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# Limitations of robust adaptive pole placement control for first order systems

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## Abstract

In this paper we investigate the limitations of pole placement control when there is an additive disturbance whose size depends on the previous input and output signals. It is shown that the control objective of robust stabilisation restricts the set of systems we can deal with. We subsequently propose and analyse an adaptive algorithm applicable to systems in this set.

**Keywords:** Pole placement control, Identification, Limitations on system parameters, Adaptive control.

## 1 Introduction

We consider the problem of robustly and adaptively stabilising a linear, first order, time-invariant, discrete time, SISO, finite dimensional system. The system is represented by

$$y_t = a_0 y_{t-1} + b_0 u_{t-1} + \delta_t \quad ; \quad t = 1, 2, \ldots \qquad (1)$$

where $u_t$ is the input, $y_t$ is the measured output. Here $\delta_t$ represents the deviation from nominal behaviour which is bounded by

$$|\delta_t| \leq \epsilon_0 + \epsilon_1 \sup_{t-k<l<t} (|u_l|, |y_l|) \qquad (2)$$

where $(|u_l|, |y_l|)$ denotes the maximum of $|u_l|$ and $|y_l|$.

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By robust stabilisation we mean that a feedback controller designed for the "nominal" system

$$y_t = a_0 y_{t-1} + b_0 u_{t-1} \qquad (3)$$

also stabilises the system (1), meaning that all signals in the controlled system remain bounded and of the order $O(\epsilon_0)$ regardless of the realization of the disturbance (2). By adaptive stabilisation we refer to the fact that the actual parameters, *i.e.* $a_0$ and $b_0$, are not known.

Robust adaptive stabilisation requires recovery of robust stability (as described above) in the limit as time tends to infinity, on the basis of input/output measurement and our prior information.

Our prior knowledge includes the parameters $\epsilon_0$ and $\epsilon_1$ of the deviation descriptions. As far as the analysis is concerned, the size of the disturbance window $k-1$, is irrelevant as long as it is finite.

Furthermore, we need a measure of the controlled system's robustness. A realization of $\delta_t$ can be written as

$$\delta_t = \epsilon_{0t} + \epsilon_{1t} \sup_{t-k<l<t} (|u_l|, |y_l|)$$

where $-\epsilon_0 \leq \epsilon_{0t} \leq \epsilon_0$, and $-\epsilon_1 \leq \epsilon_{1t} \leq \epsilon_1$. In closed loop, $y_t$ and $u_t$ can be written as $y_t = G_y \epsilon_{0t}$ and $u_t = G_u \epsilon_{0t}$. The robustness measure will be linked to the gain (the $l_\infty$ induced operator norm) of the operators $G_u$ and $G_y$. In addition to our prior knowledge concerning the deviation, we also assume that we are given a robustness margin $\epsilon_1'$. This point will be clarified in section 2.

Assume that the control designer believes that control objective of robust and adaptive stabilisation can be achieved using a pole placement controller. In particular, the designer thinks that the closed loop pole may be placed at $\sigma$. Such a pole placement controller takes the form

$$u_t = \frac{\sigma - a_0}{b_0} y_t \qquad (4)$$

In this paper we will assume that $\sigma$ can be any fixed pole belonging to a given interval $(\sigma_1, \sigma_2) \subset (-1, 1)$.

The first part of this paper deals with the following problem: We have a set of models given by (3), a controller design method (pole placement with given closed loop pole) and a control objective (stabilisation of the actual plant with a certain stability margin). The controller designed on the basis of the model is applied to the corresponding actual plant as given by (1). The question is: What kind of restrictions must we impose on the set of models in order to achieve the control objective? The restrictions on the set of models also limit the set of actual plants since the mismatch between the model and the plant is given by $\delta$. The second part of the paper is devoted to the problem: Given a plant in the aforesaid set, design an adaptive algorithm such that the control objective is achieved.

Let us point out that the more relevant (harder) problem would be to consider only $\epsilon_0$ and $\epsilon_1$ as known and search for $(a_0, b_0)$ as well as an appropriate closed loop pole $\sigma$, believing that a controller of the form (4) does exist.

We are not so much interested in the adaptive nature of the problem, rather we want to understand what type of control problem one can deal with in an adaptive context, especially the indirect philosophy of "identify" then "control" using the certainty equivalence principle [1]. The approach we take is to view the control problem from an identification perspective as introduced in the ideal case in [4] and further pursued in [5]. As it turns out, the type of control problem that seemingly can be dealt with is of a larger generality than that which is currently covered by the more classical robust adaptive control literature. This raises the question: "does there exists an adaptive algorithm that can deal with it ?" We provide an affirmative answer by constructing an adaptive algorithm, based on the previous work [3].

The paper is organized as follows: In section 2 the system is presented and analysed from a non-adaptive point of view. An adaptive algorithm is presented and analysed in section 3. Extensions of the results to AR systems is the topic of section 4, and conclusions are given in section 5.

# 2 The first order system

## 2.1 System description and robustness expectations

Let the system to be controlled be modelled by

$$y_t = a_0 y_{t-1} + b_0 u_{t-1} + \delta_t \ ; \ t = 1, 2, \cdots \quad (5)$$

$$|\delta_t| \leq \epsilon_1 \sup_{t-k<l<t} (|y_l|, |u_l|) + \epsilon_0 \ ; \ \epsilon_1, \epsilon_0 \geq 0 \quad (6)$$

Here and in the rest of the paper we will refer to $\delta_t$ as the (nominal model) mismatch.

Suppose that the system (5) can be robustly stabilized by state feedback. The pole placement controller is given by

$$u_t = \frac{\sigma - a_0}{b_0} y_t \quad (7)$$

where $\sigma$ is the desired closed loop pole. Assume further that we want every closed loop pole $\sigma \in (\sigma_1, \sigma_2) \subset (-1, 1)$ to be ***achievable*** in the sense that if we pick an arbitrary $\sigma \in (\sigma_1, \sigma_2)$, the corresponding pole placement controller will stabilize the system. For reasons of robustness we have required that all poles in an interval and not only a single pole should be achievable.

With the control based on the nominal model and no adaptation, the closed loop system can be written as

$$\begin{aligned} y_t &= \sigma y_{t-1} + \delta_t = \sigma y_{t-1} + \epsilon_{0t} + \\ &\quad \epsilon_{1t} \sup_{t-k<l<t} \left\{ \left(1, \left|\frac{\sigma - a_0}{b_0}\right|\right) |y_l| \right\} \end{aligned} \quad (8)$$

where we have substituted (7) in (5) and written $\delta_t$ as

$$\delta_t = \epsilon_{0t} + \epsilon_{1t} \sup_{t-k<l<t} \left\{ \left(1, \left|\frac{\sigma - a_0}{b_0}\right|\right) |y_l| \right\}$$

with $-\epsilon_0 \leq \epsilon_{0t} \leq \epsilon_0$ and $-\epsilon_1 \leq \epsilon_{1t} \leq \epsilon_1$. From (8) it is clear that we have to require

$$\left| \sigma \pm \epsilon_1 \left(1, \left|\frac{\sigma - a_0}{b_0}\right|\right) \right| < 1 \quad (9)$$

A natural robustness measure $\epsilon_1'$, is the deviation of $\left|\sigma \pm \epsilon_1 \left(1, \left|\frac{\sigma - a_0}{b_0}\right|\right)\right|$ from 1. Condition (9) is actually a requirement on the gain of the operator $G_y$. This gain is bounded by

$$\frac{1}{\left|\sigma \pm \epsilon_1 \left(1, \left|\frac{\sigma - a_0}{b_0}\right|\right)\right|}$$

We know the desired robustness margin $\epsilon_1'$ by assumption. It is clear that the possible range of the system parameters $(a_0, b_0)$ is limited with respect to $\epsilon_1$, $\epsilon_1'$ and $(\sigma_1, \sigma_2)$.

Let us denote the set of allowable models by $P(\epsilon_1, \sigma_1, \sigma_2, \epsilon_1')$. It is characterized by the requirement of the control objective

$$P(\epsilon_1, \sigma_1, \sigma_2, \epsilon_1') = \left\{(a_0, b_0) : \left|\sigma \pm \epsilon_1 \left(\left|\frac{\sigma - a_0}{b_0}\right|, 1\right)\right| \le 1 - \epsilon_1' \quad \forall\, \sigma \in (\sigma_1, \sigma_2)\right\} \tag{10}$$

Notice that $P$ is a union of two disjoint, convex, open sets and in particular that $b_0$ must be bounded away from zero since the inequality in (10) must be valid for all $\sigma \in (\sigma_1, \sigma_2)$.

## 2.2 Unfalsified models

Denote by $G_{y_0, \{\epsilon_{0,t}, \epsilon_{1,t}\}, \sigma}$ the set of closed loop regulated unfalsified models (compatible with both a single realization of the observed data and our prior knowledge)

$$\begin{aligned} &G_{y_0, \{\epsilon_{0,t}, \epsilon_{1,t}\}, \sigma} = \\ &\Big\{(\alpha, \beta) : (\alpha, \beta) \in P(\epsilon_1, \sigma_1, \sigma_2, \epsilon_1'), \\ &|y_t - \alpha y_{t-1} - \beta u_{t-1}| \le \\ &\epsilon_0 + \epsilon_1 \sup_{t-k<l<t}(|y_l|, |u_l|) \ \ \forall t \ge 1, \\ &u_t = \tfrac{1}{\beta}(-\alpha + \sigma) y_t \ \ \forall t \ge 0\Big\} \quad \sigma \in (\sigma_1, \sigma_2) \end{aligned} \tag{11}$$

where $\{\epsilon_{0,t}, \epsilon_{1,t}\}$ is a sequence of disturbance gains satisfying the presumed constraint, and $y_0$ is the initial condition of the system.

**Interpretation.** We are given a single realization of the disturbance gains, the initial condition and the desired closed loop pole. The set $G_{y_0, \{\epsilon_{0,t}, \epsilon_{1,t}\}, \sigma}$ contains those models $(\alpha, \beta)$ that will not be falsified if we apply the input sequence $u_t = \frac{1}{\beta}(-\alpha + \sigma) y_t$ ∎

Notice that for any model in $G_{y_0, \{\epsilon_{0,t}, \epsilon_{1,t}\}, \sigma}$ we have:

$$y_t = \sigma y_{t-1} + e_t \tag{12}$$

where

$$|e_t| \le \epsilon_0 + \epsilon_1 \sup_{t-k<l<t} (|y_l|, |u_l|) \tag{13}$$

Here $e_t$ is the prediction error $(y_t - \alpha y_{t-1} - \beta u_{t-1})$. Thus the system behaves as if it were regulated with correct pole placement and driven by some disturbance sequence (the prediction error) satisfying the a priori constraint.

**Remark.** This is an important observation since it implies that we have a model at hand that fits the data and the uncertainty, and from an identification point of view we cannot do better. ∎

Moreover, we have the following limit

$$\limsup_{t\to\infty} |y_t| \le \left[1 - |\sigma| - \epsilon_1 \left(\left|\frac{\sigma - \alpha}{\beta}\right|, 1\right)\right]^{-1} \epsilon_0 \tag{14}$$

Because $(\alpha, \beta) \in P(\epsilon_1, \sigma_1, \sigma_2, \epsilon_1')$ and (10) it follows that

$$\limsup_{t\to\infty} |y_t| \le \frac{\epsilon_0}{\epsilon_1'} \tag{15}$$

Using the same approach we can also show that

$$\limsup_{t\to\infty} |u_t| \le \frac{\epsilon_0}{\epsilon_1'} \cdot \frac{1 - \epsilon_1' - |\sigma|}{\epsilon_1} \tag{16}$$

The limits are determined solely by our prior knowledge and expectations. They also provide bounds on the gain ($l_\infty$ induced norm) of the operators $G_y$ and $G_u$ where

$$y_t = G_y(\epsilon_1')\epsilon_{0t}, \qquad u_t = G_u(\epsilon_1')\epsilon_{0t}$$

**Remark 1.** Provided the control law is based upon an unfalsified model, the actual system performs as expected on the basis of the model. Our bounds (15) and (16) indicate the worst possible behaviour consistent with our prior knowledge. ∎

**Remark 2.** In this section we have used the term unfalsified models. This does not mean that the model is unfalsifiable. If we carry out enough experiments, we will most probably find out that the model is incorrect, and hence we have falsified the model. In our case we have restricted our experiments to closed loop identification, and this is the reason why not all models which are in principle falsifiable, will be falsified. ∎

# 3 The adaptive algorithm

The main idea of the adaptive algorithm is to run a parameter estimator in each of the convex regions. We also compute a performance index for each parameter estimator, and the controller parameters are computed using the estimates from the "best" estimator. This is the same approach as used in [3], except that we now allow the convex regions to be unbounded.

### 3.1 The parameter estimator

Since the upper bound on the disturbance is known, although it is time varying and dependent on past values of $u$ and $y$, we use a projection algorithm with dead zone as described in [2]. If the estimate is outside the convex region, it will be projected back into it.

The system $y_t = a_0 y_{t-1} + b_0 u_{t-d} + \delta_t$ can be rewritten as

$$y_t = \phi_{t-1}^T \theta_0 + \delta_t$$

where

$$\begin{aligned} \phi_{t-1} &= [y_{t-1}, u_{t-1}]^T \\ \theta_0 &= [a_0, b_0]^T \end{aligned}$$

The parameter estimation algorithm is given by

$$\begin{aligned} \hat{\theta}_t &= \hat{\theta}_{t-1} + \frac{\phi_{t-1}}{||\phi_{t-1}||_2^2} f(\Delta_t, e_t) \qquad (17) \\ & \quad \text{for } ||\phi_{t-1}||_2 \neq 0 \\ \hat{\theta}_t &= \hat{\theta}_{t-1} \text{ for } ||\phi_{t-1}||_2 = 0 \qquad (18) \end{aligned}$$

where $||\cdot||_2$ denotes the Euclidian norm and

$$f(\Delta_t, e_t) = \begin{cases} e_t - \Delta_t & \text{if } e_t > \Delta_t \\ 0 & \text{if } |e_t| \leq \Delta_t \\ e_t + \Delta_t & \text{if } e_t < -\Delta_t \end{cases} \qquad (19)$$

where

$$\Delta_t = \epsilon_0 + \epsilon_1 \sup_{t-k<l<t} (|y_l|, |u_l|)$$

$e_t$ is the prediction error

$$e_t = y_t - \phi_{t-1}^T \hat{\theta}_{t-1} = \phi_{t-1}^T \tilde{\theta}_{t-1} + \delta_t$$

where

$$\tilde{\theta}_t = \theta_0 - \theta_t$$

It can be shown that the parameter estimator has the following properties

**P1** $||\tilde{\theta}_t||_2 \leq ||\tilde{\theta}_{t-1}||_2$

**P2** $\frac{f(\Delta_t, e_t)}{||\phi_t||_2} \in l_2$

Property **P2** implies that

**P3** $\lim_{t\to\infty} \frac{f(\Delta_t, e_t)}{||\phi_t||_2} = 0$

**P4** $\lim_{t\to\infty} ||\hat{\theta}_t - \hat{\theta}_{t-1}||_2 = 0$

Every new measurement defines a region in parameter space to which the true parameters $\theta_0$ belong. This region is given by

$$y_t - \Delta_t \leq \phi_{t-1}^T \theta \leq y_t + \Delta_t \qquad (20)$$

The estimation algorithm (17) projects the estimate $\hat{\theta}$ onto the boundary of this set. However, if this estimate is outside the allowable convex region, we have to project it back into the set. Since (20) describes a convex set, the intersection of (20) and the allowable convex regions are convex sets, and we will project the estimates onto these sets. By assumption there exists a region containing the true parameters, and in this region the projection will bring us closer to the true values, since we project onto a convex set containing the true parameters.

Let us denote the difference between the estimate given by the projection algorithm (17) and the intersected set by $g$. The estimation algorithm can then be rewritten as

$$\begin{aligned} \hat{\theta}_t &= \hat{\theta}_{t-1} + \frac{\phi_{t-1}}{||\phi_{t-1}||_2^2} f(\Delta_t, e_t) + g_{t-1} \qquad (21) \\ & \quad \text{for } ||\phi_{t-1}||_2 \neq 0 \\ \hat{\theta}_t &= \hat{\theta}_{t-1} \text{ for } ||\phi_{t-1}||_2 = 0 \qquad (22) \end{aligned}$$

If the intersection of the allowable region and (20) is empty then $||g_{t-1}||_2 = \infty$. Since we use the projection algorithm (17) and by convexity of the feasible regions it follows that $\phi_{t-1}^T g_{t-1} = 0$.

The estimates in the allowable region containing the true parameter estimates satisfy

$$||\tilde{\theta}_t||_2^2 \leq ||\tilde{\theta}_{t-1}||_2^2 - \frac{1}{||\phi_{t-1}||_2^2} f^2(\Delta_t, e_t) - ||g_{t-1}||_2^2 \qquad (23)$$

such that for this region both $\frac{f(\Delta_t, e_t)}{||\phi_{t-1}||_2}$ and $||g_t||_2$ are in $l_2$.

We want to choose the "best" estimate from the two available ones, and in order to decide which region contains the best estimate, we introduce a performance index and a criterion for selecting the region. The performance index is a measure of the goodness of the estimates in a region. One suitable performance index for region $i$ ($i = 1, 2$) is

$$p_{i,t} = p_{i,t-1} + \frac{f^2(\Delta_t, e_{i,t})}{||\phi_{i,t}||_2^2} + ||g_{i,t}||_2^2 \qquad (24)$$

This performance index is at least finite for the region containing the true parameters. Let $r(t)$ denote the region selected at time $t$. If $p_{r(t),t} < p_{i,t} + \gamma$ with $\gamma > 0$ (a constant) and $i \neq r(t)$, then $r(t)$ is unaltered. Otherwise $r(t) = \text{argmin}_i(p_{i,t})$, i.e. we switch regions. Along the same lines as in [3], we have the following lemma.

**Lemma 3.1** *There exists a $t_0 > 0$ such that $r(t) = r(t_0)$ for all $t \geq t_0$* ∎

## 3.2 Boundedness of the signals

The controller is now computed according to

$$u_t = \frac{\sigma - a_t}{b_t} y_t \qquad (25)$$

where $a_t$ and $b_t$ are the estimates of of $a_0$ and $b_0$ at time $t$. Using (5)-(6) the system and controller can be rewritten as

$$\begin{aligned} y_t - \sigma y_{t-1} &= e_t & (26) \\ u_t - \sigma u_{t-1} &= \frac{\sigma - a_{t-1}}{b_{t-1}} e_t + s_t & (27) \end{aligned}$$

where $e_t = (a_0 - a_t)y_{t-1} + (b_0 - b_t)u_{t-1} + \delta_t$ is the prediction error and $s_t = \left(\frac{\sigma - a_t}{b_t} - \frac{\sigma - a_{t-1}}{b_{t-1}}\right) y_t$. Since $(a_t, b_t)$ and $(a_{t-1}, b_{t-1})$ both belong to the convex regions, it follows that

$$|s_t| \leq K||\theta_t - \theta_{t-1}||_\infty |y_t|$$

for some constant $K$.

The parameter estimator has the property that for all $\epsilon$, there exists a $t_0$ such that the estimate is in the same convex region for all $t \geq t_0$ and $||\hat{\theta}_t - \hat{\theta}_{t-1}||_\infty < \epsilon$ for all $t \geq t_0$.

Redefine time origin to $t_0$ and write (26)-(27) as

$$\begin{bmatrix} y_t \\ u_t \end{bmatrix} = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix} \begin{bmatrix} y_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} 1 \\ \frac{\sigma - a_{t-1}}{b_{t-1}} \end{bmatrix} e_t + \begin{bmatrix} 0 \\ 1 \end{bmatrix} s_t \qquad (28)$$

Boundedness of $y_t$ and $u_t$ now follows along the same lines as in [3].

We now show that (15) and (16) are obtained in the limit as $t$ tends to infinity. According to (26)

$$y_t = \sigma y_{t-1} + e_t$$

Split $e_t$ into two terms: $f_t$ given by (19) and $e_t - f_t$. Finiteness of $||\phi_t||_2$, and property **P2** imply that for all $\epsilon > 0$ there exists an $N$ such that for all $t > N$, $|f_t| < \epsilon$. For $t > N$, we have

$$\begin{aligned} |y_t| \leq\ & |\sigma y_{t-1}| + \epsilon_0 + \\ & \epsilon_1 \sup_{t-k<l<t} \left(1, \left|\frac{\sigma - a_l}{b_l}\right|\right) |y_l| + \epsilon \qquad (29) \end{aligned}$$

and after some calculations it follows that

$$\limsup_{t\to\infty} |y_t| \leq \frac{\epsilon_0}{\epsilon_1'} \qquad (30)$$

The bound on $\limsup_{t\to\infty} |u_t|$ follows from $u_t = \frac{\sigma - a_t}{b_t} y_t$.

The following theorem is valid

**Theorem 3.1** *Assume that the system is given by (5)-(6) and that the controller is given by (25). Assume further that the allowable unbounded convex regions are chosen such that $|\sigma \pm \epsilon_1 (1, |\frac{\sigma - a}{b}|)| < 1 - \epsilon_1'$ is satisfied and that the nominal system parameters belong to one of the convex regions. Then the system is stable in the sense that $y_t$ and $u_t$ are bounded functions.*

*Moreover the following limits hold*

$$\begin{aligned} \limsup_{t\to\infty} |y_t| &\leq \frac{\epsilon_0}{\epsilon_1'} \\ \limsup_{t\to\infty} |u_t| &\leq \frac{\epsilon_0}{\epsilon_1'} \cdot \frac{1 - \epsilon_1' - |\sigma|}{\epsilon_1} \end{aligned}$$

**Remark 1.** As pointed out in the introduction, the more relevant and harder problem is to consider only $\epsilon_0$ and $\epsilon_1$ as prior information and search for $(a_0, b_0)$ as well as an appropriate closed loop pole $\sigma$. A possible solution is to cover the parameter space by convex sets, and to each set we assign (if possible) a closed loop pole $\sigma$ which is compatible with our control objective as specified by $\epsilon_0$, $\epsilon_1$ and the robustness margin $\epsilon_1'$. The adaptive control algorithm is then applied to those convex subsets for which a compatible closed loop pole $\sigma$ exists, running a parameter estimator in each subset. The controller is computed with the estimate from the best estimator and the $\sigma$ corresponding to this estimate.

From lemma 3.1 it follows that there exists a $t_0$ such that for all $t > t_0$, the convex region with the best parameter estimate do not change. This implies that $\sigma$ is fixed for all $t > t_0$, and the boundedness results obtained in this section are still valid. ∎

**Remark 2.** Our convex sets give a set of nominal models that we can robustly and adaptively stabilize with pole placement control when the uncertainty (disturbance) is of the form $|\delta_t| \leq \epsilon_0 + \epsilon_1 \sup_{t-k<l<t}(|y_l|, |u_l|)$. In contrast to robust control we have gained a set of possible nominal models instead of one single nominal model. However, because we have to learn about the system (identify the system parameters), we have lost the transient performance. ∎

# 4 Extensions to AR systems

The results obtained in this paper can be extended to AR systems of the form

$$A_0(q^{-1})y_t = b_0 u_{t-d} + \delta_t$$

and pole placement controllers of the form

$$C_0(q^{-1})u_t = D_0(q^{-1})y_t$$

where $C(q^{-1})$ and $D(q^{-1})$ are solutions of the Diophantine equation

$$A_0(q^{-1})C(q^{-1}) - b_0 q^{-d} D(q^{-1}) = A^*(q^{-1})$$

$A^*(q^{-1})$ is the desired closed loop polynomial.

The set of allowable models turns out to be non convex and rather complicated, but it is straight forward to find convex subsets of it to which the adaptive algorithm can be applied.

The analysis of the adaptive algorithm is similar, and a modified version of Theorem 3.1 holds. For details see [6].

# 5 Conclusion

In this paper we have investigated the problem of robustly and adaptively stabilizing a first order linear system by pole placement control. We have considered the problem from an identification point of view and showed that the control objective imposes restrictions on the system. The restrictions on the system are dependent on the magnitude of the unmodelled dynamics, $\epsilon_1$, the robustness margin, $\epsilon_1'$, and the desired closed loop pole $\sigma$.

Under the assumptions that the system satisfies these imposed restrictions and that the controller is computed on the basis of a fixed unfalsified model we have derived upper bounds on the $\infty$-norm of the signals in the control loop.

Finally, we have proposed an adaptive algorithm which deals with parameters belonging to unbounded convex sets, and we have proven boundedness of all signals in question.

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