Session TB3 (Invited) - Queues and Rare Events
Chair and organizer: Ad Ridder in A002

Analysis of a state-independent change of measure for the G/G/1 tandem queue

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In 1989, Parekh and Walrand introduced a method to efficiently estimate the probability of a rare event in a single queue or network of queueus. The event they consider is that the total number of customers in the system reaches some level N in a busy cycle. Parekh and Walrand introduce a simple change of measure, which is state-independent, in order to estimate this probability efficiently using simulation. However, they do not provide any proofs of some kind of efficiency of their method. For the single queue (with mutiple servers) it has been shown by Sadowsky, in 1991, that the change of measure as proposed by Parekh and Walrand is asymptotically efficient under some mild conditions.

In this work we study the state-independent change of measure of the $G|G|1$ tandem queue, along the lines of Parekh and Walrand, and we provide necessary conditions for asymptotic efficiency. To the best of our knowledge, no results on asymptotic effciency for the $G|G|1$ tandem queue had been obtained previously. Looking at the results for the $M|M|1$ tandem queue, it is expected that this state-independent change of measure is the only state-independent change of measure for the $G|G|1$ tandem queue that can possibly be asymptotically effcient. We show that, under some conditions, it is indeed the only exponential state-independent change of measure that can be asymptotically effcient. However, we have also identified conditions for the two node $G|G|1$ tandem queue under which the Parekh and Walrand change of measure is still *not* asymptotically effcient.

Rare event analysis and efficient simulation for a multi-dimensional ruin problem

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We look at large deviations of multivariate stochastic processes in continuous time, in particular, we consider the event that both components of a bivariate stochastic process $((A_t, B_t))_{t \geq 0}$ ever simultaneously exceed some large level; a leading example is that of two Markov fluid queues driven by the same background process ever reaching some large level u. Exact analysis being prohibitive, we resort to asymptotic techniques and efficient simulation.

The first result we present concerns various expressions for the decay rate $\lim_{u \to \infty} \frac{1}{u} \ln \mathbb{P} (\exists t > 0 : A_t > u, B_t > u)$ of the probability of interest, which are valid under Gärtner-Ellis-type conditions; these conditions are met by the bivariate Markov modulated fluid model. The first expression for the decay rate is in terms of the convex conjugate of the limiting cumulant generating function $M(\cdot, \cdot)$ of the process. It involves a trade off between the speed at which the process should reach the desired set and the 'cost' of using this speed. The second expression equals the the largest sum $\theta_1 + \theta_2$ of non-negative terms such that M evaluated at these values vanishes.

The second result is an importance-sampling-based rare-event simulation technique for the bivariate Markov modulated fluid model, which is capable of asymptotically efficiently estimating the probability of interest. We also present a technique to remedy the complication that the process can attain values in the target set while the 'embedded' process (recording values of (A_t, B_t) only at transition epochs of the background process) does not. The asymptotical efficiency of the simulation technique is illustrated with a number of numerical experiments.

A related problem is as follows. Instead of requiring that the both components of a bivariate stochastic process hit a large level at the same time, we now allow the components to exceed this level at possibly different times. We give a conjecture for the decay rate of this event and we also discuss multiple importance-sampling-based simulation techniques in order to estimate the probability of interest for the bivariate Markov modulated fluid model; these techniques differ in the number of times and moments at which a new change-of-measure should be used.