., $|a_i/b| \leq K_3$.
C4) a_n bounded away from zero, i.e., $|a_n| \geq K_4$.
C5) $a_i/a_n, i = d, \cdots, n-1$ bounded, i.e., $|a_i/a_n| \leq K_5$.

The $C(q^{-1})$ and $D(q^{-1})$ polynomials are functions of $(A(q^{-1}), b)$, and the above conditions on $(A(q^{-1}), b)$ imply that every coefficient of the $C(q^{-1})$ and the $D(q^{-1})$ polynomials is bounded, and hence the one-norm is bounded.

Notice that if we require $A^*(q^{-1})$ to be of order $n+d-1$ or lower, the two last conditions are superfluous. This follows from (25) since then the right-hand side is zero, and it follows that $c_i = 0, i = d, \cdots, n-1$. Similarly if $d = n$, then the conditions C4) and C5) do not come into play.

The set of $(A(q^{-1}), b)$ which satisfy the conditions C1)–C5) above consists of four unbounded disjoint convex subsets (two disjoint convex subset if $A^*(q^{-1})$ is of order $n+d-1$ or lower). There is one subset for each combination of signs of b and a_n, i.e., in each of the convex regions b and a_n do not change sign. These convex sets are not, however, necessarily a good approximation of the set of all $(A(q^{-1}), b)$ that give bounded $C(q^{-1})$ and $D(q^{-1})$.

Remark: There are many conditions on $a_i, i = 1, \cdots, n$ and b that guarantee boundedness of $C(q^{-1})$ and $D(q^{-1})$, and the above conditions are only one choice. Moreover, for a given bound on $(\|(C(q^{-1})/A^*(q^{-1}))\|_1, \|(D(q^{-1})/A^*(q^{-1}))\|_1)$ there will be many different combinations of $K_1, \cdots, K_5$ that guarantee this bound. ■

III. The Adaptive Algorithm

In this section we present and analyze an adaptive control algorithm which allows the true system parameters to belong to a set of unbounded convex sets. First we present the identification scheme and the control algorithm. Thereafter we show that all signals in the control loop remain bounded.

We have to introduce the additional requirement that the true system $(A_0(q^{-1}), b_0)$ belongs to one of the convex regions described by C1)–C5). This is a sufficient, but not necessary, condition for the existence of a model and a pole-placement controller such that the control objective is achievable. The main idea of the adaptive algorithm is to run a parameter estimator in each of the convex regions. We also compute a performance index for each parameter estimator, and the controller parameters are computed on the basis of the estimate from the "best" estimator. The performance index consists of the sum of the squares of the updates and a term that guarantees boundedness of the estimate. This is the same approach as used in [4], but we now allow the convex regions to be unbounded.

A. The Parameter Estimator

Since the upper bound of the disturbance is known, although time varying and dependent on past values of u and y, we will use a projection algorithm with dead zone as described in [3]. If the estimate is outside the convex region, it will be projected into it.

The system $A_0(q^{-1})y_t = b_0 u_{t-d} + \delta_t$ can be rewritten as

$$y_t = \phi_{t-1}^T \theta_0 + \delta_t$$

where

$$\phi_{t-1} = [y_{t-1}, \cdots, y_{t-n}, u_{t-d}]^T$$
$$\theta_0 = [-a_{01}, \cdots, -a_{0n}, b_0]^T.$$

The parameter estimation algorithm is given by

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \frac{\phi_{t-1}}{\|\phi_{t-1}\|_2^2} f(\Delta_t, e_t) \quad \text{for} \|\phi_{t-1}\|_2 \neq 0 \tag{27}$$

$$\hat{\theta}_t = \hat{\theta}_{t-1} \quad \text{for} \|\phi_{t-1}\|_2 = 0 \tag{28}$$

where $\|\cdot\|_2$ denotes the Euclidian norm and

$$f(\Delta_t, e_t) = \begin{cases} e_t - \Delta_t & \text{if } e_t > \Delta_t \\ 0 & \text{if } |e_t| \leq \Delta_t \\ e_t + \Delta_t & \text{if } e_t < -\Delta_t \end{cases} \tag{29}$$

where

$$\Delta_t = \epsilon_0 + \epsilon_1 \sup_{t-k<l<t} (|y_l|, |u_l|)$$

e_t is the prediction error

$$e_t = y_t - \phi_{t-1}^T \hat{\theta}_{t-1} = \phi_{t-1}^T \tilde{\theta}_{t-1} + \delta_t, \qquad \tilde{\theta}_t = \theta_0 - \hat{\theta}_t.$$

The parameter estimator has the following properties

P1) $\|\tilde{\theta}_t\|_2 \leq \|\tilde{\theta}_{t-1}\|_2$

P2) $(f(\Delta_t, e_t)/\|\phi_t\|_2) \in l_2$

and the last property implies that

P3) $\lim_{t\to\infty} (f(\Delta_t, e_t)/\|\phi_t\|_2) = 0$

P4) $\lim_{t\to\infty} \|\hat{\theta}_t - \hat{\theta}_{t-1}\|_2 = 0$.

Proof: See [3]. ■

Every new measurement defines a region in parameter space to which the true parameters, θ_0, belong. The region is given by

$$y_t - \Delta_t \leq \phi_{t-1}^T \theta \leq y_t + \Delta_t. \tag{30}$$

The estimation algorithm (27) projects the estimate $\hat{\theta}$ onto the edge of the constrained set (30). This does not mean that the new projected $\hat{\theta}$ is indeed in the convex region, and a further projection may be required. Since (30) describes a convex set, the intersection of (30) and the allowable convex regions are convex sets, and we will project the estimates onto these sets. By assumptions there exists a region containing the true parameters, and for this region the projection will bring us closer to the true values, since we project onto a convex set containing the true parameters. For the other regions we can in general not say whether this projection will bring us closer or further away from the true parameters. Notice that the intersection might be empty, but then we know that the true parameters do not belong to that allowable region.

Let us denote the difference between the estimate given by the projection algorithm (27) and the intersected set by g. Then the estimation algorithm can be rewritten as

$$\hat{\theta}_t = \hat{\theta}_{t-1} + \frac{\phi_{t-1}}{\|\phi_{t-1}\|_2^2} f(\Delta_t, e_t) + g_{t-1} \quad \text{for } \|\phi_{t-1}\|_2 \neq 0 \tag{31}$$

$$\hat{\theta}_t = \hat{\theta}_{t-1} \quad \text{for } \|\phi_{t-1}\|_2 = 0. \tag{32}$$

If the intersection of the allowable region and (30) is empty then $\|g_{t-1}\|_2 = \infty$. Since we use the projection algorithm (27) and because the allowable regions are convex, $\phi_{t-1}^T g_{t-1} = 0$.

The estimates in the allowable region containing the true parameters satisfy

$$\|\tilde{\theta}_t\|_2^2 \leq \|\tilde{\theta}_{t-1}\|_2^2 - \frac{1}{\|\phi_{t-1}\|_2^2} f^2(\Delta_t, e_t) - \|g_{t-1}\|_2^2 \tag{33}$$

such that for this region both $(f(\Delta_t, e_t)/\|\phi_{t-1}\|_2)$ and $\|g_t\|_2$ are in l_2.

To decide which region contains the best estimate, we introduce a performance index and a criterion for selecting the region. The performance index is a measure of the goodness of the estimate in a region. One performance index is the sum of the squares of the update distances, i.e., for region i we have

$$p'_{i,t} = p'_{i,t-1} + \frac{f^2(\Delta_t, e_{i,t})}{\|\phi_{i,t}\|_2^2} + \|g_{i,t}\|_2^2. \tag{34}$$

A finite performance index $p'_{i,t}$ does not, however, guarantee a bounded estimate. Hence we suggest the following modification of the performance index

$$p_{i,t} = p'_{i,t} + \max_{1\leq k\leq t} \|\hat{\theta}_{i,k}\|_2^2. \tag{35}$$

This performance index is monotonically increasing, and it is at least finite for the region containing the true parameters. Let $r(t)$ denote the region selected at time t. If $p_{r(t),t} < p_{i,t} + \gamma, \gamma > 0, i \neq r(t)$, then $r(t)$ is unaltered. Otherwise $r(t)$ =arg $\min_i (p_{i,t})$. The constant γ is introduced to prevent infinite switching between regions. The proof of the next lemma can be found in [4].

Lemma 3.1: There exists a $t_0 > 0$ such that $r(t) = r(t_0)$ for all $t \geq t_0$. □

B. Boundedness of the Signals

The pole-placement controller is given by

$$C_t(q^{-1})u_t = D_t(q^{-1})y_t \tag{36}$$

where $C_t(q^{-1})$ and $D_t(q^{-1})$ is computed according to

$$A_t(q^{-1})C_t(q^{-1}) - q^{-d}b_t D_t(q^{-1}) = A^*(q^{-1}). \tag{37}$$

$A_t(q^{-1})$ and b_t are the current estimate of $A_0(q^{-1})$ and b_0. By Lemma 3.1 and property P4) of the parameter estimator there exists a t_0 such that the estimate is bounded and in the same convex region for all $t \geq t_0$, and $\|\hat{\theta}_t - \hat{\theta}_{t-1}\|_2 < \epsilon_2$ for all $t \geq t_0$.

We rewrite the system and the controller as

$$A^*(q^{-1})\begin{bmatrix} y_t \\ u_t \end{bmatrix} = \begin{bmatrix} C_{t-1}(q^{-1}) \\ D_{t-1}(q^{-1}) \end{bmatrix} e_t + s'_t \tag{38}$$

where s'_t is the error that occur due to the fact that $A_t(q^{-1}), b_t, C_t(q^{-1})$ and $D_t(q^{-1})$ are time varying. It is bounded by

$$s'_t \leq K \max_{1\leq k<n} \|\theta_t - \theta_{t-k}\|_2 \max_{t-2n+1\leq l<t} (|u_l|, |y_l|). \tag{39}$$

Now redefine the time origin to t_0, and introduce the state-vector

$$x_t = [y_t, \cdots, y_{t-2n+2}, u_t, \cdots, u_{t-2n+2}]^T$$

and the system matrix

$$\overline{A}^* = \begin{bmatrix} \overline{A}^{*'} & 0 \\ 0 & \overline{A}^{*'} \end{bmatrix}$$

where

$$\overline{A}^{*'} = \begin{bmatrix} -a_1^* & -a_2^* & \cdots & -a_{2n-2}^* & -a_{2n-1}^* \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & & & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & & & 1 & 0 \end{bmatrix}$$

and the vector

$$F_t(q^{-1}) = [C_t(q^{-1}), 0, \cdots, 0, D_t(q^{-1}), 0, \cdots, 0]^T.$$

Then rewrite the system and the controller as

$$x_{t+1} = \overline{A}^* x_t + F_t(q^{-1})e_{t+1} + s_{t+1} \qquad (40)$$

where s_{t+1} takes care of the swapping error. This gives

$$x_t = \overline{A}^{*t} x_0 + \sum_{k=1}^{t} \overline{A}^{*t-k}(F_{k-1}(q^{-1})e_k + s_k) \qquad (41)$$

Since A^* is exponentially stable we have

$$\|x_t\|_2 \le k_0 + \sum_{k=1}^{t} k_1 \lambda^{t-k} \left\|F_k(q^{-1})e_k + s_k\right\|_2 \qquad (42)$$

λ and k_1 are known since $A^*(q^{-1})$ is known. e_k is split in two terms, f_k and $e_k - f_k$. The term $e_k - f_k$ is less than $\epsilon_0 + \epsilon_1 \sup_{\tau<k} \|x(\tau)\|_2$, and $f_k/\|\phi_k\|_2 \in l_2$. This gives

$$\|x_t\|_2 \le k_0 + \sum_{k=1}^{t} k_1 \lambda^{t-k} \left\|F_{k-1}(q^{-1})(e_k - f_k)\right\|_2 + \sum_{k=1}^{t} k_1 \lambda^{t-k} \left\|F_{k-1}(q^{-1})f_k\right\|_2 + \sum_{k=1}^{t} k_1 \lambda^{t-k} \|s_k\|_2. \qquad (43)$$

Denote the upper bound of the one-norm of the $C(q^{-1})-$ and $D(q^{-1})-$polynomial by k_2' and let $k_2 = \sqrt{2}k_2'$. This bound is known since the system belongs to an allowable convex region. Provided $(k_1k_2/1 - \lambda)\epsilon_1$ is strictly less than one boundedness follows along the same lines as in [4].

Bounds similar to (14) and (15) in Section II are obtained in the limit as t tends to ∞. We use (38) as a starting point

$$y_t = \frac{1}{A^*(q^{-1})}\begin{bmatrix} C_t(q^{-1}) \\ D_t(q^{-1}) \end{bmatrix} e_t + \frac{1}{A^*(q^{-1})} s_t'. \qquad (44)$$

By splitting e_k in $e_k - f_k$ and f_k as before and utilizing the facts that $A^*(q^{-1})$ is stable and x_t is bounded (and hence f_k is in l_2) we can show that

$$\limsup_{t\to\infty} |y_t| \le \frac{k_C(\epsilon_1')}{1 - \epsilon_1(k_C(\epsilon_1'), k_D(\epsilon_1'))}\epsilon_0$$

$$\limsup_{t\to\infty} |u_t| \le \frac{k_D(\epsilon_1')}{1 - \epsilon_1(k_C(\epsilon_1'), k_D(\epsilon_1'))}\epsilon_0$$

where $k_C(\epsilon_1')$ and $k_D(\epsilon_1')$ are upper bounds on $\|(C_t(q^{-1})/A^*(q^{-1}))\|_1$ and $\|(D_t(q^{-1})/A^*(q^{-1}))\|_1$ valid for parameter values belonging to the allowable convex regions.

We have the following theorem

Theorem 3.2: Assume that the system is given by (1)–(3) and that the controller is given by (36)–(37). Assume further that the allowable (possibly unbounded) convex regions satisfying condition C1)–C5) in Section II-C are such that $(k_1k_2/1 - \lambda)\epsilon_1 < 1 - \epsilon_1'$ and that the nominal system parameters belong to one of the allowable convex regions. Then the system is stable in the sense that y_t and u_t are bounded functions.

The following limit results are valid

$$\limsup_{t\to\infty} |y_t| \le \frac{k_C(\epsilon_1')}{1 - \epsilon_1(kDC(\epsilon_1'), k_D(\epsilon_1'))}\epsilon_0$$

$$\limsup_{t\to\infty} |u_t| \le \frac{k_D(\epsilon_1')}{1 - \epsilon_1(k_C(\epsilon_1'), k_D(\epsilon_1'))}\epsilon_0$$

where $k_C(\epsilon_1')$ and $k_D(\epsilon_1')$ are upper bounds for $\|(C_t(q^{-1})/A^*(q^{-1}))\|_1$ and $\|(D_t(q^{-1})/A^*(q^{-1}))\|_1$. ∎

Remark 1: The quantity $(k_1k_2/1 - \lambda)\epsilon_1$ might be interpreted as a robustness measure similar to (16) in Section II, and we can choose the convex sets such that $(k_1k_2/1 - \lambda)\epsilon_1 < 1 - \epsilon_1'$ where ϵ_1' is a given robustness margin. ∎

Remark 2: Two other noise descriptions are

$$|\delta_t| \le \epsilon_0 + \epsilon_1 \sup_{l<t}(|y_{t-k-1}|, |u_{t-k-1}|) \qquad t \ge 0 \qquad (45)$$

and

$$|\delta_t| \le \epsilon_0 + \epsilon_1 \sum_{k=0}^{\infty} \beta^k(|y_{t-k-1}|, |u_{t-k-1}|), \qquad 0 < \beta < 1. \qquad (46)$$

Boundedness of the signals follows as before, except for that description (46) the robustness criterion is now of the form

$$\frac{1}{1-\beta}\frac{k_1k_2}{1-\lambda}\epsilon_1 < 1 - \epsilon_1'.$$

For description (46) we have the following limit result

$$\limsup_{t\to\infty} |y_t| \le \frac{k_C(\epsilon_1')}{1 - \frac{\epsilon_1}{1-\beta}(k_C(\epsilon_1'), k_D(\epsilon_1'))}\epsilon_0.$$

For description (45) we have no similar limit result. The reason is that our initial model will most likely be falsified by later data, hence we have to change our parameters. While noise descriptions (3) and (46) forget what has happened during the initial phases, noise description (45) never does, and large values of $|y_t|$ and $|u_t|$ encountered during the initial phases (maybe as a consequence of inappropriate control action) will forever affect the possible size of the disturbance. ∎

Remark 3: A more relevant and harder problem is to consider only $(n, d, \epsilon_0, \epsilon_1)$ as prior information and search for $(A_0(q^{-1}), b_0q^{-d})$ as well as an appropriate $A^*(q^{-1})$. A possible solution is to cover the parameter space by convex sets, and to each set we choose (if possible) for $A^*(q^{-1})$ which is compatible with our control objective as specified by $(n, d, \epsilon_0, \epsilon_1)$ and the robustness margin ϵ_1'. The adaptive control algorithm is then applied to those convex subsets for which an $A^*(q^{-1})$ compatible with our control objective exists, running a parameter estimator in each one of them. The controller is computed with the estimate from the best estimator and the $A^*(q^{-1})$ corresponding to this estimate.

From Lemma 3.1 it follows that there exists a t_0 such that for all $t > t_0$, the convex region with the best parameter estimate do not change. This implies that $A^*(q^{-1})$ is fixed for all $t > t_0$, and the boundedness results obtained in this section are still valid. ∎

IV. Conclusion

In this paper we have considered the problem of robustly and adaptively stabilizing a linear time invariant, discrete time, SISO, finite dimensional system by pole placement control.

We have considered the problem from an identification point of view and shown that the control objective imposes restrictions on the system. $(A_0(q^{-1}), b_0)$ cannot be arbitrary in view of the proposed control strategy. The restrictions on the system are dependent on the magnitude of the unmodelled dynamics, ϵ_1, the robustness margin, ϵ_1', and the desired closed-loop polynomial $A^*(q^{-1})$.

Under the assumptions that the system satisfies these imposed restrictions and that the controller is computed on the basis of a fixed unfalsified model, we have derived upper bounds on the $\infty-$norm of the signals in the control loop.

Finally, we have proposed an adaptive algorithm which deals with parameters belonging to unbounded convex sets and boundedness of all signals in the control loop is proven. Further we have found unbounded convex sets of systems which satisfy the aforesaid restrictions.

ACKNOWLEDGMENT

The authors would like to thank the anonymous reviewers for helping in improving the paper.

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A Note on Lyapunov Stability for an Adaptive Robot Controller

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***Abstract*—Stability in the sense of Lyapunov for the adaptive robot controller proposed by Slotine and Li is proved in this note. The result is a generalization of previous work, where the feedback gain matrix was assumed to be constant and diagonal, while in this paper the feedback gain matrix is only assumed to be uniformly positive definite.**

I. INTRODUCTION

In [1] an adaptive robot controller was proposed, and boundedness of signals and convergence of position and velocity errors to zero were shown. Lyapunov stability, however, was not established. In [2] the adaptive robot controller of [1] was proved to be stable in the sense of Lyapunov, but in the proof the feedback gain matrix was assumed to be constant and diagonal, whereas in [1] the feedback gain matrix was only assumed to be uniformly positive definite.

In this paper we generalize the results of [2] by proving Lyapunov stability for the controller in [1] under the assumption that the feedback gain matrix is uniformly positive definite and possibly time varying. To this end the passivity properties of the system [3] are used to construct a Lyapunov function from the dissipation inequalities of the appropriate passive mappings.

In the following, the definitions of passivity and strict passivity and the notation concerning inner products and norms on the function space L^2_{ne} are taken from [4].

II. SYSTEM DESCRIPTION

The closed loop dynamic system studied in [1] and [2] is given by

$$\dot{\tilde{\boldsymbol{a}}} = -\boldsymbol{\Gamma}\boldsymbol{Y}^T\boldsymbol{s} \tag{1}$$

$$\dot{\tilde{\boldsymbol{q}}} = \boldsymbol{s} - \boldsymbol{\Lambda}\tilde{\boldsymbol{q}} \tag{2}$$

$$\boldsymbol{H}\dot{\boldsymbol{s}} + (\boldsymbol{K}_D + \boldsymbol{C})\boldsymbol{s} = \boldsymbol{Y}\tilde{\boldsymbol{a}} \tag{3}$$

where the notation is as defined in [1]. Here $\tilde{\boldsymbol{q}} = \boldsymbol{q} - \boldsymbol{q}_d$ is the position error vector and $\tilde{\boldsymbol{a}} = \hat{\boldsymbol{a}} - \boldsymbol{a}$ is the parameter error vector. The matrix $\boldsymbol{K}_D$ is the feedback gain matrix, and $\boldsymbol{Y} = \boldsymbol{Y}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{q}_d, \dot{\boldsymbol{q}}_d, \ddot{\boldsymbol{q}}_d)$ is the regressor. The matrix $\boldsymbol{H} = \boldsymbol{H}(\boldsymbol{q})$ is the uniformly positive definite inertia matrix of the manipulator, while $\boldsymbol{C} = \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is a matrix defined by the Coriolis and centrifugal terms which appear in the dynamic model

$$\boldsymbol{H}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{g} = \boldsymbol{\tau}. \tag{4}$$

It is assumed that the matrix $\dot{\boldsymbol{H}} - 2\boldsymbol{C}$ is skew symmetric. $\boldsymbol{g} = \boldsymbol{g}(\boldsymbol{q})$ is the vector of gravity terms, and $\boldsymbol{\tau}$ is the vector of input generalized forces. In [2] the matrices $\boldsymbol{\Gamma}$, $\boldsymbol{\Lambda}$ and $\boldsymbol{K}_D$ were assumed to be constant, diagonal and positive definite. In this note we assume that $\boldsymbol{\Gamma}$, $\boldsymbol{\Lambda}$ are constant, symmetric, and positive definite, while the feedback gain

Manuscript received January 21, 1993; revised September 15, 1993.
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IEEE Log Number 9402700.