Numerical simulation of astigmatic liquid lenses tuned by a stripe electrode

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Abstract: We propose a new design for tuning the astigmatism of liquid micro-lenses using electric field and hydrostatic pressure as control parameters. We explore the feasibility and operating range of the lens with a self-consistent numerical calculation of the electric field distribution and the shape of the two-phase interface. Equilibrium shapes, including surface profiles parallel and perpendicular to a stripe electrode, are extracted to determine the astigmatism. The wavefronts are decomposed into Zernike polynomials under zero defocus conditions using a commercial ray-tracing software. We observe that the global curvature of the lens is primarily controlled by the hydrostatic pressure, while asphericity and astigmatism are controlled by the electric field. For optimized electrode geometries and simultaneous control of pressure and electric fields the astigmatism can be tuned from $Z6 = 0...0.38 \mu m$ with minor changes in the focal length.

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OCIS codes: (230.0230) Optical devices; (000.4430) Numerical approximation and analysis; (230.2090) Electro-optical devices; (220.1080) Active or adaptive optics.

References and links

Adaptive micro-optical systems have drawn attention over the past years due to their wide applicability in mobile phones, military equipment, lab-on-a-chip devices and medical devices. Amongst such systems, adaptive lenses are of particular interest because of their wide range of tunability, reconfigurability, smooth interfaces and high response speeds [1–3]. Over the years, several approaches have been developed to tune fluidic lenses in order to increase their optical power and to correct different types of aberrations. In an out-of-plane configuration [2], the focal length of liquid lenses can be controlled by changing the curvature of the lens. For liquid micro-lenses, the curvature was tuned by varying the hydrostatic pressure [4], hydrodynamically [5], by thermal response [6], electro-optic tuning [7] and by electrowetting [8]. The latter approach was shown to provide particularly efficient and reversible control of both the refractive power and the aperture of adaptive lenses video rate with faster actuation speeds [1, 9–11].

Most of the methods mentioned above are only able to control the global curvature of the lens while preserving its spherical shape. However, asphericity and astigmatism are common aberrations in optical systems that need to be corrected [12]. For instance in ophthalmic applications, where phoropters based on adaptive liquid lenses were shown to enable quick identification of lens error such as astigmatism and defocus with a wide tuning range [13].
Some approaches using e.g. pre-shaped membranes, non-circular apertures or anisotropic strains provide tunable asphericity and/or astigmatism [14–18]. Alternatively, position-dependent electric fields applied between the deformable interface, between a non-conductive and a conductive fluid and a flat electrode placed on top, enable the controlled formation of aspherical shapes [4, 19–21]. Compared to some of the other approaches such electrically actuated lenses have the advantage of fast reversible tunability while preserving circular apertures. Moreover, the purely electrical actuation without mechanical actuators enables compact device geometries [8, 22].

In this study, we make use of this flexibility of electric fields to develop a novel concept of adaptive lenses with tunable astigmatism. The source of astigmatism in our device is provided by a stripe-shaped electrode. With the aid of CFD (computational fluid dynamics) simulations, using the software OpenFOAM, we perform a full three-dimension simulation to predict the equilibrium shape of the lens. Subsequently, we analyze the optical properties of the lens using the commercial optical design program Zemax. The optical analysis is made in terms of the orthonormal Zernike polynomials. We focus in particular on the term of astigmatism (Z6), on the focal length (described by the front focal length), which is obtained by optimizing the system to zero defocus and on the optical modulation transfer function (MTF)

The paper is structured as follows: first, we introduce the setup used to perform the simulations. We then validate it against known analytical solutions for the pressure and against results from the literature for the voltage (considering a flat electrode). Finally, we present and discuss the results obtained by applying a voltage to a stripe electrode of variable width. A brief conclusion presents the main aspects of the work as well as suggestions and ideas for the future in the area. The electrohydrodynamic model employed in the problem is presented in the appendix.

2. Methods

Fig. 1. (a) Sliced computational mesh. The image shows a local refinement for the stripe on the top boundary and a dynamic refinement on the lens (interface between both fluids). (b) View of the setup. A hydrostatic pressure $\Delta P_w$ is applied between the lower plates (water phase) to induce global curvature changes on the lens with a circular aperture of radius $a$. The voltage $U$ is applied between the upper plates (oil phase), separated by a distance $h$, using a stripe electrode (red stripe) of width $d$. When the voltage is applied, different profiles on the lens are achieved on the $x$ and $z$ planes (white and black cross lines, respectively). The optical axis of the system is represented by the dashed arrow.

The general configuration of the setup was based on the previous work of Mishra et al. [4], in which two immiscible liquids (namely, water and oil), located between parallel plates, are separated by a middle plate with a circular aperture of radius $a = 0.5$ mm. The space between the two lower plates is filled with electrically conductive water and the one between the upper plates is filled with a dielectric oil. The water phase as well as the middle plate are kept at zero voltage, while a variable potential can be applied to an electrode kept at a distance $h$.
above the aperture plane. While Mishra et al. [4] used a planar electrode to generate axisymmetric lenses with tunable spherical aberration, we use a stripe electrode, in order to break the rotational symmetry and to induce cylindrical deformations of the liquid surface, as shown Fig. 1(b). As in [4], a hydrostatic pressure can be applied in our model between the lower plates to control global curvature changes, while the electrostatic force will provide the astigmatic characteristics by inducing two different curvatures along the \( x \) and the \( z \) axis on the lens. The oil phase is always kept at atmospheric pressure. As the lens starts to deform, the oil-water interface remains pinned to the edge of the circular aperture thereby guaranteeing a fixed aperture diameter.

Numerical simulations of the equilibrium surface shape were performed using the free open-source CFD package OpenFOAM. An electrohydrodynamic model, based on the leaky dielectric model [23, 24], was used to deal with the coupling of fluid mechanics and Maxwell's equations. The governing equations and the basic solution strategy are described in the appendix.

For the simulations, a full three-dimensional mesh was built, with a thin local refinement on the top boundary to delimit the stripe, as presented in Fig. 1(a). The mesh was validated by considering three meshes with different refinements and comparing their results with the most refined one. The mesh with the best results together with the lower computational cost was chosen. The latter corresponds to the mesh with a linear resolution of 30 cells/mm. A dynamic meshing tool was also used on the interface to reduce even more the computational cost and to provide a narrow transition between the two fluids.

When the equilibrium between the Laplace pressure and the electrostatic force is reached, the surface is extracted from OpenFOAM and fitted in Matlab using the biconic equation, given by:

\[
y(x, z) = \frac{c_x x^2 + c_z z^2}{1 + \sqrt{1 - (1 + k_x) c_x^2 x^2 - (1 + k_z) c_z^2 z^2}},
\]

where \( c_x, c_z \) are the the curvatures on the apex and \( k_x, k_z \) the conic constants of the lens for the \( x \) and \( z \) planes, respectively. Each pair of curvature and conic constant describes one conic section of the lens. Whenever necessary, a polynomial correction, of the form \( \sum_{i=0}^{5} (\alpha_i x^i + \beta_i z^i) \), was used on Eq. (1) to improve the quality of the fitting [25].

The parameters of the biconic equation are then exported to Zemax, where the optical analysis is performed. For each lens, the setup is optimized by translating a point object along the optical axis such that the Zernike coefficient for defocus is adjusted to zero. This approach is commonly used to adjust the position of the objective in order to produce a collimated output after the lens under test [4, 14, 16]. The focal length can then be obtained by measuring the distance between the point object and the first optical surface, \( i.e. \) the top electrode in our geometry. The ratio between the refractive indexes from the water and the oil phase was taken to be \( n = 1.10 \).

3. Results and discussion

To check the consistency of the numerical code, a validation of the deformation of the lens, shown in Fig. 2, was performed for both the applied pressure (at zero voltage) and the electric field (without applying a pressure) for a homogeneous electrode instead of the stripe. The former can be described by the Young-Laplace equation \( \Delta P = \gamma \kappa \), where \( \gamma \) is the surface tension and \( \kappa \) is the mean curvature of the interface, which, for a sphere, is given by \( \kappa = 2/R \) (\( R \) being the radius of curvature of the sphere). In Fig. 2(a), a good agreement between the profile extracted from the numerical simulation and the predicted analytic profile can be observed. The influence of the electric field was compared to the results of Oh et al. [26] by considering a planar electrode, \( i.e. \) \( d = 1 \) mm. In this case, the comparison was made.
by the variation of the non-dimensionalized equilibrium deflection of the lens \( \xi_0 = \zeta_0 / h \) with respect to the electrocapillary number \( \Lambda = (a/h)(\varepsilon \varepsilon_0 U^2/(2\gamma h))^{1/2} \) for two different electrode distances \( H = 1.0 \) and 2.0; where \( H \) is the ratio between the electrodes distance \( h \) and the radius of the aperture \( a \) and \( \zeta_0 \) is the deflection at the center of the surface.

The open symbols in Fig. 2(b) represents the regions where instabilities of the lenses were observed. The electrocapillary number describes the relative strength of the electrostatic force with respect to the Laplace pressure [26]. Therefore, by increasing the voltage past a critical value, the Laplace pressure cannot balance the Maxwell stress anymore, and thus, the system becomes unstable. In some cases, the tip of the drop may also break up into small droplets. This type of instabilities can be encountered in many different situations, such as electrohydrodynamic atomization and the Cassie-to-Wenzel transitions [26–29]. Ultimately, such effects limit the accessible range of optical properties that can be addressed by varying the applied voltage.

![Fig. 2.](image)

In the stable regions presented in Fig. 2(b), the agreement with the results from Oh et al. [26] is satisfying. The deviations in the unstable regions can be probably related to a different numerical implementation of the interface, resulting in a different threshold for the production of satellite droplets. As a result, the exact values of the stability limits should be taken with some caveats.

Despite the deviations on the unstable regions presented in Fig. 2(b), when considering optical systems, we are only interested in stable solutions (filled symbols in Fig. 2(b)) since the instabilities can generate oscillations and internal flow patterns, which are not desirable for a lens. For the lens geometry considered in this work, i.e. \( H = 1 \) and \( d = 0.5 \) mm (see discussion below), a maximum value of \( \Lambda = 0.58 \) prevents instabilities. This leads to a maximum safe voltage of approximately 700 V (depending on the applied pressure).

For large values of \( H \), the system behaves as a parallel plate capacitor, because the meniscus of the lens will no longer affect the electric field. Thus, for \( H >> 1 \), the critical deflection will tend to remain constant [26]. For small values of \( H \), when the instability is reached, the meniscus might touch the top electrode, running into the regime that was used for a tunable aperture in [10]. The optimum value for \( H \) is then governed by making it as small as possible without getting into the snapping interface regime. Therefore, \( H = 1 \) is a reasonable choice, because we can avoid the instabilities and reach higher deformations of the lens with lower voltages.
To analyze the optical performance of our system regarding astigmatism, we need to consider the anisotropy of the lens shape in the presence of the stripe electrode. This is most easily visualized by considering the radii of curvature in two orthogonal directions of the lens. Since we are interested in the vertical astigmatic aberration, namely the one described by the Z6 Zernike mode, the orthogonal directions must be considered as the tangential and the sagittal planes, in this case the \( z \) and \( x \) planes, respectively (see Fig. 1(b)). Figure 3 shows the different profiles of the lens obtained when the voltage is increased from 300 V to 700 V considering a hydrostatic pressure of 30 Pa and a stripe electrode of 0.5 mm width placed in the \( z \) direction.

![Fig. 3. Lenses modulated by a pressure of 30 Pa and using a stripe electrode of 0.5 mm width for three different voltages 300 V, 500 V and 700 V. White and black lines correspond to the \( x \) and \( z \) sections, respectively. (Left) Front and side views of the lenses. (Right) Comparison of the \( x \) and \( z \) profiles normalized by the radius of the aperture.](image)

As the voltage increases, the difference between the \( x \) and \( z \) sections increases as well. Since the stripe is located along the \( z \) direction, the electric force acts only on a segment of the \( x \) section, generating a large curvature on its apex (open symbols in Fig. 4(a)). Figure 4(b) shows that this curvature will give rise to a closer (paraxial) back focal point \( f_\rho \) on the sagittal plane \( (x \) plane) compared to the one of the tangential plane \( (z \) plane), which, as a consequence, will provide a positive value of the Z6 coefficient. The paraxial back focal length can be calculated as \( f_\rho = 1/c(n-1) \) [30], where \( c \) is the curvature on the apex and \( n \) is the ratio between the refractive indexes from the water and the oil phase.
Fig. 4. (a) Normalized curvatures $c_x$ (open symbols) and $c_z$ (filled symbols) as a function of the voltage considering a stripe electrode of 0.5 mm width for three different pressures, 10 Pa, 30 Pa and 50 Pa. (b) Curvature for 30 Pa and the related variation of the paraxial back focal length as a function of the voltage. Open symbols correspond to the $x$ plane and filled symbols correspond to the $z$ plane.

Fig. 5. (a) Influence of the stripe width $d$ on the vertical astigmatism aberration $Z_6$ considering two different voltages 500 V (black columns) and 700 V (grey columns) at zero hydrostatic pressure. (b) Corresponding $x$ and $z$ profiles, normalized by the radius of the aperture, considering three stripe widths 0.1, 0.5 and 0.9 mm for two voltages, 500 and 700 V.

Different stripe widths will provide different curvatures of the lens and, as a consequence, different ranges of astigmatism. In order to optimize the stripe width such that higher ranges could be achieved, measures of the Zernike coefficient $Z_6$ were made considering 9 different stripes ranging from 0.1 mm to 0.9 mm for two different applied voltages, 500 V and 700 V, and zero hydrostatic pressure. The results presented in Fig. 5(a) show that, for both voltages, as $d$ increases, the astigmatism gradually increases until it reaches a peak value for a stripe of 0.5 mm. Beyond this value, i.e. when the width of the stripe approaches the diameter of the aperture, the $Z_6$ coefficient decreases until it finally reaches zero for a planar electrode, for which rotational symmetry is reestablished. Thus, in this work, in order to achieve the maximum tuning range of astigmatism we choose half the aperture diameter as optimum stripe width.

#254866

Received 2 Dec 2015; revised 1 Feb 2016; accepted 1 Feb 2016; published 19 Feb 2016

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22 Feb 2016 | Vol. 24, No. 4 | DOI:10.1364/OE.24.004210 | OPTICS EXPRESS 4216
For lower values of $d$, the electric stresses are not strong enough to deflect the surface significantly in order to reach a considerable difference between the two sections of the lens. On the other hand, for large values of $d$, the stresses provided by the electric field affect almost the entire surface equally (Fig. 5(b)). This reduces the asymmetric deformation and, as a consequence, the astigmatism. Interestingly, however, the values of the astigmatism are not symmetric around $d = 0.5$ mm. This asymmetry arises because the different deflections achieved for different stripe widths give rise to a non-linear enhancement of the electrical stresses.

![Graphs showing the transition of image spots and OTF curves](image)

Fig. 6. (a) Transition of the image spot when increasing the voltage from 0 to 400V, considering a pressure of 30 Pa. The airy disk is represented by the black circle. Note the different scales as indicated on the left axes. (b) Plots showing the divergence between the tangential (blue curve) and the sagittal (red curve) MTF curves as the voltage increases. The black line represents the diffraction limited curve.

Figure 6(a) shows the transition of the image spot from spherical at zero voltage to cylindrical shapes at finite voltages as observed in the image plane. It also demonstrates that the astigmatism becomes more pronounced at higher voltages, as expected based on the Z6
coefficient (see Fig. 5). At zero voltage, the spot is contained within the airy disk. This is due to the fact that optical aberrations are insubstantial due to a considerably low curvature of the lens at 30 Pa. As the voltage is increased from 0 V to 400 V, the root mean square (RMS) spot size increases from 0.135 mrad to 0.778 mrad, eventually spanning outside the confines of the airy disk. Similar result can also be interpreted from the MTF plots, as shown in Fig. 6(b). The MTF plot represents the holistic picture of all optical aberrations presented in the system. At 30 Pa and zero voltage, the curves, corresponding to sagittal (red curve) and tangential (blue curve) planes and representing the MTF of our optical system, overlap the diffraction limited black curve. In this case, aberrations are insignificant due to low curvature. As the voltage is applied, this axisymmetry breaks and the two curves start to diverge from each other. It is evident from the MTF plots that the deviation between the tangential and sagittal MTF curves increases as the voltage is increased. This increasing deviations corroborates the sharp increase in astigmatism at elevated voltages.

By applying a voltage, we tune not only the astigmatism, but unavoidably also the focal length. To be able to tune both parameters independently, it is necessary to make use of the pressure as a second control parameter. Yet, despite the fact that higher deflections are achieved using higher voltages, the average deformation of the lens is primarily influenced by the applied pressure (see Fig. 2). Therefore, the pressure has a stronger effect on the average focal length. This is clearly shown in Fig. 7, where the map of the front focal length (shown in Fig. 7(a)) and the Z6 coefficient (shown in Fig. 7(b)) are plotted by simulating several combinations of pressures and voltages for a device with a stripe width $d = 0.5$ mm. The almost horizontal lines in Fig. 7(a) indicate that the front focal length $f$ is indeed mainly influenced by the pressure. A similar observation can be made in Fig. 7(b), in which the astigmatism is basically tuned by the variation of the voltage.

![Fig. 7. Color maps for both the front focal length (a) and the Zernike coefficient Z6 (b) for different combinations of voltages and pressures considering a stripe width of 0.5 mm. The small kinks in the lines result from the discretization of the data and the white space on the bottom left corner represents the regions where the surface remained almost flat.]

Although the focal length and the astigmatism are mostly influenced by the pressure and the voltage, respectively, the decoupling of the dependences is not complete. A device, for which the focal length and/or the astigmatism are to be tuned completely independently will therefore require a simultaneous actuation of both voltage and pressure to follow iso-astigmatism or iso-focal length paths in the pressure/voltage control parameter space. Such paths can be readily extracted from Fig. 7. Overall, by varying the hydrostatic pressure and the electric field simultaneously we can access, a tuning range of $f = 5.5$ mm – 142 mm and $Z6 = 0.001 \mu m$ - 0.38 $\mu m$. It is important to notice that, by applying a voltage, other aberrations are also induced, specially the spherical aberration given by the 11th Zernike
Coefficient (Z11). In our case, when considering a pressure of 30 Pa, for example, the value of the Z11 coefficient also changes from 0.0139 μm at zero voltage to −0.5815 μm at 600V.

The level of the astigmatism tuning reached by the design proposed in this work is comparable to the ones of other approaches used in the literature. For example, similar levels of astigmatism were achieved using pre-molded membranes tuned via injected liquid volume in circular apertures [14, 31]. By applying a strain in elastomeric lenses, one can reach higher levels of astigmatism [16]. In particular, for high strains, a value of the Z6 coefficient around 6 times greater than the one obtained in this work was achieved. However, using mechanical motions is not ideal for micro-optics systems, as compactness, low prices and high actuation speeds are desirable features.

4. Conclusion

By means of numerical simulations and optical analysis, we were able to predict the asymmetric deformation and the astigmatism of liquid lenses modulated by a stripe electrode. Two control parameters, namely the hydrostatic pressure and the voltage, were used to generate different equilibrium shapes of the lens. In order to reach higher levels of astigmatism, several widths of stripes were tested and, in this case, an optimum width was shown to be half the aperture diameter. Moreover, our simulations show that the focal length and the astigmatism can be tuned largely independently from each other, by tuning the applied pressure and voltage, respectively. The absence of membranes, non-circular apertures, and macroscopic mechanical actuators makes our system more flexible and make integration into optical microsystems and lab-on-a-chip systems more easy. In addition to assigning all these characteristics, our design is not only restricted to stripes. One could think of an array of individually addressable electrodes to generate arbitrary lens shapes, adjusting it, accordingly, to specific types of aberrations. Finally, our results provide an optimized setup with a high range of results that can be used in future experiments and equipments. Since our simulations are based on previous experiments from [4], ideas and details for producing our design are also provided. Therefore, the simulation proved to be an effective tool, providing a new method as well as baseline results for tuning astigmatism in liquid micro-lenses.

Appendix

For an incompressible and isothermal problem, the fluid flow field is given by the mass and momentum (Navier-Stokes) conservation equations (Eq. (2) and Eq. (3), respectively):

\[ \nabla \cdot \mathbf{u} = 0, \quad \text{(2)} \]

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \left[ \mu \left( \nabla \mathbf{u} + \nabla \mathbf{u}^T \right) \right] + \rho \mathbf{g} + \mathbf{F}_E + \mathbf{F}_S, \quad \text{(3)} \]

where \( \rho \) is the liquid density, \( \mathbf{u} \) is the velocity vector, \( p \) is the pressure, \( \mu \) is the dynamic viscosity and \( \mathbf{g} \) is the vector of acceleration due to gravity. The additional terms \( \mathbf{F}_E \) and \( \mathbf{F}_S \) represent the forces due to the electric field and the surface tension, respectively. By solving the Gauss’s law and the charge conservation equations (Eqs. (4) and (5)), the electric force (presented in Eq. (6)) can then be obtained by taking the divergent of the Maxwell stress tensor:

\[ \nabla \cdot (\epsilon \mathbf{E}) = \rho_e, \quad \text{(4)} \]

\[ \frac{\partial \rho_e}{\partial t} + \nabla \cdot (\sigma \mathbf{E} + \rho_e \mathbf{u}) = 0, \quad \text{(5)} \]

#254866

Received 2 Dec 2015; revised 1 Feb 2016; accepted 1 Feb 2016; published 19 Feb 2016

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22 Feb 2016 | Vol. 24, No. 4 | DOI:10.1364/OE.24.004210 | OPTICS EXPRESS 4219
where $\varepsilon$ is the dielectric constant, $\vec{E}$ is the electric field, given by $\vec{E} = -\nabla U$, with $U$ being the electric potential, $\rho_e$ is the electric charge density, $\sigma$ is the electric conductivity and $I$ is the identity tensor. The force related to the surface tension is given by $\gamma \nabla \alpha_i$, where $\gamma$ is the surface tension, $\kappa$ is the curvature of the interface and $\alpha_i$ is the dimensionless volume fraction. The latter is an indicator function provided by the VOF (Volume of Fluid) method, which is used to deal with two-phase problems [32]. It consists on the definition of a function $\alpha_i$, which defines one phase given by $\alpha_i = 0$ (e.g. the oil phase) and the other phase by $\alpha_i = 1$ (the water phase). The interface between the two liquids is the transition from $0 < \alpha_i < 1$. In this work, the surfaces were extracted using the value of $\alpha_i = 0.5$. The properties of both liquids can be given as a weighted averaging with the phase fraction, given by Eq. (7):

$$\theta = \alpha_i \theta_{\text{phase1}} + (1 - \alpha_i) \theta_{\text{phase2}},$$

where $\theta$ represents the physical properties $\rho$, $\mu$ and $\varepsilon$. For the conductivity, a harmonic weighted averaging was used to smooth the electric charge density on the interface [33]. Finally, the indicator function $\alpha_i$ must satisfy the following governing equation:

$$\frac{\partial \alpha_i}{\partial t} + \nabla \cdot (\alpha_i \vec{u}) + \nabla \cdot \left[ \alpha_i (1 - \alpha_i) \vec{u}_i \right] = 0,$$

where $\vec{u}_i = \vec{u}_1 - \vec{u}_2$ is the velocity between phase 1 and phase 2. The last term in the left-hand side of Eq. (8) is only activated at the interface region. Details regarding the implementation and the equations of the model can be found in the works of Roghair et al. [34] and Lima & d’Avila [35].

**Acknowledgments**

The present work was performed with the support of CNPq, Conselho Nacional de Desenvolvimento Científico e Tecnológico - Brasil. We gratefully acknowledge the Dutch Science Foundation NWO and the Foundation for Technical science STW for financial support within the VICI program. We also thank Prof. Dr. Marcos Akira d’Avila and Dr. Ivo Roghair for the support and for the collaboration on this work. We extend our thanks to Arjen Pit for the help with editing the text and figures.