



International Conference on the Technology of Plasticity, ICTP 2017, 17-22 September 2017,  
Cambridge, United Kingdom

## Modelling of anelastic deformation in dual-phase steel for improved springback simulation

A. Torkabadi\* , E.S. Perdahcıođlu, A.H. van den Boogaard

*University of Twente, Chair Nonlinear Solid Mechanics, P.O. Box 217, 7500 AE Enschede, The Netherlands*

---

### Abstract

Classical elasto-plastic models assume linear elastic stress-strain relations for all stresses within the yield surface. Closer examination discloses a nonlinear relation in the elastic domain that is dependent on the prior plastic deformation. The ‘unloading strain’ can be decomposed in a linear elastic contribution and an anelastic contribution that is related to reversible dislocation movement in the crystal lattice. The anelastic contribution in the total recovered strain upon unloading is significant and therefore should be considered in accurate springback predictions.

Modelling of this phenomenon with E-modulus degradation is fundamentally incorrect and only gives a fair strain prediction after completely unloading the material. In springback situations, inner fibres of the sheet material are partly unloaded and outer fibres are even reloaded in compression. Therefore, a model is required that includes the amount of plastic pre-loading and the amount of unloading separately. For implementation of the model in a finite element code, it needs to be formulated in the complete 6-dimensional stress space and not only for uniaxial stresses. A model is presented that can be applied for arbitrary strain paths and that is consistent with the main observations in uniaxial loading-unloading-reloading experiments.

© 2017 The Authors. Published by Elsevier Ltd.

Peer-review under responsibility of the scientific committee of the International Conference on the Technology of Plasticity.

*Keywords:* Springback; Anelasticity; Dislocations; Nonlinear unloading; AHSS

---

---

\* Corresponding author. Tel.: +31-(0)53-489-8955.

*E-mail address:* [a.torkabadi@utwente.nl](mailto:a.torkabadi@utwente.nl)

## 1. Introduction

Unlike what is generally accepted, experimental evidence shows that assuming a Hookean behaviour to describe unloading is not realistic. It has been observed experimentally that a plastically deformed material shows a nonlinear unloading/reloading behaviour. This is caused by an extra reversible strain recovered during unloading along with the pure elastic strain. The root cause of this phenomenon is the short-range reversible movement of dislocations known as anelasticity [1-3]. The dislocation structures which are impeded by the pinning points or piled up before the grain boundaries can move to a new equilibrium upon the relaxation of the lattice stress, contributing to some extra microscopic strain. This results in the nonlinear unloading/reloading behaviour and the reduction of the effective modulus. A number of authors have claimed that the magnitude of the anelastic strain is related to the dislocation density in the material [3-6]. In a recent work, Arechabaleta et al. [6] used XRD method to quantify the dislocation density in pure iron and a low-alloy steel. The results confirmed the proportionality between the dislocation density and the anelastic strain.

From an industrial perspective, modeling of the above mentioned phenomenon is essential for an accurate springback simulation, since the magnitude of the springback is governed by the total recovered strain upon unloading of the deformed part [7]. To overcome this issue, an approach commonly referred to as the “E-modulus degradation” has been taken by various authors [8-10]. In this approach the E-modulus of the material is made a function of the equivalent plastic strain. In this way the nonlinear unloading curve is approximated by the chord modulus which is measured from the experiments. It has been shown that adopting the E-modulus degradation approach significantly improves the springback prediction accuracy by the FEM. However, It has been shown by Ghaei et al. [11] that the springback angle is over-predicted when the E-modulus degradation approach is taken.

Recently, few attempts have been made in order to model the nonlinear unloading behaviour. Eggertsen and Mattiasson [12] and Sun and Wagoner [13] have taken similar approaches based on the two-yield-surface plasticity theory and proposed two-surface constitutive models in which the inner surface defines the transition between the linear and nonlinear elasticity and the outer surface gives the yield criteria. In that way, as long as the stress state is within the inner surface, the stress-strain relation remains linear. This was found in contradiction with the experimental observations. On top of that, such models are built based on mathematical convenience rather than capturing the underlying physics of the phenomenon.

In this work by quantifying the anelastic strain, based on the physics of the phenomenon, a one dimensional model for describing the nonlinear unloading behaviour is presented. This one dimensional uniaxial model is generalized to a three dimensional constitutive model incorporating elastic, anelastic and plastic strains. The performance of the model is evaluated by comparing the predicted cyclic unloading/reloading stress-strain curves with the experimental ones.

## 2. Experimental

In order to determine the parameters for the anelastic model, cyclic uniaxial loading/unloading/reloading (LUR) experiments were conducted on a DP800 steel with a thickness of 1 mm. In such experiments the material was loaded to a certain force and then unloaded to zero force. In the subsequent cycles the loading force was increased by increments of 500 N. In order to determine the hardening parameters of the material, a monotonic tensile experiment was conducted. In this experiment, the specimen was strained up to 15% engineering strain. All the experiments were conducted at a constant crosshead speed of 5 mm/min resulting in a strain rate of  $0.0005 \text{ s}^{-1}$ . The experimental stress-strain curves resulting from the monotonic and LUR experiments are presented in Fig. 1.

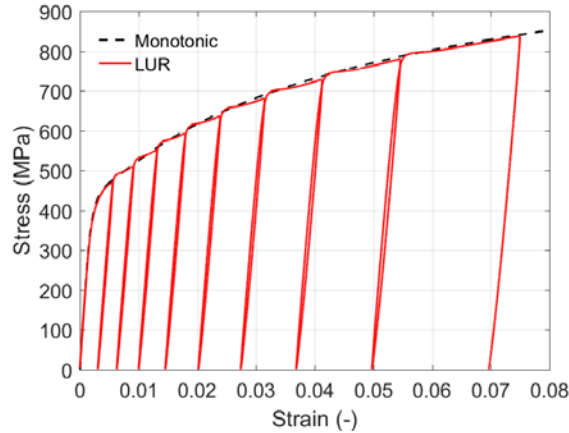


Fig. 1 Experimental stress strain curves.

### 3. The model

The model, presented in this work, is based on the anelastic theory proposed by Zener [1] which was quantified by Hull and Bacon [14] according to

$$\varepsilon^{an} = \rho b \bar{x} \tag{1}$$

where  $\rho$  is the dislocation density,  $b$  is the Burgers vector and  $\bar{x}$  is the average distance moved by dislocations. As the material plastically deforms, the dislocation density increases which leads to an increase in the flow stress. The dislocation density in the material can be correlated to the flow stress of the material according to the Taylor relation given as

$$\sigma_f = \sigma_0 + \bar{M} \alpha G b \sqrt{\rho} \tag{2}$$

where  $\bar{M}$  is the Taylor factor,  $\alpha$  is the dislocation strengthening parameter and a material related constant,  $G$  is the shear modulus of the material,  $\rho$  is the dislocation density and  $\sigma_0$  is the lattice friction stress of the material in the absence of dislocation interactions. In order to replace  $\sigma_0$  with the flow stress of the as-received material  $\sigma_{y0}$ , Equation (2) can be rewritten as [15]

$$\sigma_f = \sigma_{y0} + \bar{M} \alpha G b (\sqrt{\rho} - \sqrt{\rho_0}) \tag{3}$$

where  $\rho_0$  is the dislocation density of the material when it becomes plastically deformed at  $\sigma_{y0}$ .

In this way as the material hardens during the plastic deformation, the density of the dislocation which can contribute to the recovered anelastic strain increase. Therefore, upon unloading, a larger anelastic strain is expected to be recovered. After rewriting Equation (3) in combination with Equation (1) the total recoverable anelastic strain as a function of the work hardening behaviour of the material is obtained as [16]

$$\varepsilon^{an} = (K(\sigma_f - \sigma_{y0}) + \sqrt{\varepsilon_{pre}^{an}})^2 (\sinh(\alpha \cdot s) / \sinh(\alpha)) \tag{4}$$

where  $K$  is a material parameter,  $\sigma_f$  is the flow stress,  $\sigma_{y0}$  is the yield stress of the material,  $\varepsilon_{pre}^{an}$  is value of the anelastic strain at the yield stress,  $\alpha$  is a fitting parameter and  $s$  is the dimensionless normalized stress defined as

$$s = |\Delta\sigma| / \sigma_f \tag{5}$$

here  $\Delta\sigma$  is the stress decrement upon unloading from the flow stress.

In order to obtain the parameters for the anelastic model, Equation (4) is fitted to the experimental data as shown in Fig. 2a and b. The obtained values for the anelastic model parameters are given in Table 1.

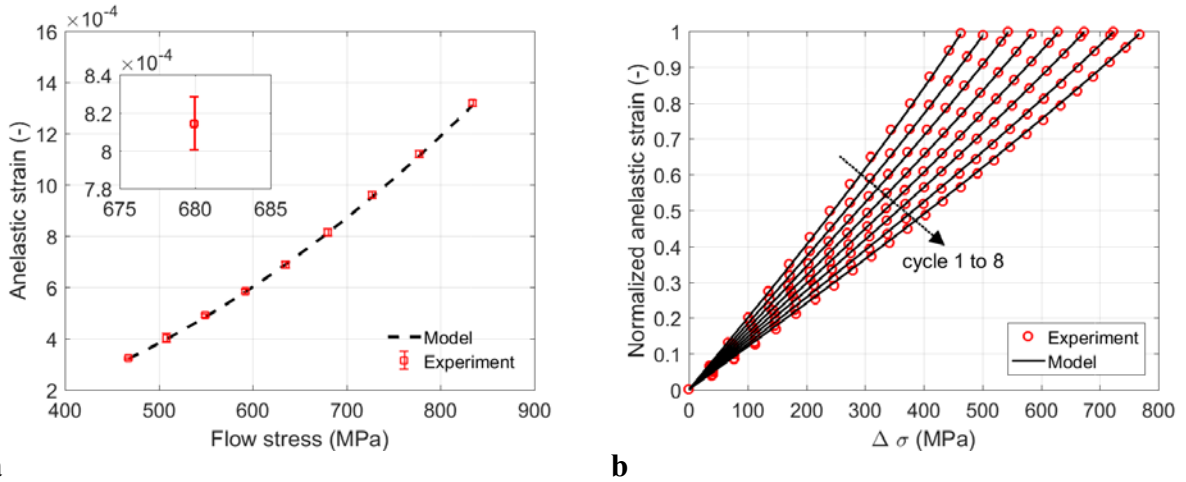


Fig. 2 (a) Variation of the total recovered anelastic strain with unloading stress; (b) Normalized anelastic strain as a function of unloading stress.

In order to implement the model in a FEM code, the one-dimensional anelastic model should be extended to a three-dimensional formulation and the constitutive rate equations are required. Additive decomposition of the total strain increment into elastic  $\dot{\epsilon}^e$ , anelastic  $\dot{\epsilon}^{an}$  and plastic  $\dot{\epsilon}^p$  strain rates gives

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^{an} + \dot{\epsilon}^p \tag{6}$$

The rate of the elastic strain is given according to

$$\dot{\epsilon}^e = \bar{E}^{-1} : \dot{\sigma} \tag{7}$$

and the plastic strain rate is

$$\dot{\epsilon}^p = \dot{\lambda} N \tag{8}$$

where  $N = \partial \varphi / \partial \sigma$  is the normal vector to the yield surface which depends on the derivative of the yield function ( $\varphi$ ) with respect to stress. Assuming the same flow potential for the anelastic strain, the rate of the anelastic strain is given as

$$\dot{\epsilon}^{an} = \dot{\xi} N \tag{9}$$

where the scalar multiplier  $\dot{\xi}$  can be written as

$$\dot{\xi} = \frac{3}{2} \dot{\epsilon}^{an} \tag{10}$$

here  $\dot{\epsilon}^{an}$  is the time derivative obtained from Equation (1).

In order to model the hysteretic behaviour observed in unloading/reloading cycles, a load reversal criterion is introduced. Accordingly, in the rate formulation the value of  $s$  is evaluated as

$$s = \int \dot{s} dt \tag{11}$$

where the integral is evaluated from the time when the load reversal takes place (i.e. from unloading to reloading or vice versa). This is when the relative angle between the two successive stress increments exceeds 90 degrees.

For the implementation of the model an implicit, backward Euler type numerical scheme is used for return mapping algorithm. More details on the formulations can also be found in [17].

Table 1 Material parameters.

Swift hardening law			Anelastic model			
$\sigma_f = C(\epsilon_0 + \epsilon_{eq}^p)^n$			$\epsilon^{an} = (K(\sigma_f - \sigma_{y0}) + \epsilon_{pre}^{an 0.5})^2 (\sinh(\alpha \cdot s) / \sinh(\alpha))$			
$C$ (MPa)	$\epsilon_0$	$n$	$K$ (MPa <sup>-1</sup> )	$\sigma_{y0}$ (MPa)	$\epsilon_{pre}^{an}$	$\alpha$
1710	0.0072	0.28	$5.1 \cdot 10^{-5}$	430	$3 \cdot 10^{-4}$	0.68

Clearly, this formulation is independent of the flow function and the hardening law; however, for demonstration of the model the von Mises flow function and the Swift hardening law are incorporated in the following. The parameters used for the Swift hardening law are given in Table 1.

#### 4. Model validation

In order to validate the performance of the presented model, the prediction of the model and the LUR experiment described in Section 2 are compared. The experimental stress strain curve and the prediction of the model are plotted in Fig. 3. A magnified view of the 7<sup>th</sup> unloading/reloading cycle is plotted in Fig. 3b. As can be noted from Fig. 3b, the proposed model can accurately predict the strain throughout the complete unloading/reloading path.

For a better comparison, the unloading path predicted by the E-modulus (i.e. 204 GPa) and the chord modulus are plotted in Fig. 3b as well. The unloading path obtained by E-modulus and unloading using the chord modulus respectively tend to underpredict and overpredict the strain during unloading. This result is consistent with the claim of Ghaei et al. [11] regarding to the underprediction of the springback angle by the E-modulus and overprediction of it taking the E-modulus degradation approach. Therefore, a more accurate springback prediction is expected by employing the anelastic model in the FEM simulations.

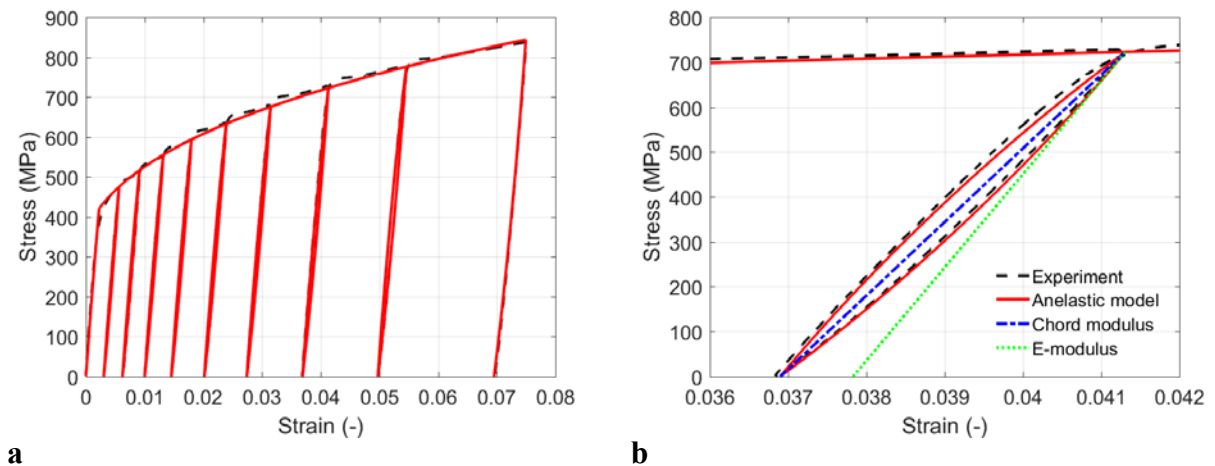


Fig. 3 Stress-strain response of DP800; (a) model prediction (red) and experimental data (black); (b) magnified view of the 7<sup>th</sup> cycle.

#### 5. Conclusions

In this study a model was presented to capture the nonlinear unloading/reloading behaviour in AHSS. The model was calibrated to the experimental data obtained from DP800 steel using cyclic unloading/reloading experiments. To that end, four parameters were fitted to the experimental data. A comparison between the experimental results and the model shows a good correlation for the whole unloading/reloading path. Hence, using the proposed model in FEM simulations of forming processes will result in better prediction of the springback behaviour of complex parts.

#### Acknowledgements

This research was carried out under project number S22.1.13494a in the framework of the Partnership Program of the Materials innovation institute M2i ([www.m2i.nl](http://www.m2i.nl)) and the Technology Foundation STW ([www.stw.nl](http://www.stw.nl)), which is part of the Netherlands Organization for Scientific Research ([www.nwo.nl](http://www.nwo.nl)).

## References

- [1] Zener C M and Siegel S, *Elasticity and Anelasticity of Metals*. The Journal of Physical and Colloid Chemistry, 1949. **53**(9): p. 1468-1468.
- [2] Ghosh A K, *A physically-based constitutive model for metal deformation*. Acta Metallurgica, 1980. **28**(11): p. 1443-1465.
- [3] van Liempt P and Sietsma J, *A physically based yield criterion I. Determination of the yield stress based on analysis of pre-yield dislocation behaviour*. Materials Science and Engineering: A, 2016. **662**: p. 80-87.
- [4] Mott N F, *CXVII. A theory of work-hardening of metal crystals*. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 1952. **43**(346): p. 1151-1178.
- [5] Friedel J, *XLVI. Anomaly in the rigidity modulus of copper alloys for small concentrations*. The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 1953. **44**(351): p. 444-448.
- [6] Arechabaleta Z, van Liempt P, and Sietsma J, *Quantification of dislocation structures from anelastic deformation behaviour*. Acta Materialia, 2016. **115**: p. 314-323.
- [7] Wagoner R H, Lim H, and Lee M-G, *Advanced Issues in springback*. International Journal of Plasticity, 2013. **45**(0): p. 3-20.
- [8] Morestin F and Boivin M, *On the necessity of taking into account the variation in the Young modulus with plastic strain in elastic-plastic software*. Nuclear Engineering and Design, 1996. **162**(1): p. 107-116.
- [9] Yoshida F, Uemori T, and Fujiwara K, *Elastic-plastic behavior of steel sheets under in-plane cyclic tension-compression at large strain*. International Journal of Plasticity, 2002. **18**(5-6): p. 633-659.
- [10] Chongthairungruang B, Uthaisanguk V, Suranuntchai S, and Jirathearanat S, *Experimental and numerical investigation of springback effect for advanced high strength dual phase steel*. Materials & Design, 2012. **39**(0): p. 318-328.
- [11] Ghaei A, Green D E, and Aryanpour A, *Springback simulation of advanced high strength steels considering nonlinear elastic unloading-reloading behavior*. Materials & Design, 2015. **88**: p. 461-470.
- [12] Eggertsen P A, Mattiasson K, and Hertzman J, *A Phenomenological Model for the Hysteresis Behavior of Metal Sheets Subjected to Unloading/Reloading Cycles*. Journal of Manufacturing Science and Engineering, 2011. **133**(6): p. 061021-061021.
- [13] Sun L and Wagoner R H, *Complex unloading behavior: Nature of the deformation and its consistent constitutive representation*. International Journal of Plasticity, 2011. **27**(7): p. 1126-1144.
- [14] Hull D and Bacon D J, *Chapter 3 - Movement of Dislocations, in Introduction to Dislocations (Fifth Edition)*. 2011, Butterworth-Heinemann: Oxford. p. 43-62.
- [15] Viguier B, *Dislocation densities and strain hardening rate in some intermetallic compounds*. Materials Science and Engineering: A, 2003. **349**(1-2): p. 132-135.
- [16] Torkabadi A, Perdahcıođlu E S, Meinders V T, and van den Boogaard A H, *On the nonlinear anelastic behavior of AHSS*. International Journal of Solids and Structures.
- [17] Simo J C and Hughes T J R, *Computational Inelasticity*. Interdisciplinary Applied Mathematics Series. 2000: Springer.