Sound Attenuation: Implementation of the Biot Theory

F.J.M. van der Eerden, R.M.E.J. Spiering, H. Tijdeman

University of Twente
Department of Mechanical Engineering, Laboratory for Applied Mechanics
P.O. Box 217, 7500 AE Enschede, the Netherlands
☎ +31 (0) 53 489 4368, 🖷 +31 (0) 53 489 3900, ✉ F.J.M.vanderEerden@wb.utwente.nl

Summary

Sound absorption can be modelled as a surface boundary condition by means of the impedance elements in B2000 or as a volume by means of the Limp elements. Both methods are simplifications of a porous material. The Biot theory allows a more complete description of the sound absorbing material and models the acousto-elastic interaction of the elastic frame (skeleton) and the fluid inside. This theory is implemented in B2000 and the first results are presented. A short overview of the experimental sound reduction techniques at the laboratory is presented as well.

1 Introduction

The reduction of sound becomes more and more important. This can be seen for instance in the automobile, aircraft and railway industry. So with an accurate description of sound reducing techniques time and money can be saved in an early stage.

At the University of Twente, The Netherlands, two techniques to reduce the sound level are investigated. The first one reduces the transmission of sound through a light weight double wall panel with a thin air layer in between. The second technique aims to develop an accurate description of sound absorbing material. For the latter case several descriptions of sound absorbing material are already present. An overview of the existing finite elements for sound absorbing material in B2000 is given is section 2.2.

There are two ways to describe a sound absorbing material: as a surface boundary condition or as a volume. In the volume method there is an interaction between the frame and the fluid inside the absorbing material. The volume description of the Biot theory, and the implementation in B2000, will be discussed in more detail. This theory is a rather complete description of sound absorbing material. In section 3 a Finite Element formulation of the Biot theory and some characteristics of the FEM matrices are given and a few preliminary results are presented. Conclusions and notes on further research are listed in section 4.

2 Sound attenuation

An important condition for an accurate description of sound reducing methods is a good understanding of the acousto-elastic interaction because in general a two-way coupling of the two media exists. For the double wall panels there is an interaction between two elastic panels and a trapped fluid, for the sound absorbing material the two media are the skeleton of the elastic material and the fluid inside, which is usually air.

The research at the Laboratory for Applied Mechanics, Department of Mechanical Engineering at the University of Twente, focuses on this acousto-elastic interaction [1]. In section 2.1 the double wall panels are discussed shortly in order to show the similarity of the two sound reducing techniques. Section 2.2 describes the effect of the acousto-elastic interaction for the sound absorbing materials.

2.1 Double wall panels

When a thin air layer is trapped between two elastic panels not only the acousto-elastic interaction becomes important, but also the viscothermal effects. These effects can result in quite high damping coefficients.

A model, which includes both effects, was developed [1] and implemented in B2000. The new viscothermal acoustic elements, **Q4.VISC** and **Q8.VISC**, have a constant pressure over the air layer. In the complete system continuity of the velocity at the interface is demanded. This results in a in a fully coupled, complex and frequency dependent finite element formulation.

$$-\omega^2\begin{bmatrix} M_s & 0 \\ M_c(\omega) & M_a(\omega) \end{bmatrix}\begin{Bmatrix} U \\ P \end{Bmatrix}+\begin{bmatrix} K_s & -K_c \\ 0 & K_a \end{bmatrix}\begin{Bmatrix} U \\ P \end{Bmatrix}=\begin{Bmatrix} F_u \\ F_p \end{Bmatrix} \tag{2.1}$$

With ω the angular frequency, {U} the vector with structural degrees of freedom and {P} the vector with acoustic pressures. M_s and K_s are the structural mass and stiffness matrices, $M_a(\omega)$ and K_a are the acoustic mass and stiffness matrices. M_c and K_c are the coupling matrices. The excitation forces are on the right-hand side.

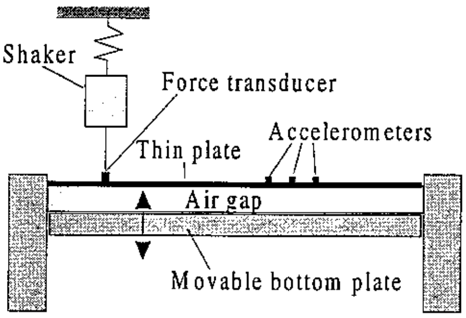

Figure 1. Acousto-elastic interaction of a thin elastic plate and an air gap.

The model was validated with experiments (see Figure 1). The results showed a good agreement. As a next step the transmission loss of a double wall panel was investigated. The effect of the viscosity on the transmission loss becomes especially important at the eigenfrequencies of the panels [2,3]. For the double wall panel also the energy losses are calculated. The results of these calculations will be checked with an experimental set-up where the radiated sound energy will be measured with a sound intensity probe. The optimisation of the light weight double panels for specific applications is the next step.

2.2 Sound absorbing materials

Another technique to reduce the amount of noise is the use of sound absorbing materials. However it is difficult to predict the sound level or amount of sound absorption if such material is present in a room. The main reason for this is that an accurate description of the behaviour of sound absorbing material is not an easy task. In the next section the finite elements in B2000 to describe sound absorbing material are presented.

2.2.1 FEM elements in B2000

Roughly there are two ways to describe a sound absorbing material: via a boundary condition at the surface or as a volume. Both methods are well suited for implementation in a Finite Element Formulation.

The surface method is usually applied in the form of a impedance condition. The perturbations of the pressure p and the normal velocity v_n are related through the complex normal impedance.

$$Z_n = \frac{p}{v_n} \tag{2.2}$$

On the basis of the interface elements of B2000 (Q4.INT and Q16.INT [4]) the surface impedance elements **Q4.IMP** and **Q8.IMP** were implemented [5]. This results in an a-symmetric set of equations and because the impedance is a complex quantity the system of matrices becomes complex as well. Furthermore the system is frequency dependent and has to be solved in an iterative way or by a direct harmonic response calculation.

The impedance model provides a fast numerical method, compared to the volume description. However, each time measurements are needed to determine the normal impedance values as a function of the frequency if for instance the thickness of the material is varied. This is usually done for normal incident waves in an impedance tube (see Figure 2).

In the volume method there is an interaction between the frame and the fluid inside the sound absorbing material. A distinction can be made in increasing order of complexity: the 'Rigid' theory treats the frame as infinitely stiff, the 'Limp' theory treats the frame without stiffness and the 'Biot' theory includes the stiffness of the frame and the acousto-elastic coupling. Table 1 gives an overview of the volume descriptions.

	Rigid	Limp	Biot
Approach	Volume	Volume	Volume
Frame	Infinitely stiff (no mass)	No stiffness (but with mass)	Poisson's ratio Young's modulus
Mechanisms			
Inertia of fluid	√	√	√
Inertia of frame		√	√
Viscous drag	√	√	√
Thermal effects			√
Wave types	1x dilatational	1x dilatational	2x dilatational 1x shear

Table 1. Comparison of models for sound absorbing material.

From Table 1 can be seen that only in the Biot theory three types of waves can propagate. This is the result of the extra degrees of freedom for the frame. In the Rigid and Limp theories the pressure P for the fluid is the only degree of freedom. Measurements have indicated that indeed three types of waves propagate in materials with an elastic frame and a fluid phase [6]. Although the Biot theory is the most complete the computational effort and the variety of parameters to describe the material are drawbacks.

For the Limp theory the global system of matrices is [5]

$$\left[-\omega^2 f(\omega)\left[M_{Limp}\right] + \left[K_{Limp}\right]\right]\{P\}=\{F\} \tag{2.3}$$

With $[M_{Limp}]$ and $[K_{Limp}]$ the 'mass' and 'stiffness' matrices of ordinary acoustic elements and f(ω) a complex, frequency dependent function

$$f(\omega)=Ks-\frac{i\phi h}{\omega\rho_0}\bigg/\left(1-\frac{i\phi h}{\omega(1-h)\rho_s}\right) \tag{2.4}$$

In (2.4) Ks is a structure factor which represents the tortuosity of the material. It is often close to 1. ϕ is the flow resistivity for the fluid in the frame, h is the porosity (ratio of the volume of the fluid to total volume) and ρ_0 and $(1-h)\rho_s$ the mean densities of the fluid and frame respectively. For the Rigid theory the density of the frame can be chosen infinitely high in equation (2.4).

The Limp theory and the coupling with structural elements were implemented in B2000 in the form of the **HE8.LIMP**, **HE20.LIMP** and **Q16.INT.IMP** elements. However the coupling with conventional acoustic elements has still to be done and requires an interface element for the normal velocities as well as a global transition matrix to couple the pressures.

For the Biot theory the finite element description of the **HE8.BIOT** element is given in section 3.

2.2.2 Alternative Biot theory

Another way of describing sound absorbing material, slightly similar to the theory of Biot, is a description of the behaviour of the fluid in the cylindrical pores. In such a pore viscous, thermal and mass effects are present [7].

To start with a simple sound absorbing material, perforated samples are used. An advantage of such samples is that complicated parameters, for example a structure factor or the tortuosity, are not present. This results in an accurate repeatability of the measurements. The important parameters for the perforations in the samples are

- *radius*, influences the amount of viscous losses
- *length*, determines the frequency range
- *surface porosity*, optimises the absorption

Samples of such perforated material are placed in an impedance tube (see Figure 2). The transfer function between the two microphones is used to calculate the impedance and absorption coefficient α of the sample as a function of the frequency. Variations of the radius, the length and the surface porosity are investigated numerically and experimentally [8,9]. The results show a good agreement for the complete frequency range (see Figure 3).

The tube elements were also implemented in B2000 as **T2.VISC** and **T3.VISC** [1]. However for simple cases the analytical equations are more efficient.

As a next step, oblique incident sound will be investigated as well as an elastic frame to account for the acousto-elastic coupling.

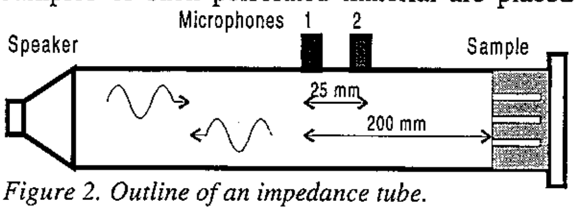

Figure 2. Outline of an impedance tube.

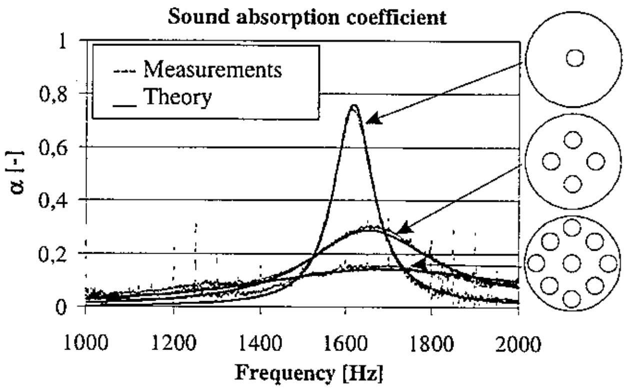

Figure 3. Sound absorption for different surface porosities.

3 Implementation of the Biot theory in B2000

Biot formulated his equations for sound absorbing material as a function of the displacements of the elastic frame **u** and the displacements of the fluid inside **U**. Figure 4 is presented for a schematic view and suggests an an-isotropic behaviour. However Biot's theory is isotropic.

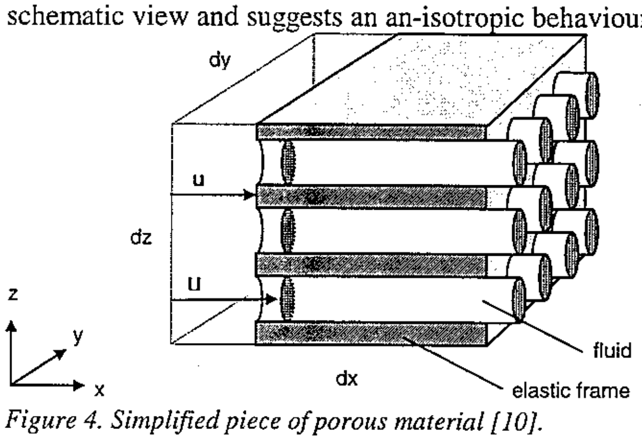

Figure 4. Simplified piece of porous material [10].

Through the stress-strain relations and Lagrange's equations Biot's theory becomes

$$N\nabla^2\mathbf{u}+(A+N)\nabla(\nabla\cdot\mathbf{u})+Q\nabla(\nabla\cdot\mathbf{U})=\frac{\partial^2}{\partial t^2}(\rho_{11}\mathbf{u}+\rho_{12}\mathbf{U})+b\frac{\partial}{\partial t}(\mathbf{u}-\mathbf{U}) \tag{3.1}$$
$$Q\nabla(\nabla\cdot\mathbf{u})+R\nabla(\nabla\cdot\mathbf{U})=\frac{\partial^2}{\partial t^2}(\rho_{12}\mathbf{u}+\rho_{22}\mathbf{U})-b\frac{\partial}{\partial t}(\mathbf{u}-\mathbf{U})$$

In (3.1) the *elastic coefficients* are *N*, *A*, *Q* and *R*. They can be seen as the equivalent elastic constants in case the frame is consisted of a homogeneous medium. *N* is the shear modulus (one of the constants of Lamé) of the elastic frame which can be measured in a vacuum. The others are defined as follows

$$A=K_b-\frac{2}{3}N+\frac{(1-h)^2}{h}K_f \tag{3.2}$$
$$Q=(1-h)K_f$$
$$R=hK_f$$

With h the porosity, K_f the bulk modulus of the fluid and K_b the bulk modulus of the elastic frame. The Poisson's ratio v_b of the frame can also be measured in a vacuum.

$$K_b=\frac{2(1+v_b)}{3(1-2v_b)}N \tag{3.3}$$

Notice the resemblance of the elastic coefficients with Hooke's law. '*A*' corresponds to one of the constants of Lamé with an extra term for the stress caused by the fluid. The coefficient Q represents the coupling.

Biot's *dynamic coefficients* for the inertia forces are

$$\rho_{11}=(1-h)\rho_s+\rho_a \tag{3.4}$$
$$\rho_{12}=-\rho_a$$
$$\rho_{22}=h\rho_f+\rho_a$$

With ρ_a a mass coupling factor due to the forced movement of the fluid in the pores.

The viscous effects are accounted for in the viscous factor *b*.

$$b=\phi\, h^2 F(\omega) \tag{3.5}$$

It is a function of the flow resistivity ϕ and the porosity. For higher freqencies the factor is adapted by the function $F(\omega)$ because the assumption of a Poiseuille flow breaks down for higher frequencies.

A finite element formulation of Biot's theory results in 6 degrees of freedom per node, which is rather large. Furthermore the formulation in terms of displacements suffers from spurious modes in modal calculations. Therefore the equations were rewritten in terms of the pressure p for the fluid [11,12] which results in 4 degrees of freedom per node. A small harmonic fluctuation of the displacements and pressure is assumed.

$$N\nabla^2\mathrm{u}+(N+k_{ps})\nabla(\nabla\cdot\mathrm{u})+\left(k_{pspf1}+k_{pspf2}\right)\nabla p+\left(\omega^2 m_{ps}+i\omega c_{ps}\right)\mathrm{u}=0 \tag{3.6}$$
$$\nabla^2 p+\left(\omega^2 m_{pf}+i\omega c_{pf}\right)p+\left(\omega^2 m_{pfps}+i\omega c_{pfps}\right)\nabla\cdot\mathrm{u}=0$$

The mass, damping and stiffness coefficients are defined as

$$m_{ps}=\frac{\omega^2(\rho_{11}\rho_{22}-\rho_{12}{}^2)}{\omega^2\rho_{22}+i\omega b}\qquad c_{ps}=\frac{\omega^2\rho b}{\omega^2\rho_{22}+i\omega b}\qquad k_{ps}=A-\frac{1-h}{h}Q \tag{3.7}$$

$$m_{pfps}=\frac{1-h}{h^2}\rho_{22}-\frac{\rho_{12}}{h}\qquad c_{pfps}=b\left(\frac{1-h}{h^2}+\frac{1}{h}\right)\qquad k_{pspf1}=-(1-h)$$

$$m_{pf}=\frac{\rho_{22}}{R}\qquad c_{pf}=\frac{b}{R}\qquad k_{pspf2}=h\frac{\omega^2\rho_{12}-i\omega b}{\omega^2\rho_{22}+i\omega b}$$

With $\rho=\rho_{11}+\rho_{12}+\rho_{22}$ and the subscripts '*ps*' for the solid part of the porous material, '*pf*' for the fluid part and '*pfps*' for the coupling terms.

3.1 Finite element formulation

A weak formulation of equation (3.6) results in the following system of matrices for a porous material (with a $e^{-i\omega t}$ convention). The boundary conditions are not considered in detail here.

$$\left(-\omega^2\begin{bmatrix}M_{ps} & 0\\ M_{pfps} & M_{pf}\end{bmatrix}-i\omega\begin{bmatrix}C_{ps} & 0\\ C_{pfps} & C_{pf}\end{bmatrix}+\begin{bmatrix}K_{ps} & K_{pspf}\\ 0 & K_{pf}\end{bmatrix}\right)\begin{Bmatrix}U_{ps}\\ P_{pf}\end{Bmatrix}=\begin{Bmatrix}F_{ps}\\ F_{pf}\end{Bmatrix} \tag{3.8}$$

As could already be seen from (3.7) the 'mass', 'damping' and 'stiffness' matrices are complex and frequency dependent. The whole system is a-symmetric as well. In the present version of B2000 a damping matrix cannot be used separately. Therefore the [C] matrix is included in the mass matrix [M] for the **HE8.BIOT** element because of the similarity between these two matrices.

3.2 Test results

As a first check the **HE8.BIOT** element was tested for two extreme cases. If the porosity is one the porous material behaves as ordinary air and can be compared to the **HE8.ACOU** element. The results for a slender tube filled with air are presented in Table 2.

Mode	Eigenfrequency (Hz) Analytical	ACOU and LIMP (ϕ=0) and BIOT (ϕ=0, h=1, ρ_a=0)
1	149.13	149.14
2	298.26	298.30
3	447.39	447.52
4	596.52	596.82
5	745.65	746.23

Table 2. Eigenfrequencies of a closed tube filled with air (length=1.15 m, c_0=343 ms^{-1}, 115 elements).

If the porosity is zero and the parameters for a solid material are used in the Biot element the axial eigenfrequencies of a clamped slender solid rod can be calculated and compared to the analytical values ($f_n=n/2\ \sqrt{(E/\rho_s L^2)}$, $n=1,2,\ldots$).

Mode	Eigenfrequency (Hz) Analytical	BIOT, solid
1	2109.01	2113.17
2	4218.01	4226.62
3	6327.02	6340.62
4	8436.02	8455.46
5	10545.03	10571.41

Table 3. Axial eigenfrequencies of a solid rod (length=1.15 m, E=6.8 10^{10} Nm^{-2}, ρ_s=2890 kgm^{-3}, ν_b=0.3, 115 elements).

The comparison of the Biot theory to the Limp theory is not complete at this moment. For an impression of the complex eigenfrequencies the values for the Limp case are listed in Table 4.

Mode	Eigenfrequency (Hz) Limp
1	95.15 + 0.56i
2	190.3 + 2.27i
3	285.6 + 5.11i
4	381.1 + 9.08i
5	476.7 + 14.2i

Table 4 Eigenfrequencies of a tube filled with porous material (h=0.97, ϕ=20000 Nsm^{-4}, ρ_0=1.19 kgm^{-3}, ρ_s=30 kgm^{-3}, c_o=290 ms^{-1}.

As a next step the influence of the stiffness of the frame was investigated through a variation of the shear modulus N. This was done by using the analytical equations for a one dimensional case. Figure 5 shows a frequency response function of a tube filled with porous material.

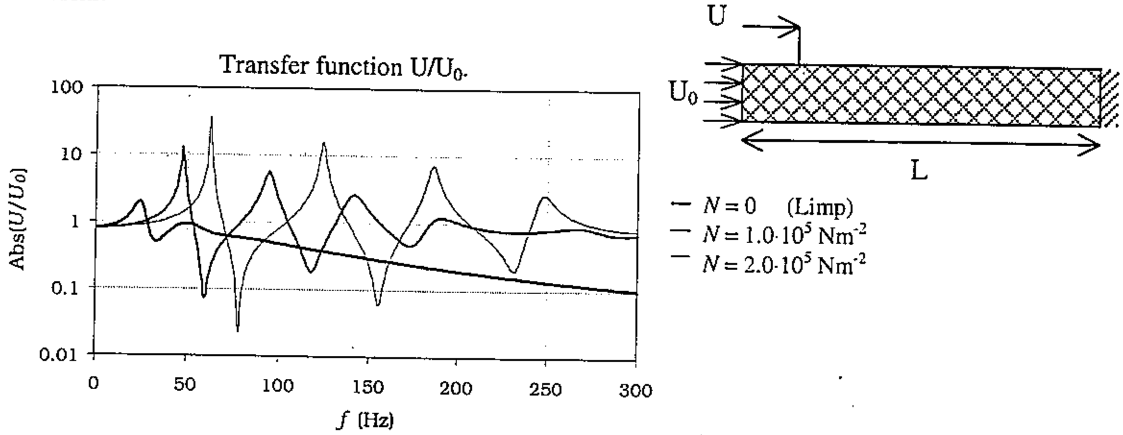

Figure 5. Effect of variation of the shear modulus N.

The damping is clearly present in Figure 5. The presence of the elasticity of the frame results in more eigenfrequencies in this frequency range.

4 Conclusions and Further Research

Acousto-elastic interaction plays an important role in sound reduction. This was seen in the double wall panels and the sound absorbing materials. An overview of the elements in B2000 to model sound absorbing material was presented and a new element, on the basis of the Biot theory, was introduced. The Biot theory contains the acousto-elastic interaction of the elastic frame (skeleton) and the fluid inside and therefore three types of waves can propagate in the material. As a rather complete model the Biot theory is very interesting, but there are a lot of parameters which do not have a clear physical meaning. Moreover, the computational effort is large.

The equations of Biot were written in terms of displacements for the frame and pressure for the fluid to reduce the number of degrees of freedom and to avoid spurious modes. The first results showed two extreme cases for the porous material, i.e. the material was compared to a fluid and to solid material. The comparison with the existing Limp elements is not ready yet, but the influence of the stiffness of the frame was indicated already.

Future work involves:

- Completion of the tests for the Biot element. A comparison with: Limp elements, results from the literature and analytical results.
- The calculation of the frequency dependent factors for the element matrices only once for more efficiency.
- The more conventional **u-U** formulation can be implemented without much work to compare the results with the **u**-p formulation. The effect of the spurious modes can also be investigated.
- The development of an alternative model on the basis of the viscothermal wave propagation of the fluid (see section 2.2.2).

Acknowledgement

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