The Use of Modal Derivatives in Determining Stroke-Dependent Frequencies of Large Stroke Flexure Hinges

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Abstract

Nowadays, a lot of use is made of large stroke flexure hinges in precision engineering. However, these large stroke flexure hinges typically lose stiffness in supporting direction during deflection. The lowest natural frequency is a commonly used measure for this property. Therefore, in shape and topology optimization, the decrease of this first parasitic frequency is often minimized. These optimizations are typically very time consuming, due to the large number of design evaluations. In this paper, a method is presented for determining stroke-dependent frequencies of large stroke flexure hinges. This method makes use of derivatives of mode shapes with respect to modal coordinates. Therefore, geometrical nonlinearities can be taken into account. Using these modal derivatives, frequency derivatives can be determined, making it possible to determine natural frequencies for any given deflection without having to linearize for every load step. For demonstration, the method is used to determine the first parasitic frequency of a single leaf spring as a function of the deflection. The results show that the decrease of this parasitic frequency has the shape of a bell-shaped curve, as commonly described in literature and found in experiments.

Keywords: modal derivatives, frequency derivatives, floating frame of reference, large stroke flexure hinges

1. Introduction

Flexure hinges are frequently used in precision engineering for their deterministic behavior, due to the absence of friction, hysteresis and backlash [1]. For their common applications, flexure hinges are compliant in driving directions, while constraining motion in other directions. Figure 1 illustrates this concept for a single leaf spring. This property requires a high support stiffness throughout the entire range of motion. A commonly used measure for the support stiffness of a flexure hinge, and therefore the performance, is its first parasitic frequency, i.e. the lowest natural frequency in the support direction. Because the support stiffness typically decreases rapidly with deflection, flexure hinges tend to have a reduced performance in their deflected state [1].

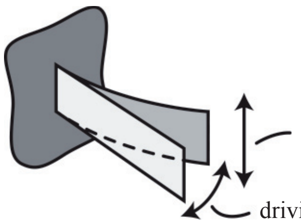

Figure 1: Single leaf spring flexure.

From literature [1,2], we know that the decrease of the first parasitic frequency, and therefore the stiffness in support direction, is typically described by a bell-shaped curve as shown in Figure 2. This figure shows a comparison between results from software package SPACAR and results from FEM for a flexure hinge. This means that already for small deflections, the stiffness decreases significantly. Recent development is aimed at the design of large stroke mechanisms for which this performance reduction is minimized. This would result in the bell-shaped curve to become flatter. Figure 3 shows the result of such an optimized large stroke flexure hinge, for which the performance is improved. For deflections of $\pm 45°$, the first parasitic frequency only drops approximately 25%.

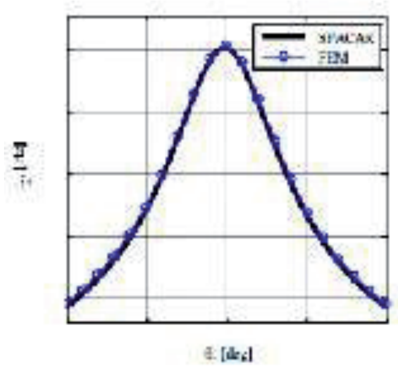

Figure 2: Natural frequency of a flexure hinge as a function of the angle of deflection, determined by SPACAR and finite element method (FEM) [2].

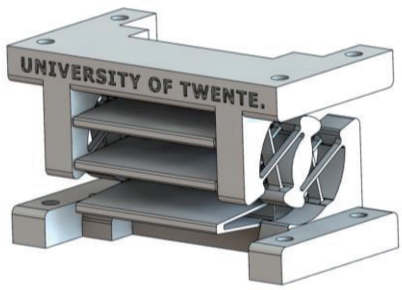

Figure 3: Large stroke flexure hinge for which performance reduction is minimized [3].

Shape and topology optimization are used to design these sophisticated mechanisms, for which large stroke flexure hinges are included in geometrically nonlinear multibody analyses. As these forms of optimization require many design evaluations, model order reduction (MOR) is used to reduce computational costs. In [4], Wu and Tiso present a MOR technique suitable for multibody systems in the floating frame of reference (FFR) formulation using modal derivatives (MDs). The MDs are presented as the derivatives of the mode shapes $\boldsymbol{\phi}_i$ with respect to the modal coordinates η_j.

In the FFR formulation, the configuration of a flexible body is written as a combination of its global rigid body motion and a local elastic displacement field. When applying MOR, this local displacement field $\mathbf{q}$ is expressed as a linear combination of a small number of mode shapes, e.g. Craig-Bampton modes: $\mathbf{q} = \boldsymbol{\Phi}\boldsymbol{\eta}$. The modal coordinates $\boldsymbol{\eta}$ describe how the mode shapes behave in time. Geometric nonlinear effects in the displacement field are taken into account by the MDs, which are static corrections on the mode shapes.

In this work, this MD-based technique is extended to determine stroke-dependent natural frequencies. To this end, the frequency derivatives (FDs) are introduced as the derivatives of the natural frequencies squared ω_i^2 with respect to the modal coordinates η_j. The derivations of both MDs and FDs are presented in chapter 2. Using the FDs, the natural frequencies for any given configuration can be determined. For demonstration purposes, this method is used to determine the parasitic frequency as a function of the deflection in driving direction for the single leaf spring shown in Figure 1, using 3D beam elements. Chapter 3 shows this example. Finally, the conclusions are presented in chapter 4.

2. Method

In order to derive MDs, the full nonlinear Green-Lagrange strain expression [5] is taken into account:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_j}\frac{\partial u_k}{\partial x_i}\right). \quad (1)$$

Using this expression, the configuration dependent stiffness matrix can be determined by differentiating the strain energy twice with respect to the generalized coordinates:

$$\mathbf{K}(\mathbf{q}) = \frac{\partial^2 U}{\partial \mathbf{q} \partial \mathbf{q}^T}. \quad (2)$$

To this end, the strain energy [5] is expressed as:

$$U = \frac{1}{2}\int_V \boldsymbol{\varepsilon}^T \boldsymbol{\sigma} dV, \quad (3)$$

in which $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the stress and strain vector, respectively, including all terms of the stress and strain matrix. The eigenvalue problem for free vibrations around the undeformed equilibrium configuration is then given as:

$$(\mathbf{K} - \omega_i^2\mathbf{M})\boldsymbol{\phi}_i = \mathbf{0}, \tag{4}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrix, respectively, and $\boldsymbol{\phi}_i$ is the natural mode shape corresponding to natural frequency ω_i.

Differentiation of the eigenvalue problem with respect to the modal coordinates η_j yields:

$$\left(\frac{\partial\mathbf{K}}{\partial\eta_j} - \gamma_{ij}\mathbf{M}\right)\boldsymbol{\phi}_i + (\mathbf{K} - \omega_i^2\mathbf{M})\boldsymbol{\theta}_{ij} = \mathbf{0}, \tag{5}$$

where $\boldsymbol{\theta}_{ij}$ are the MDs, defined as the derivate of $\boldsymbol{\phi}_i$ with respect to η_j, and γ_{ij} are the FDs, defined as the derivatives of ω_i^2 with respect to η_j.

The inertia terms in (5) can be neglected [6], therefore the expression for the modal derivatives yields:

$$\boldsymbol{\theta}_{ij} = -\mathbf{K}^{-1}\frac{\partial\mathbf{K}}{\partial\eta_j}\boldsymbol{\phi}_i. \tag{6}$$

Since the MDs are now known, it is possible to determine the system's mode shapes in its deflected state. The local elastic displacement field is now written as a combination of mode shapes and modal derivatives [7]:

$$\mathbf{q} = \boldsymbol{\phi}_i\eta_j + \boldsymbol{\theta}_{ij}\eta_i\eta_j. \tag{7}$$

From (5), the FDs can be derived. In order to obtain a scalar equation, (5) is pre-multiplied with $\boldsymbol{\phi}_i^T$:

$$\boldsymbol{\phi}_i^T\left(\frac{\partial\mathbf{K}}{\partial\eta_j} - \gamma_{ij}\mathbf{M}\right)\boldsymbol{\phi}_i + \boldsymbol{\phi}_i^T(\mathbf{K} - \omega_i^2\mathbf{M})\boldsymbol{\theta}_{ij} = \mathbf{0}. \tag{8}$$

The FDs can then be solved as:

$$\gamma_{ij} = \frac{\boldsymbol{\phi}_i^T\frac{\partial\mathbf{K}}{\partial\eta_j}\boldsymbol{\phi}_i + \boldsymbol{\phi}_i^T(\mathbf{K} - \omega_i^2\mathbf{M})\boldsymbol{\theta}_{ij}}{\boldsymbol{\phi}_i^T\mathbf{M}\boldsymbol{\phi}_i^T}. \tag{9}$$

The natural frequencies can now be written as a combination of the natural frequencies in undeformed equilibrium configuration and frequency derivatives:

$$\omega_i^2 = (\omega_i)_0^2 + \gamma_{ij}\eta_j. \tag{10}$$

3. Results

To demonstrate the method presented in chapter 2, the 3D example of the single leaf spring in Figure 1 is analyzed using ten 3D beam elements. For numerical computations, the leaf spring is given length 0.1 m, height 0.04 m and thickness 0.0005 m. The material is given the following properties: Young's modulus 210 GPa, shear modulus 79 GPa and density 7800 kg/m^3. The first natural frequency of the system corresponds to a bending mode shape in the driving direction. When the system is deflected in this first mode, the natural frequency in the support direction is determined using the method as explained in chapter 2. The system is deflected until the angle of the end node reaches $\pm45°$.

Figure 4 shows the decrease of this parasitic frequency. It can be seen that already for relatively small deflections, the reduction in performance is indeed significant: for a deflection of $\pm10°$, the decrease is already near 50%. The results match the typical bell-shaped curve as shown in Figure 2 and presented in literature [1,2]. This means that the behavior of the natural frequencies can indeed be predicted in a reliable way using FDs.

In current optimization processes, software packages such as SPACAR are used. SPACAR uses a flexible multibody approach with nonlinear beam elements, which can also include large deformations. A detailed description of the formulation is found in [8]. To produce similar results, SPACAR needs to solve a linearized eigenvalue problem for each incremental step in the deflection. The method presented here does not require this, which makes it attractive to apply on computationally expensive processes such as shape and topology optimizations.

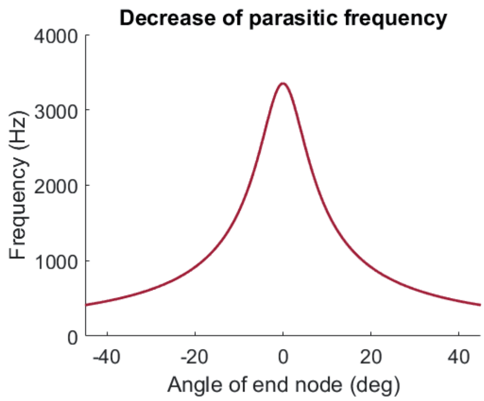

Figure 4: Decrease of parasitic frequency of a single leaf spring for increasing deflection in the first mode shape.

4. Conclusions

This paper presents a new method to determine stroke-dependent natural frequencies during large deflections, based on a model order reduction technique using modal derivatives as earlier presented by Wu and Tiso in [4]. Both the modal derivatives and frequency derivatives are determined by differentiating the eigenvalue problem for free vibrations with respect to the modal coordinates.

The frequency derivatives are used to describe the stroke-dependent natural frequencies as a combination of natural frequencies in undeformed equilibrium configuration and frequency derivatives. This method is applied on a 3D example of a single leaf spring and has proven to give realistic results, as the behavior of the first parasitic frequency was found to be similar to the typical bell-shaped decrease of the first parasitic frequency as presented in literature before.

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