A Four-phase Approach to a Timetabling Problem in Secondary Schools

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1 Introduction

Timetabling problems are present in all types of schools. The research in this area is still very active; of the 19 selected contributions of PATAT 2004 ([1]), 12 are dedicated to Educational Timetabling. These problems can often be modeled by a graph coloring problem. Here the vertices represent the events (lessons, courses, exams) to be scheduled, the (vertex)colors the available timeslots, and the edges express incompatibilities between two events. Typically incompatibilities are caused by the effect that resources related to an event (teachers, students, rooms) can attend at most one event at the same time.

Apart from the basic model of graph coloring, extra conditions are common. Typical conditions are (room) capacities, resource (room, teacher) availabilities, and precedence relations among events.

The subject of our study is a High School Timetabling Problem as it is common in the Netherlands. Beforehand it is decided which teachers give which lessons. Hence the events are the lessons, and, in principle, the problem can be stated as a graph coloring problem with extra conditions on availability of resources (rooms, teachers). However there is an extra dimension: the quality of the constructed timetable. A feasible solution, though necessary, is absolutely not sufficient: we need to improve the feasible timetable to a schedule that is acceptable to use.

The size of the graph involved, and the extra efforts to improve the quality are the main reasons for our 4-phase approach: we try to control the quality by a preprocessing phase, and a post-processing phase. In the preprocessing phase, we cluster events in so-called clusterschemes. These clustered events can be considered as the new events to be scheduled. In the second and third phase a feasible timetable is constructed. In the fourth phase a Tabu Search is used to improve the best schedule found. The developed approach is tested by using data from the Kottenpark, which is one of the locations of 'Het Stedelijk Lyceum' in Enschede, the Netherlands.

2 Problem description

In the Kottenpark the timetable is still made by hand, and checked by computer. The reason for not using the automatic planner, that their commercial package provides, is mainly quality: this package is not able to generate any complete solution, and moreover the part that is generated is of bad quality.

In 2004, the Kottenpark had around 1000 students, 36 school classes, 71 teachers, and 40 rooms. There are 1049 lessons to be scheduled. As such it is a school of average size in the Netherlands. A bottleneck is the number of rooms for physical exercise, which is only 2, accounting a 100% occupation. All lessons have to be scheduled in 37 timeslots: 8 timeslots on Monday, Tuesday and Wednesday, 6 on Thursday (2 hours are reserved for staff meetings), and 7 on Friday. The classes have a timeslot occupation ranging from 73% up to 97%.

There is one major complication in the Dutch situation; for students in the upper years, 2/3 of their lessons are in optional subjects. This means that the basic class-teacher model (see for example [4]) can not be used anymore. We cope with this situation by using clusterschemes, see Phase 1 in the next section.

The most important aspects, which were included in our research, are the following:

- The lessons of a subject should be on different days.
- Some lessons should be given as a block. A block means that the subject is scheduled on two consecutive timeslots on the same day. This is often the case for physical exercise, but can also be the case for other subjects.
- Some teachers are not available on specific days, or parts of days.
- The lessons of some teachers should be concentrated on a limited number of days.
- In the lower grades the schedule must be compact (without free periods); in the higher grades free periods should be avoided.
- Free periods should be avoided for (most) teachers.

3 The four phase approach

Our approach consists of four principal phases.

Phase 1. Constructing the clusterschemes.
Students in the upper grades have optional subjects. All the groups for the optional subjects of one grade (and level), are put in a clusterscheme. This clusterscheme is divided in clusterlines, and students are to be placed in groups, such that the groups within one line have no students in common. Consequently all the subject groups in one line can be scheduled at the same time. Working with clusterschemes is not new (see for instance [2, 3]), and is in use at (nearly) all secondary schools in the Netherlands. We use a branch & bound algorithm to solve this problem.

Phase 2. Assigning lessons to day-parts.
In second phase the lessons of clusters of subject groups are assigned to day-parts one by one. The method we use is dynamic priority rule. At each stage we estimate the difficulty for a cluster to be scheduled. This estimate is mainly based on the availability of the resources. We schedule the cluster with the lowest availability first. If a resource gets tight on a day-part, we construct a graph with lessons of this resource on this day-part, and check whether these lessons can still be scheduled.

If this scheduling process breaks down (a particular cluster can not be assigned anymore), the heuristic lowers the availability (or increases the priority) of this cluster, and starts all over again. After a few tries, all lessons are assigned to day-parts, and we can proceed to the next phase. Because of the checks in this phase we expect that there exists a feasible solution for Phase 3.

Phase 3. Scheduling the day-parts.
The third phase schedules the lessons on day-parts to timeslots. Hence the original graph with all lessons is broken down to 10 smaller subgraphs. We use a graph coloring heuristic, which colors the points one by one (first fit). Originally we use the degree to sort the points, but we also use random orders to obtain several schedules.

Phase 4. Improving the schedule.
We use a Tabu Search to improve the best schedule found in Phase 3. For this we determine the worst resource (teacher or school class). Here 'worst' is measured by the objective function. For this resource we consider all lessons and free timeslots and determine the 10 best (lesson, new timeslot) combinations, as far as the worst resource is concerned. Starting with such a combination we consider an ejection chain: rescheduling a lesson of the worst resource to a new timeslot may be infeasible regarding the *other* resources of this lesson. We try to lift this infeasibility, by shifting the conflicting lesson to another timeslot, which can cause infeasibility, etcetera. We do not use branching of possibilities here, and stop if the chain of shifts reached a given length.

If one or more of the (lesson, new timeslot) combinations can be rescheduled, we perform the best chain of moves. We use tabu lists for lessons and resources to make sure that the next step does not undo the current step.

4 Results and conclusions

The presented study is performed with data from a specific Dutch school, but we believe that this data is representative for many schools in the Netherlands. Unfortunately not all constraints were incorporated yet, which makes comparison to the real timetable not completely fair. Comparing what *was* included we see a huge improvement in quality; for instance the number of free periods for teachers drops from 128 (hand-made) to 48. Using additional interactive methods, the quality can be improved to a level hard to obtain by manual scheduling.

References

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