IMA Journal of Mathematical Control and Information (2020) **00**, 1–23
doi:10.1093/imamci/dnaa018
f.califano@utwente.nl

Twenty years of distributed port-Hamiltonian systems: a literature review

RAMY RASHAD* AND FEDERICO CALIFANO
Robotics and Mechatronics Group, University of Twente, Enschede, The Netherlands
*Corresponding author: ramy.abdelmonem@gmail.com f.califano@utwente.nl

ARJAN J. VAN DER SCHAFT
Bernoulli Institute for Mathematics, Computer Science and AI, University of Groningen, The Netherlands
a.j.van.der.schaft@rug.nl

AND

STEFANO STRAMIGIOLI
Robotics and Mechatronics group, University of Twente, Enschede, The Netherlands, and ITMO University, Saint Petersburg, Russia
s.stramigioli@utwente.nl

[Received on 15 November 2019; revised on 1 April 2020; accepted on 2 July 2020]

The port-Hamiltonian (pH) theory for distributed parameter systems has developed greatly in the past two decades. The theory has been successfully extended from finite-dimensional to infinite-dimensional systems through a lot of research efforts. This article collects the different research studies carried out for distributed pH systems. We classify over a hundred and fifty studies based on different research focuses ranging from modeling, discretization, control and theoretical foundations. This literature review highlights the wide applicability of the pH systems theory to complex systems with multi-physical domains using the same tools and language. We also supplement this article with a bibliographical database including all papers reviewed in this paper classified in their respective groups.

Keywords: port-Hamiltonian theory; distributed parameter systems; multi-physical systems; energy-based control; spatial discretization.

1. Introduction

The port-Hamiltonian (pH) systems paradigm is a framework for the modeling, analysis, design and control of complex dynamical systems. The mentioned complexity arises due to multi-physical domains, inter-domain couplings, nonlinearities and distributed sensing and actuation. The framework's key features include the emphasis on power flow between subsystems, the separation of the interconnection structure of the system from its components' constitutive relations and the exploitation of this separation in the analysis and control of the system. The pH system theory has been successfully extended from the lumped parameter (finite-dimensional) systems to distributed parameter (infinite-dimensional) systems in the seminal work of van der Schaft & Maschke (2002).

For the past two decades, the distributed port-Hamiltonian (dpH) systems theory underwent a huge evolution that branched in different directions and is still an active area of research. The first research direction in the literature was in implementing the generic dpH theory for modeling distributed parameter systems in different physical domains. The second direction was the analysis and design

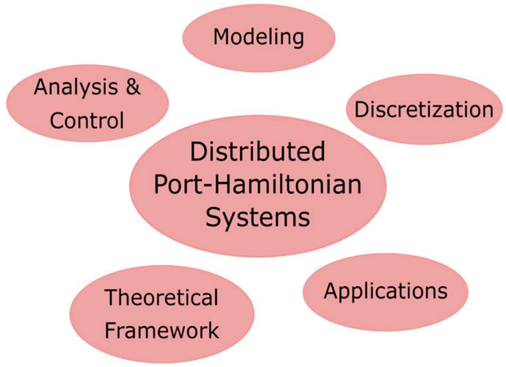


FIG. 1. The classification of research studies surveyed in this paper.

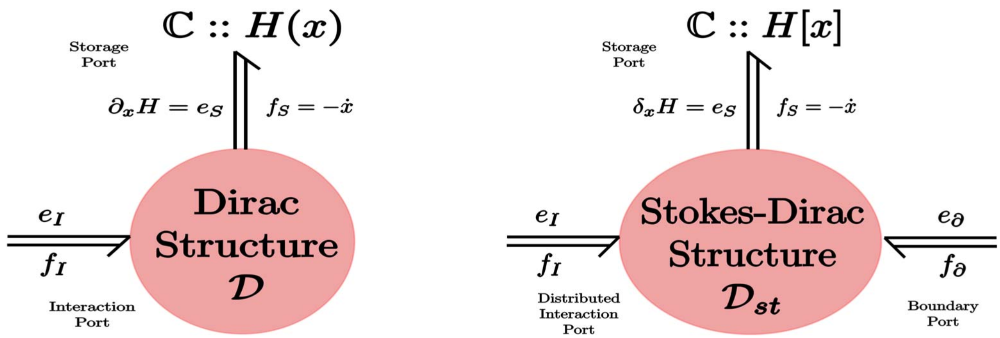
of control techniques suitable for pH models by extending the well-established techniques for finite-dimensional pH systems, like energy shaping and damping injection. Another research direction was focused on generating finite dimensional approximations of the dpH models or the infinite-dimensional controllers by structure-preserving discretization methods. Finally, there has been also a lot of research efforts in different theoretical formulations of pH systems and extensions to the original framework in van der Schaft & Maschke (2002).

In this article we present a review of the research studies carried out in the past 20 years (approximately) relying on the work of van der Schaft & Maschke (2002). We classify over 150 studies into different classes depending on their respective research focus, shown in Fig. 1. The research directions considered include modeling, analysis and control, discretization, theoretical framework and applications. The last class includes papers that combined other different techniques.

We aim with this literature review to show the wide applicability of the pH theory to complex systems in multi-physical domains. We also highlight one of the key benefits of the pH paradigm which is modeling a wide range of systems in different fields with the same tools and language at all stages, starting from the theoretical modeling to the practical implementation. This aforementioned advantage can be clearly observed in the *applications* class. Moreover, we highlight the main methodologies and techniques developed in the different subfields of pH system theory, which helps identifying the existing gaps. Finally, we provide in supplement to this article a bibliographical database[1] that includes the papers reviewed in this article, classified in their respective groups. We hope this could guide new researchers in the field and accelerate the research and development of this powerful paradigm.

The rest of the paper is organized as follows: we first start in Section 2 by a summary of the pH formulation of van der Schaft & Maschke (2002), followed by a discussion of the other studies in the literature related to that *theoretical framework* In Section 3, we present the *modeling* class that includes the implementation of the framework to a wide range of physical domains. In Section 4, we present the *analysis and control* class followed by the *discretization* class in Section 5. Finally, we conclude the article in Section 6.

[1] http://dx.doi.org/10.17632/wz6h2xpvg9.1



(a) Finite-dimensional pH system (b) Infinite-dimensional pH system

FIG. 2. pH systems bond graph representations.

2. Theoretical Framework

In this section we provide an overview of the different formulations of dpH systems introduced in the literature. We stress that this classification is not so neat in the development of the various frameworks that have followed in the past 20 years and many sub-formulations valid for different classes of systems are present in the literature. Indeed we are conscious of the arbitrariness of the proposed macro-classifications of the dpH formulations, that we choose in order to be pedagogical with the reader, in the sense of dividing the formulations in a functional, and not chronological way.

We recognize and briefly address three main formulations of dpH systems: the *Stokes–Dirac structure approach*, the *functional analytic approach* and the *jet bundle approach*. Then we present a collection of different results extending these approaches in different directions.

2.1. *Stokes–Dirac structure formulation*

As previously mentioned, a key characteristic of the pH paradigm is the modeling of a physical system ensuring the separation of the interconnection-structure and the constitutive relations of the system's components linked together using the concept of ports. A port is defined by a pair (f, e) of a flow and an effort, which are power conjugated.

In the lumped parameter case, shown in Fig. 2a, the aforementioned power-continuous interconnection structure is mathematically modeled by a Dirac structure $\mathscr{D}$. A finite-dimensional pH system is given by the tuple $(\mathscr{X}, H, \mathscr{I}, \mathscr{D})$, where the state space $\mathscr{X}$ is a finite-dimensional manifold, and the Hamiltonian $H : \mathscr{X} \to \mathbb{R}$ is a function of the state x. The pair $\mathscr{X}$ and H define the energy storage port of the system $(f_S, e_S) = (-\dot{x}, \partial_x H)$, where $\dot{x}$ denotes the time derivative of x and $\partial_x H$ denotes the partial derivative of H with respect to x. The interaction port $(f_I, e_I) \in \mathscr{I}$ is a general port that can be used to incorporate all other effects like internal energy dissipation, external sources and external interaction with the environment. The absence of the interaction port is what identifies classical Hamiltonian systems, in which the underlying mathematical object is a Poisson structure. The implicit pH system dynamics and energy balance are then given by

$$(-\dot{x}, \partial_x H, f_I, e_I) \in \mathscr{D}, \qquad \dot{H} = e_I^\top f_I. \tag{2.1}$$

In the foundational work of van der Schaft & Maschke (2002), the power-continuous interconnection structure of distributed parameter systems was mathematically modeled by a Stokes–Dirac structure (SDS) denoted by $\mathscr{D}_{st}$, shown in Fig. 2b. A dpH system is given by the tuple $(M, \mathscr{X}, H, \mathscr{I}, \partial, \mathscr{D}_{st})$, where M is the finite-dimensional manifold representing the spatial domain, with boundary ∂M. The state space $\mathscr{X}$ is composed by a subspace of differential forms over M. The space ∂ is a subspace of differential forms over ∂M. Finally $H : \mathscr{X} \to \mathbb{R}$ is a functional of the state x. The energy storage port of the system is defined by $(f_S, e_S) = (-\dot{x}, \delta_x H)$, where $\delta_x H$ denotes the variational derivative of the functional H with respect to x. Energy-flow through the boundary into the system is modeled by utilizing Stokes' theorem which defines the boundary port variables $(f_\partial, e_\partial) \in \partial$ used to specify the boundary conditions freely. Distributed port-variables, that allow for energy flow within the spatial domain, can also be included through the interaction port $(f_I, e_I) \in \mathscr{I}$. The implicit pH system dynamics and energy balance are then given by

$$(-\dot{x}, \delta_x H, f_I, e_I, f_\partial, e_\partial) \in \mathscr{D}_{st}, \qquad \dot{H} = \int_{\partial M} e_\partial \wedge f_\partial + \int_M e_I \wedge f_I. \tag{2.2}$$

After the seminal work of van der Schaft & Maschke (2002) some extensions of dpH formulations were carried out. The work in Macchelli *et al.* (2004b) was motivated by the need to find a clearer formulation for infinite-dimensional pH systems, not relying on the definition of the SDS introduced in van der Schaft & Maschke (2002), which was introduced as fundamental building block, but whose generalization for more complex systems was not provided. In particular the definition of a novel class of the SDS was formalized by a proper generalization of constant (i.e. not depending on the state variables) Dirac structures of finite-dimensional pH systems. This has been done by generalizing the matrix operators (representing interconnection, damping and interaction) of finite-dimensional pH systems, to multivariable differential operators suitable for the infinite-dimensional case. The key result is the constructive definition of a SDS starting from a formally skew-adjoint operator acting on vector-valued smooth functions. This represents a further extension with respect to van der Schaft & Maschke (2002) where only scalar-valued smooth functions on the spatial manifold were described as zero differential forms. This generalization is referred to as *multi-variable dpH system* in Macchelli *et al.* (2004b). Using Stokes theorem, a linear operator induced on the spatial boundary is formally introduced and depends on the skew-adjoint operator. This operator-based approach has the advantage to describe broader classes of PDEs with respect to the original formulation[2] , e.g. the controversial example of the *heat equation* is described as dpH system in Macchelli *et al.* (2004b) by introducing a self-adjoint differential operator representing distributed dissipation for the diffusive process.

2.2. *Functional analytic formulation*

In the following we describe the field that put together concepts coming from functional analysis and dpH systems. Actually, the class of dpH systems that are considered in this framework is very small if compared to the general class presented in van der Schaft & Maschke (2002) and therefore strictly speaking it should not be considered as a separate formulation, but as a subclass. Nevertheless the contributions and the connections with the PDE community have been of such importance and impact, that we consider it valuable to keep this distinction.

[2] Nevetherless some systems (e.g. n-dimensional fluid dynamic equation) can not be described by such operator due to their non-constant SDS.

The seminal work for this approach is Le Gorrec *et al.* (2005), which had tremendous impact on the future research and represents one of the most important contributions for dpH systems. The framework introduced in this paper describes the connection between Dirac structures, Boundary Control Systems (BCS) and skew adjoint-operators using functional analytic tools. This framework would allow to handle rigorously *well-posedness* of the dpH system, in the sense of existence and smoothness of solutions[3] . In principle this step is necessary to make a control design based on Lyapunov-like arguments, since variations of energy have to be computed *along solutions*, for which these existence and smoothness properties must be known. The price to pay to achieve such a level of mathematical rigour is to diminish dramatically the class of considered systems to linear, one-dimensional spatial domain. The whole approach is based on the definition of the operator $\mathscr{J}$

$$\mathscr{J}e = \sum_{i=0}^{N} P(i)\frac{d^i e}{dz^i}(z), \qquad z \in [a,b], \tag{2.3}$$

where conditions on matrices $P(i)$ are explicitly assumed in order to make the operator formally skew-adjoint. In this context, Dirac structures have been defined on Hilbert spaces and consequently state variables are not more identified with differential 1-forms as in van der Schaft & Maschke (2002), but with vector-valued L^2 functions. The port variables living on the boundary of the spatial domain (introduced by means of Stokes theorem like in the previous approach) are included in the SDS and parametrized in such a way that the differential operator $\mathscr{J}$ restricted to a proper domain generates a contractive C_0- semigroup. Consequently, a rich description of these systems as BCS was possible, introducing input and output variables as specific linear combinations of state variables restricted on the boundary of the spatial domain. In particular, connection to system theoretic properties like *passivity* and more in general *dissipativity* are formalized.

The definition of dpH systems was done by introducing a quadratic energy functional $H(x) = \frac{1}{2}\langle x, \mathscr{L}(z)x\rangle$ with $\mathscr{L}(z)$ a coercive, bounded operator depending in general on the spatial variable z. In this situation the dpH can be abstractly written as

$$\dot{x} = \mathscr{J}\mathscr{L}x, \tag{2.4}$$

with the appropriate boundary conditions. Here the energy function is the norm of a Hilbert space induced by $\mathscr{L}$ itself. The structure of the system led to important results like the possibility of valuating properties which are complex and abstract in principle (e.g. generation of contraction semigroup) by means of simple matrix inequalities and the fact dpH systems of the considered class are dissipative with respect to quadratic supply rates, by considering energy as storage functional. Summarizing, this formulation bridged the more geometric and physical approach started in van der Schaft & Maschke (2002) with a more system and control-theoretic approach.

The contribution after Le Gorrec *et al.* (2005) grew fast in the relatively small system theoretic community working on the subject and in Section 4.4 we are briefly going through the main results developed in this framework in terms of both analysis and control.

[3] This is only a rough definition aiming at giving an insight of the problem to the reader. The rigorous definition of well-posedness, often not shared among authors, needs to be carefully addressed in the specific framework that is considered.

2.3. *Jet bundle formulation*

The SDS formulation of dpH systems in van der Schaft & Maschke (2002) relies on a choice of proper *energy variables* (efforts and flows) such that the Hamiltonian energy density does not depend on their spatial derivatives. However, this choice is not unique at all and in some cases there are convincing arguments that suggest different choices. An important example of this different way of thinking led to a formulation that is based on the jet-bundle structure of the distributed system (Ennsbrunner & Schlacher, 2005; Schöberl & Siuka, 2014). To give an exhaustive overview of this formulation is out of the scope of this literature review but we briefly highlight the main idea behind this approach, which makes use of differential geometric tools like jet bundles (see, e.g., Schlacher (2007) for an introduction to this topic). The main differences between the SDS formulation and the jet-bundle formulation is that in the former differential operators appear in the interconnection map and that the variational derivative of the Hamiltonian plays a different role. These differences are caused by a different choice of the independent variables for the system, sometimes called *evolutionary variables*, in contrast to the energy variables. This approach bridges theory of dPH systems with classical Hamiltonian field theory developed in physics. A detailed comparison of the two approaches can be found in Schöberl & Siuka (2013a) applied on the Mindlin plate as an example. Moreover, the jet-bundle formulation has been extended to second order field theories in Schöberl & Schlacher (2015a,b, 2018).

2.4. *Theoretical extensions*

There have been many efforts in the past two decades to mathematically explore and extend the fundamental building block in van der Schaft & Maschke (2002), which is the SDS. We briefly list them in this section, for the sake of completeness and in order to provide references for the interested reader.

In Vankerschaver *et al.* (2010), a method for systematically deriving the SDS for a given system was presented based on symmetry reduction of a canonical Dirac structure on the co-tangent bundle, i.e., the unreduced phase space. In Nishida *et al.* (2015, 2008a), the authors studied an extended SDS with distributed port variables, which makes it a boundary non-integrable structure. Moreover, the authors show that systems with distributed energy flow can be transformed to standard ones, without distributed energy flows. This allows the application of boundary control techniques to the transformed system. The work of Nishida *et al.* (2015, 2008a) was extended to manifolds with non-trivial topology in Nishida & Maschke (2018).

The pH formulation in van der Schaft & Maschke (2002) was defined on a fixed spatial domain. A relaxation of this assumption for the one-dimensional case was presented in Diagne & Maschke (2013), where the authors studied two systems coupled via a moving boundary interface.

A unified modeling procedure of dpH systems for field equations was presented in Nishida & Yamakita (2005) where a high-order SDS on variational complexes of jet bundles was used. This class of dpH systems is referred to as field port-Lagrangian systems (Nishida & Yamakita, 2005). It has been shown in Nishida *et al.* (2006) how field pL systems can be derived systematically from conservation laws with a variational symmetry. Moreover, the authors of Nishida *et al.* (2006) present a strategy for observing and detecting the breaking of the variational symmetry for field port-Lagrangian systems. The practical significance of such symmetry observer have been discussed in Nishida *et al.* (2009). An extension of the work of Nishida *et al.* (2006) to the case of infinite-dimensional symmetry of bi-Hamiltonian systems can be found in Nishida *et al.* (2007).

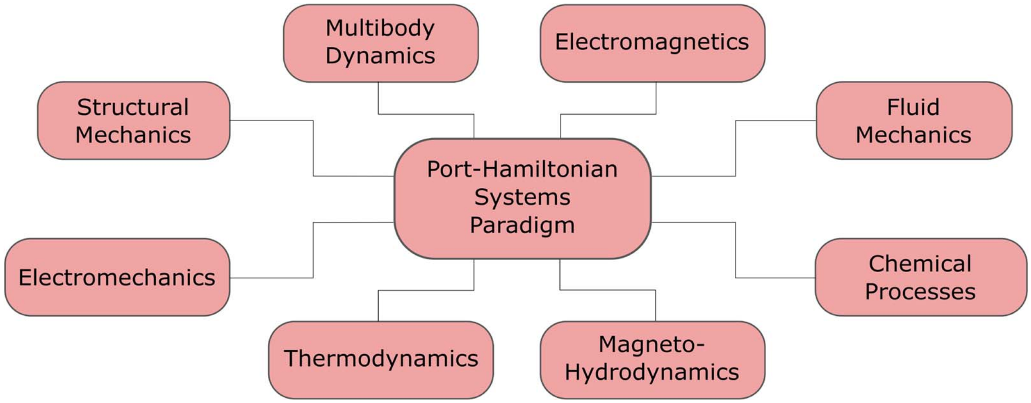


FIG. 3. The wide variety of physical domains in which the pH paradigm was applied to in the literature.

3. Modeling

There have been many efforts in the pH community for the past two decades to implement the generic dpH framework for modeling distributed-parameter systems in a wide variety of multi-physical domains including mechanical, electrical, magnetic, thermal and chemical domains, as shown in Fig. 3. In this section we survey the different physical domains of the distributed parameter systems modeled in the pH framework in the literature.

3.1. *Structural mechanics*

The first class of distributed parameter systems presented is structural mechanical systems with its mechanical energy comprising of kinetic and potential elastic energy. In van der Schaft & Maschke (2002), a model of a vibrating string was presented with the energy variables chosen as the strain and momentum. The underlying SDS of the model was a canonical one with the exterior derivative describing effort-flow relations. In the case of higher-order models like that of the Euler–Bernoulli (Nishida & Yamakita, 2004) or the Timoshenko beam (Macchelli & Melchiorri, 2004), the underlying SDS of the model describes high-order effort-flow differential-relations. The extended structures consist of compositions of the exterior derivative and the Hodge star operator. In Brugnoli *et al.* (2019a,b) a formulation of the thick plates (using respectively Midlin and Kirchoff models) that employs tensor variables is given for a coordinate-free pH description. Nonlinear phenomena can be easily incorporated in the framework through the constitutive relations, as in Trivedi *et al.* (2016), where a nonlinear Euler-Bernoulli beam model was presented including two-dimensional stress-strain relations modeling large deflections of the beam. Another approach for modeling planar beams with large deformations is the work of Golo *et al.* (2003), in which screw theory was used. Under the assumption of small deviations around an equilibrium configuration, the longitudinal part of the developed dpH model in Golo *et al.* (2003) is shown to correspond to the rod equation, while the transversal part corresponds to the Timoshenko beam model. The work of Golo *et al.* (2003) was extended in Macchelli *et al.* (2007a, 2006) to describe flexible links with one-dimensional spatial domain deforming in three-dimensional space, where gravity was also included through the use of a distributed port. In Heidari & Zwart (2019) two different pH models for studying longitudinal vibrations in a nanorod are provided.

In the context of mixed pH systems, i.e. systems merging from the interconnection of finite- and infinite-dimensional pH systems, an inverted pendulum on a cart was modeled in Trivedi *et al.* (2011), in which the pendulum was described by a flexible beam with a tip mass. Moreover, the work of Macchelli *et al.* (2007a) was extended to the case of a complete complex multi-body system in Macchelli *et al.* (2009) with both rigid and flexible links connected through kinematic pairs. An example of such systems is a robot manipulator with flexible links. Along the same line, a simplified model for a rotating flexible spacecraft was presented in Aoues *et al.* (2017), where the model consists of a center rigid hub, two flexible (Euler–Bernoulli) beams each connected to a tip mass. Finally, in Ramirez *et al.* (2013), an underactuated simplified model for a flexible nano-gripper for DNA manipulation was presented. The model comprised of a flexible (Timoshenko) beam connected to a network of mass-spring-dampers modeling the DNA bundle, suspension and actuation mechanisms.

Several pH models of mechanical systems have been also presented using the jet-bundle formalism including the Euler–Bernoulli beam (Schöberl & Schlacher, 2015a), Kirchhoff plate (Schöberl & Schlacher, 2015a, 2018), Timoshenko beam (Schöberl & Siuka, 2014) and Mindlin plate (Schöberl & Schlacher, 2015b; Schöberl & Siuka, 2013a).

3.2. *Electro-mechanics*

The second class of infinite-dimensional systems presented is electro-mechanical systems, which consists of two different physical domains interacting with each other in addition to mechanical and electrical ports defined on the boundary of the domain. In Macchelli *et al.* (2004a), the dynamics of a piezoelectric material, connected to a flexible beam, is modeled with the assumptions of quasi-static electric field, linear behavior and negligible thermal effects. Under the same assumptions, another model of the piezoelectric dynamics has been presented in Schöberl *et al.* (2008) using jet variables. An extended model for a piezoelectric Timoshenko beam has been studied in Voß & Scherpen (2014), which can represent nonlinear beam deformations as well as dynamic electric fields. The model studied was used to describe inflatable space structures. Another type of electro-mechanical system that was studied is the ionic polymer-metal composite (Nishida *et al.*, 2008b, 2011), which is a type of electro-active polymer consisting of a mechanical flexible beam with large deformations in addition to an electric double layer of polymer and metal electrodes. The mechanical and electric parts are coupled through electro-stress diffusion. The presented model incorporated the inherent nonlinearities as well as the multi-spatial-scale structure of the system. A very original example of mixed finite/infinite-dimensional pH description of an electro-mechanical system is present in Falaize & Hélie (2017) where a *Rhodes Piano* is modeled and simulated.

3.3. *Fluid mechanics*

Fluid-dynamical systems equations are characterized by two conservation laws: the balance of mass and momentum of the fluid. Several models of fluid-dynamical systems have been studied in the literature. The shallow-water equations (*aka* Saint-Venant equations) have been used in Hamroun *et al.* (2006); Pasumarthy & van der Schaft (2006b) for the (one-dimensional) modeling of open-channel irrigation systems. An extended model is presented in Hamroun *et al.* (2007) to include interconnected reaches with slopes and bed frictions. In one of the earliest work on distributed pH systems, the dynamics of an three-dimensional compressible inviscid isentropic fluid was presented in van der Schaft & Maschke (2001). The same results were also included in the original paper of van der Schaft & Maschke (2002) with the energy variables chosen as the mass density and Eulerian velocity field. The SDS describing the system is nonlinear; however, it was also shown that for irrotational flow, the underlying SDS becomes the canonical one. An extended model describing the case of non-isentropic fluids is presented in Polner & van der Vegt (2014) in terms of the vorticity-dilatation variables. Moreover, for the spatial one-dimensional case, different pH models of fluid-thermal equations can be found in Lopezlena & Scherpen (2004b), a dpH of Navier–Stokes equations for reactive flows can be found in Altmann & Schulze (2017), while jet-bundle formulations of the Kortweg-de Vries and Boussinesq equations can be found in Maschke & van der Schaft (2013).

Some work on fluid-structure interaction has also been conducted in the dpH framework. In Lequeurre & Tucsnak (2015), the one-dimensional Navier–Stokes equation is coupled with a point mass to describe a simplified model of a gas/piston system. Moreover, in Cardoso-Ribeiro *et al.* (2017), the shallow water equations in a moving tank are coupled to a flexible beam with piezoelectric actuators. The whole setup is a model for the fuel sloshing in a tank connected to very flexible wings in aeronautical applications.

3.4. *Magneto-hydrodynamics*

Magneto-hydrodynamical systems are ones in which the fluid's magnetic behavior and electrical conductance is considered. Examples of such systems include electrolytes, liquid metals and plasmas. Such systems comprise of two physical domains interacting together, i.e. electromagnetism, governed by Maxwell equations, and fluid mechanics, governed by Euler equations of ideal isentropic fluids. The coupling term is a function of the free current density and the magnetic field induction. pH models of the magneto-hydrodynamics equations based on the jet-bundle formalism can be found in Siuka *et al.* (2010) and Nishida & Sakamoto (2010). Moreover, if the fluid thermal behavior is taken into consideration, then such models are called thermo-magneto-hydrodynamical systems. Several research studies have been conducted in Vu & Lefèvre (2013); Vu *et al.* (2012, 2016) for pH modeling of plasma in nuclear fusion reactors. The work has also been extended in Vincent *et al.* (2017) for modeling burning plasma models.

3.5. *Chemical processes*

The dpH formalism was also applied fo the modeling of chemical processes in which heat and mass transport occurs along with chemical reactions. In Eberard & Maschke (2004), the dpH framework was extended to included irreversible thermodynamic processes. Then in Eberard *et al.* (2005), the diffusion process in a heterogeneous mixture is modeled, with an example of the pressure swing adsorption process.

In Baaiu *et al.* (2009a), the heat and mass transport phenomena with multiscale coupling is modeled. The hydrodynamics of the fluid is not modeled explicitly but is handled by the introduction of a moving reference frame. Several studies have been also devoted for reaction-diffusion systems in which there is coupling between mass transport and chemical reactions. Different pH models can be found in Šešlija *et al.* (2010, 2014b); Zhou *et al.* (2017, 2012, 2015). A different way to model multi-scale systems stemming from the combination of hyperbolic and diffusive processes is studied in Le Gorrec & Matignon (2013), where fractional integrals and derivatives are used.

4. Analysis and Control

Here we summarize the main control techniques that have been developed for dpH systems. As preamble it is worth mentioning that when dealing with infinite-dimensional systems, some intrinsic difficulties arise in the proofs of stability of equilibria. This is mainly due to the fact that all norms are not equivalent in infinite-dimensional spaces and in principle a norm has to be chosen in the specific stability argument (Jacob & Zwart, 2012). The pH modeling for infinite-dimensional systems does not solve these issues, which are intrinsic in distributed parameter systems, but provides a physical understanding of the model, in which properties of the energy of the system are encoded in the dynamic equations. Normally, the Hamiltonian energy appearing in the system is a good choice for a Lyapunov functional, which can be used for designing the control law in a more intuitive way. In other words the idea of the control techniques developed in this context aim at generalizing the machinery of *energy shaping* and *damping injection* established for finite-dimensional pH systems in a *control by interconnection* paradigm.

The rest of this section is organized in the following way: in Section 4.1 the control techniques that arise as extension of control by interconnection ideas from the finite-dimensional case are presented. In Section 4.2 control designs based on spatial discretization of dpH are listed while in Section 4.3 the rest of the relevant control ideas are present. In Section 4.4 the control techniques related to the framework described in Section 2.2 are summarized, by presenting first well-posedness and stabilizability results, and then design of controllers for such systems. Basically our choice is to distinguish the techniques that have been developed for dpH systems as BCS and the rest of the approaches, which are in principle applicable to general dpH systems.

4.1. *Extension of control by interconnection*

In Macchelli & Melchiorri (2004) the damping injection technique is applied to the dpH Timoshenko model to stabilize the beam both at the distributed and boundary ports. In the same work, control by interconnection to perform energy shaping of the beam is extended to the infinite-dimensional case following the ideas presented in Rodriguez *et al.* (2001). This has been done by introducing a finite-dimensional controller and a tip mass on the extremities of the beam and generalizing the concept of Casimir functions for the resulting closed-loop system. In the same spirit as in Macchelli & Melchiorri (2004), the dpH model and damping injection control is performed to the two-dimensional case of the Mildlin plate in Macchelli *et al.* (2005a).

In Macchelli & Melchiorri (2005) control by interconnection was formalized for a class of mixed pH systems, in the sense of a power conserving interconnection between an infinite-dimensional system, ‘sandwiched’ between two finite-dimensional ones, all of them in pH form. The control by interconnection by means of Casimir generation has been extended to this class of systems, allowing for a structural state feedback law able to shift the equilibrium of the closed-loop system if appropriate conditions on the Casimirs are fulfilled. In Macchelli *et al.* (2005b) the same idea was applied for closed-loop systems consisting on the interconnection of two systems only, one finite-dimensional and the other distributed. A related work is Pasumarthy & van der Schaft (2005) where only one-dimensional systems are considered. In Pasumarthy & van Der Schaft (2007) the problem of characterizing achievable Casimirs is analysed for systems merging form the interconnection of finite-dimensional systems at the boundary of infinite-dimensional ones. Distributed and boundary dissipation are included in the analysis which is substantially limited to one-dimensional spatial domain dpH systems in which the SDS is defined on the space of differential forms. An attempt to tackle the two-dimensional case has been done in Macchelli *et al.* (2015b) where control by interconnection has been applied on dpH systems with rectangular domain.

4.2. *Control design based on spatial discretization*

Another approach is to take advantage of the spatial discretization algorithms for dpH systems and design the regulator on the basis of the finite-dimensional approximation. This has been the strategy in Macchelli (2011, 2012b); Macchelli & Melchiorri (2009, 2010) where the Casimirs for the closed-loop system were chosen on the basis of the finite-dimensional closed-loop approximation and where the connection with the corresponding infinite-dimensional Casimirs is studied. In Kotyczka (2014); Kotyczka & Brandst (2014) the feedforward control problem is studied for spatially discretized dpH systems consisting in hyperbolic systems of two conservation laws. The obtained results rely on the explicit expression of the inverse dynamics, which is available since the discretized system of the considered class possesses a non zero feedthrough term. In Toledo *et al.* (2019) an observer-based controller is designed on the basis of a spatial discretization of a dpH system in the form (4.1) in such a way that it stabilizes the infinite-dimensional system as well, avoiding spillover effects. In Cardoso-Ribeiro *et al.* (2019) the nonlinear two-dimensional shallow water equations are discretized and controlled at the boundary. The problem is tackled after a reduction to a one-dimensional system performed by means of symmetry conditions and use of polar coordinates.

4.3. *Other control methods*

In Malzer *et al.* (2019); Rams & Schöberl (2017); Schöberl & Siuka (2011, 2013b), control by interconnection in terms of Casimir generation is analysed in the context of dpH systems described using the jet bundle formulation, and not through skew-adjoint operators. In Nishida *et al.* (2013) optimal control for dpH systems has been used to derive passivity based control laws in a differential geometric setting. A numerical algorithm is then proposed to control a flexible beam with large deformations. In Kosaraju *et al.* (2017); Macchelli (2016a) a Brayton–Moser formulation is given for the dpH systems in the form (4.1) and a boundary control algorithms is proposed to shape the mixed-potential function to overcome the dissipation obstacle. In Trang Vu *et al.* (2017); Trenchant *et al.* (2017b) control by structural invariant is extended to shape not only the energy of the closed-loop system, but also its structure. It is shown how a one-dimensional hyperbolic dpH system of two conservation laws can be shaped in a parabolic one. A similar idea was followed in Vu *et al.* (2017a) where the IDA-PBC synthesis is explicitly addressed for a dpH system with spatial symmetries.

4.4. *Well-posedness, stabilization and control of dpH systems as BCS*

4.4.1. *Well-posedness and Stabilization.* The following contributions rely on the functional analytic formulation of Le Gorrec *et al.* (2005). It is important to realize that the machinery connecting dpH systems and BCS was not yet available at the time at which some of the previously listed control techniques were introduced. As a consequence, in the first works extending control by interconnection to dpH systems, the closed-loop system merging from the designed controller is ‘solution free’ (Macchelli & Melchiorri, 2004), in the sense that existence of trajectories of the closed-loop system was not addressed. This does not take away importance to these works, that in principle can be applied to more general systems than those for which exhaustive functional analysis has been applied on later. The idea of section 4.1 is indeed to collect control by interconnection for dpH systems in this ‘solution free’ fashion, while in the following we discuss results relying directly on the functional analytic approach. As previously mentioned, this framework is characterized by the following trade-off: the control design gained more rigour from a functional analytic point of view in the sense that existence of solutions for closed-loop the system is addressed rigorously (the design of the controller is not anymore ‘solution free’) with a dramatic decrease of the class of systems that are considered. In fact besides of minor variations, the plants on which controllers have been designed are substantially represented by the PDE (4.1). An isolated exception is represented by the work of Kurula & Zwart (2015), where well-posedness of the linear wave equation is studied in an n-dimensional spatial domain. It is important to cite Le

Gorrec *et al.* (2006) where distributed dissipation was added to the framework by means of a symmetric operator.

The work on stabilization of dpH systems by means of finite-dimensional boundary controllers started in Villegas *et al.* (2005) where *asymptotic* stability results based on static and dynamic feedback of BCS considered in Le Gorrec *et al.* (2005) were carried out by means of frequency based arguments. It started to be clear that by restricting the class of systems described by the differential operator (2.3) to the case $N = 1$ technical issues necessary for stability were automatically satisfied, leading to the definition of the abstract class of linear dpH systems on one-dimensional spatial domain

$$\dot{x} = P_1 \frac{\partial}{\partial z}(\mathscr{L}(z)x) + (P_0 - G_0)\mathscr{L}(z)x, \tag{4.1}$$

with P_1 and G_0 being symmetric matrices and P_0 skew-symmetric. Here $z \in [a, b]$ and x lives in the functional space equivalent to $L^2([a, b], \mathbb{R}^n)$ endowed with the inner product that makes its induced norm equal to twice the energy functional. It is clear that the names P_1, P_0 derive from the indices of the truncated series in (2.3) and G_0 is the operator describing distributed dissipation. In the PhD thesis of Villegas (2007) a great number of results in terms of stability and stabilization have been formalized for systems in form (2.4)–(4.1). In this thesis arguments highlighting the difficulty of extending the functional analytic approach to higher dimensional spatial domains are present. The key result in Villegas *et al.* (2009) provided an *exponential* stability argument for BCS systems in pH form (4.1) by means of adding dissipation on the boundary of the spatial domain. This result has been of great importance since it provided a practical check to evaluate exponential stability and was instrumental for future stabilizability results. In Zwart *et al.* (2009) results on well-posedness of systems in the form (4.1) are carried out even for the case in which the autonomous system does not generate a contraction semigroup. At this point the richness of the framework was enough to publish the monograph Jacob & Zwart (2012), where the important results for systems (4.1) were collected and developed.

Further important research involving exponential stabilization of BCS in pH form is present in Ramirez *et al.* (2014) where the methodology developed in Villegas *et al.* (2009) has been used to study stabilizability of (4.1) by means of finite-dimensional boundary controllers, here considered to be finite-dimensional pH systems with non zero feedthrough term. It is worth mentioning that practically every result in stabilizability of systems in form (2.4) or (4.1) make the assumption of a particular input–output parametrization, i.e. impedance passivity (Le Gorrec *et al.*, 2005). In Macchelli & Califano (2018) the results are extended to any possible parametrization introduced in Le Gorrec *et al.* (2005) such that the autonomous BCS generates a contraction semigroup and consequently the supply rate can assume any quadratic form of the input-output pair. Here the finite-dimensional controllers are assumed to be general linear-time-invariant systems, and not more in pH form, in order to deal with general supply rates. This result has been instrumental to handle more general systems of coupled PDEs and ODEs and as result *repetitive control systems* (Califano *et al.*, 2017; Macchelli & Califano, 2018) have been cast in this rigorous framework to derive novel stability conditions. In Jacob & Zwart (2018) the operator based analysis to dpH systems is extended to address control theoretic properties like *controllability* and *input-to-state stability*. It is also important to cite the PhD thesis of Augner (2018), where some stability results for the second order case are present.

We conclude this dense list of results by mentioning studies aiming at interconnecting linear dpH systems with nonlinear controllers, ending up with the difficult task of handling a nonlinear, infinite-dimensional closed-loop systems. This scenario is particularly challenging and only few recent results are present in terms of well-posedness and stabilization. In particular in Ramirez *et al.* (2017a) a class of nonlinear passive systems stabilizing first asymptotically and then exponentially dpH in form (4.1) is presented. This work has inspired Califano *et al.* (2018) where stability analysis for nonlinear repetitive control schemes has been performed for a class of systems, subsequently extended in Califano & Macchelli (2019). In Augner (2018) a nonlinear semigroup approach is used to prove exponential stability of dpH systems by means of nonlinear dissipative feedback. In Hastir *et al.* (2019) a different approach is used to study the interconnection of infinite-dimensional linear systems interconnected with a static nonlinearity. The result can be applied in this framework to show well-posedness of a vibrating string in pH form with a nonlinear damper at the boundary.

4.4.2. *Control of dpH systems as BCS.* In this context aspects related to Casimir generation have been reinvestigated for systems in the form (4.1) in order to properly address well-posedness (i.e. existence and smoothness of solutions) for the closed-loop system.

In Macchelli (2012a) control by interconnection by means of Casimir generation is revisited in this framework without dissipation ($G_0 = 0$ in (4.1)) where the closed-loop system merges form the interconnection of the dpH and a finite-dimensional boundary regulator in pH form. Conditions for existence of Casimirs and asymptotic stability conditions are given. In Macchelli (2013, 2014) the class of stabilising controllers has been enlarged by relying on the parametrisation of the system dynamics provided by the image representation of the SDS to overcome the *dissipation obstacle*, i.e. the problem of steering a dynamical system in a state that dissipates energy. In this way the boundary control is not generated implicitly by means of Casimirs of the closed-loop system but directly as feedback control law. The methodology is applied to the whole class of (4.1) in Macchelli *et al.* (2015a). In Macchelli (2015) the dissipation obstacle is overcome in a different way: by defining a new passive output and applying control by interconnection to the new input-output pair of the dpH system it was shown how to overcome the dissipation obstacle by means of Casimir generation for the new closed-loop-system. In Le Gorrec *et al.* (2014) asymptotic stability for (4.1) is proven by means of a boundary finite-dimensional pH controller and Casimir generation for the closed-loop system. The methodology is applied to a nanotweezer DNA-manipulation device. In Macchelli (2016b) the control by interconnection paradigm is augmented with an output feedback control loop providing exponential stability of the closed-loop system of the internally shaped equilibrium. An important contribution summarizing the ideas about boundary control laws for (4.1) is Macchelli *et al.* (2017b), where Casimir generation, state feedback control laws able to overcome dissipation obstacle, and asymptotic stabilization with damping injection are extensively addressed.

In Ramirez *et al.* (2017b) a backstepping boundary controller is designed for a simple one-dimensional hyperbolic lossless dpH system, showing that pH structure is able to simplify the control design process for a target system with dissipation. In Macchelli *et al.* (2017a) the same idea to map system (4.1) into a target one is explored through a proper coordinate transformation preserving the Hamiltonian structure.

5. Discretization

For numerical simulation and control synthesis, it is necessary to have finite approximations of the infinite-dimensional pH system models or infinite-dimensional controllers discussed earlier. These finite-dimensional approximations should maintain the ‘openness’ property to be able to interconnect it via its ports to other systems. The conventional numerical algorithms emanating from the numerical analysis field fail to preserve the intrinsic pH system structure and properties, such as passivity, symplecticity, and conservation laws (Šešlija *et al.*, 2012).

TABLE 1. *Structure-preserving discretization methods for pH systems.*

Discretization Method	**References**
Finite element	Baaiu *et al.* (2006, 2009b); Bassi *et al.* (2007); Eberard *et al.* (2007); Golo *et al.* (2002); Macchelli *et al.* (2007b); Pasumarthy *et al.* (2012); Talasila *et al.* (2002); Voß & Weiland (2011); Wu *et al.* (2015)
Finite difference	Lopezlena & Scherpen (2004a), Trenchant *et al.* (2018a), Trenchant *et al.* (2017a), Trenchant *et al.* (2018b)
Finite volume	Kotyczka (2016), Serhani *et al.* (2018)
Partitioned finite element	Cardoso-Ribeiro *et al.* (2018), Serhani *et al.* (2019c), Serhani *et al.* (2019b), Serhani *et al.* (2019a)
Pseudo-spectral	Harkort & Deutscher (2012), Moulla *et al.* (2011), Moulla *et al.* (2012), Vu *et al.* (2013a), Vu *et al.* (2017b)
Discrete exterior calculus	Kotyczka *et al.* (2018), Šešlija *et al.* (2011), Šešlija *et al.* (2014a),Šešlija *et al.* (2012)

TABLE 2. *Application research papers combining different aspects of the pH framework. The aspects include Modeling, Discretization, Control, and Experimental Validation.*

	Modeling	Discretization	Control	Experiment
Banavar & Dey (2010)	✓		✓	
Hamroun *et al.* (2010)	✓	✓	✓	✓
Siuka *et al.* (2011)	✓		✓	
Voß & Weiland (2011)	✓	✓	✓	
Nishida *et al.* (2012)	✓		✓	✓
Ramirez *et al.* (2013)	✓		✓	
Vu *et al.* (2013b)	✓	✓	✓	
Šešlija *et al.* (2014b)	✓	✓		
Kotyczka & Blancato (2015)	✓	✓	✓	
Trenchant *et al.* (2015)	✓		✓	
Trivedi *et al.* (2016)	✓	✓		
Aoues *et al.* (2017)	✓		✓	✓
Cardoso-Ribeiro *et al.* (2017)	✓	✓	✓	✓
Brugnoli *et al.* (2019a,b)	✓	✓		

This has motivated research for the past two decades in developing structure-preserving discretization techniques of dpH systems, summarized in Table 1. These methods, similar to traditional ones, are based either on the approximation of equations or the approximation of solutions (Baaiu *et al.*, 2009b). The first category includes finite-differences methods and finite-volume methods, while the second category includes weighted residual methods and finite-element methods.

One of the earliest works in structure-preserving discretization techniques was the work of Golo *et al.* (2002, 2003, 2004); where a mixed finite-elements method was presented. The method of Golo *et al.* (2004) considered the approximation of the differential forms, corresponding to the port variables in Fig. 2b, by Whitney forms. The method was used to discretize the telegrapher's equation (Golo *et al.*, 2002), wave equation (Golo *et al.*, 2003), chemical adsorption column (Baaiu *et al.*, 2006), Maxwell's equations (Eberard *et al.*, 2007) and a vibro-acoustic system (Wu *et al.*, 2015).

The work of Golo *et al.* (2004) was extended to the case of a non-constant SDS in Pasumarthy *et al.* (2012); Pasumarthy & van der Schaft (2006a) for the shallow water equations and extended to irreversible pH systems in Baaiu *et al.* (2009b); Voß & Weiland (2011). Another method that was inspired by Golo *et al.* (2004) is Bassi *et al.* (2007), which was formulated for the functional analytic formulation of pH systems discussed in section 2.2. The algorithm described in Bassi *et al.* (2007) was implemented in Macchelli *et al.* (2007b) for a flexible link.

Structure-preserving discretization methods based on finite differences and finite volumes have also been presented in the literature. In Clemente-Gallardo *et al.* (2002); Lopezlena & Scherpen (2004a), a method based on discretizing the spatial domain into a grid of nodes where finite differences are used to approximate the differential form variables of the pH system. A drawback of this method, unlike the method of Golo *et al.* (2004), is that only uniform grids can be used. This problem was avoided in Trenchant *et al.* (2017a, 2018b) by using staggered-grids finite differences thus allowing the use of different grids. Rectangular grids have been used in Trenchant *et al.* (2017a) while cylindrical grids have been used in Trenchant *et al.* (2018a). A finite-volume scheme based on the generalized leapfrog method has been introduce for one-dimensional systems in Kotyczka (2016) and extended to two-dimensional systems in Serhani *et al.* (2018).

A promising and very recent structure-preserving numerical method is the *Partitioned Finite Element Method* (PFEM). It has been introduced in Cardoso-Ribeiro *et al.* (2018), developed in Serhani *et al.* (2019a,b,c) and successfully applied for discretization and simulation of thick plate models in the already referred papers Brugnoli *et al.* (2019a,b). The method consists in rewriting the system in a weak-form where only some of the equations are integrated by parts. As consequence the SDS and the constitutive equations are discretized separately, preserving the power balance of the open system (including boundary control and observation) at the discrete level.

Another class of discretization methods that has been used in the literature includes pseudo-spectral methods, which can be considered a generalization of mixed finite element methods when low-order polynomials are used for approximation. In Moulla *et al.* (2011) a method based on polynomial approximation bases, with Lagrange interpolation, is introduced and used to discretize the lossless transmission line (Moulla *et al.*, 2011) and the shallow water equations (Moulla *et al.*, 2012). The aforementioned method was extended in Vu *et al.* (2013a, 2017b) using Bessel function bases and applied to a one-dimensional Tokamak model. Another pseduo-spectral method was introduced in Harkort & Deutscher (2012) using a generalized Galerkin projection method for discretizing linear pH systems in the functional analytic approach.

A natural approach to the structure-preserving discretization of pH systems is to mirror the continuous exterior calculus formulation using *discrete exterior calculus*. The framework replaces the smooth structures in exterior calculus by their discrete analogues, e.g. replacing the smooth manifold by a simplicial complex and replacing the differential forms by co-chains. The conventional discrete exterior calculus methods were extended from the Hamiltonian setting to the pH setting in the work of Šešlija *et al.* (2011, 2012) in an algebraic topology setting. The main results of Šešlija *et al.* (2011, 2012) have been made more accessible in Šešlija *et al.* (2014a) by being rewritten in matrix representations instead of the algebraic topology nomenclature. A closely-related work to the discrete exterior calculus formulation is Kotyczka *et al.* (2018) which was based on the weak-form of the SDS of the pH system.

6. Conclusion

In this survey paper we reviewed the topic of dpH systems that started about 20 years ago. We analyzed and classified over 150 studies that we considered relevant. Our pedagogical classification is based on conceptual approaches and not on chronological order. The aim of this review is to help researchers in consulting and examining relevant references, and also in understanding the conceptual backbone of the topic, together with their variations. This manuscript highlights the fact that the dpH framework provides a deep understanding of the different multi-physical natural phenomena by explicitly separating energetic and interconnection properties of the system. One of the key benefits of the pH framework is the unified language and conceptual insight that can be applied for the synthesis of a distributed parameter system, namely the modeling, discretization, analysis and control, in addition that these powerful techniques are directly usable at the practical level. Table 2 lists the research studies that applied the dpH framework through the combination of the aforementioned steps. Taking into consideration also that the pH framework can incorporate both finite and infinite dimensional systems in a similar manner conceptually, we believe this framework to be very fruitful in the near future.

Funding

PortWings project funded by the European Research Council (grant agreement no. 787675).

References

Altmann, R. & Schulze, P. (2017) A port-Hamiltonian formulation of the Navier stokes equations for reactive flows. *Syst. Control Lett.*, **100**, 51–55.

Aoues, S., Cardoso-Ribeiro, F. L., Matignon, D. & Alazard, D. (2017) Modeling and control of a rotating flexible spacecraft: a port-Hamiltonian approach. *IEEE Trans. Contr. Syst. Technol.*, **27**, 355–362.

Augner, B. (2018). Stabilisation of infinite-dimensional port-Hamiltonian systems via dissipative boundary feedback. *Ph.D. Thesis*, Universität Wuppertal, Fakultät für Mathematik und Naturwissenschaften.

Baaiu, A., Couenne, F., Eberard, D., Jallut, C., Lefevre, L., Le Gorrec, Y. & Maschke, B. (2009a) Port-based modelling of mass transport phenomena. *Math. Comput. Model. Dyn.*, **15**, 233–254.

Baaiu, A., Couenne, F., Lefevre, L., Le Gorrec, Y. & Tayakout, M. (2006) Energy based discretization of an adsorption column. *IFAC Proceedings Volumes*, **39**, 753–758.

Baaiu, A., Couenne, F., Lefevre, L., Le Gorrec, Y. & Tayakout, M. (2009b) Structure-preserving infinite dimensional model reduction: application to adsorption processes. *J. Process Control*, **19**, 394–404.

Banavar, R. & Dey, B. (2010) Stabilizing a flexible beam on a cart: a distributed approach port-Hamiltonian approach. *J. Nonlinear Sci. Appl.*, **20**, 131–151.

Bassi, L., Macchelli, A. & Melchiorri, C. (2007). An algorithm to discretize one-dimensional distributed port-Hamiltonian systems. *Lagrangian and Hamiltonian Methods for Nonlinear Control 2006*. Berlin, Heidelberg: Springer, pp. 61–73.

Brugnoli, A., Alazard, D., Pommier-Budinger, V. & Matignon, D. (2019a) Port-Hamiltonian formulation and symplectic discretization of plate models, Part I: Mindlin model for thick plates. *Appl. Math. Model.*, **75**, 940–960.

Brugnoli, A., Alazard, D., Pommier-budinger, V. & Matignon, D. (2019b) Port-Hamiltonian formulation and symplectic discretization of plate models, Part II: Kirchhoff model for thin plates. *Appl. Math. Model.*, **75**, 961–981.

Califano, F., Bin, M., Macchelli, A. & Melchiorri, C. (2018) Stability analysis of nonlinear repetitive control schemes. *IEEE Control Syst. Lett.*, **2**, 773–778.

Califano, F. & Macchelli, A. (2019) A stability analysis based on dissipativity of linear and nonlinear repetitive control. *IFAC-PapersOnLine*, **52**, 40–45.

Califano, F., Macchelli, A. & Melchiorri, C. (2017). Stability analysis of repetitive control: The port-Hamiltonian approach. *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, vol. 2018. Melbourne, VIC, Australia: IEEE, pp. 1894–1899.

Cardoso-Ribeiro, F. L., Brugnoli, A., Matignon, D., & Lefèvre, L. (2019). Port-Hamiltonian modeling, discretization and feedback control of a circular water tank. *2019 IEEE 58th Conference on Decision and Control (CDC)*. Nice, France: IEEE, pp. 6881–6886.

Cardoso-Ribeiro, F. L., Matignon, D. & Lefèvre, L. (2018) A structure-preserving partitioned finite element method for the 2D wave equation. *IFAC-PapersOnLine*, **51**, 119–124.

Cardoso-Ribeiro, F. L., Matignon, D. & Pommier-Budinger, V. (2017) A port-Hamiltonian model of liquid sloshing in moving containers and application to a fluid-structure system. *J. Fluid Struct.*, **69**, 402–427.

Clemente-Gallardo, J., Lopezlena, R. & Scherpen, J. (2002). Geometric discretization of fluid dynamics. *Proceedings of the 41st IEEE Conference on Decision and Control*, vol. 4. Las Vegas, NV, USA: IEEE pp. 4185–4190.

Diagne, M. & Maschke, B. (2013) Port Hamiltonian formulation of a system of two conservation laws with a moving interface. *Eur. J. Control*, **19**, 495–504.

Eberard, D., Lefevre, L. & Maschke, B. (2005). Multiscale coupling in heterogeneous diffusion processes: a port-based approach. *Proceedings of 2005 International Conference Physics and Control*. Saint Petersburg, Russia: IEEE pp. 543–547.

Eberard, D. & Maschke, B. (2004) Port Hamiltonian systems extended to irreversible systems: the example of the heat conduction. *IFAC Proceedings Volumes*, **37**, 243–248.

Eberard, D., Maschke, B. & van der Schaft, A. (2007). On the interconnection structures of discretized port-Hamiltonian systems. *PAMM: Proceedings in Applied Mathematics and Mechanics*, vol. 7. Weinheim: WILEY-VCH Verlag GmbH & Co. KGaA , pp. 3030005–3030006.

Ennsbrunner, H. & Schlacher, K. (2005). On the geometrical representation and interconnection of infinite dimensional port controlled Hamiltonian systems. *Proceedings of the 44th IEEE Conference on Decision and Control*. Seville, Spain: IEEE, pp. 5263–5268.

Falaize, A. & Hélie, T. (2017) Passive simulation of the nonlinear port-Hamiltonian modeling of a Rhodes piano. *J. Sound Vibration*, **390**, 289–309.

Golo, G., Talasila, V. & van der Schaft, A. (2002). Approximation of the Telegrapher's equations. *Proceedings of the 41st IEEE Conference on Decision and Control*, vol. 4. Las Vegas, NV, USA: IEEE pp. 4587–4592.

Golo, G., Talasila, V., van der Schaft, A. & Maschke, B. (2004) Hamiltonian discretization of boundary control systems. *Automatica J. IFAC*, **40**, 757–771.

Golo, G., van der Schaft, A. & Stramigioli, S. (2003) Hamiltonian formulation of planar beams. *IFAC Proceedings Volumes*, **36**, 147–152.

Hamroun, B., Dimofte, A., Lefèvre, L. & Mendes, E. (2010) Control by interconnection and energy-shaping methods of port Hamiltonian models. Application to the shallow water equations. *Eur. J. Control*, **16**, 545–563.

Hamroun, B., Lefèvre, L. & Mendes, E. (2006) Port-based modelling for open channel irrigation systems. *Trans. Fluid Mech.*, **1**, 995–1009.

Hamroun, B., Lefèvre, L., & Mendes, E. (2007) Port-based modelling and geometric reduction for open channel irrigation systems. *Proceedings of the IEEE Conference on Decision and Control*. New Orleans, LA, USA: IEEE pp. 1578–1583.

Harkort, C. & Deutscher, J. (2012) Stability and passivity preserving Petrov-Galerkin approximation of linear infinite-dimensional systems. *Automatica J. IFAC*, **48**, 1347–1352.

Hastir, A., Califano, F. & Zwart, H. (2019) Well-posedness of infinite-dimensional linear systems with nonlinear feedback. *Syst. Control Lett.*, **128**, 19–25.

Heidari, H. & Zwart, H. (2019) Port-Hamiltonian modelling of nonlocal longitudinal vibrations in a viscoelastic nanorod. *Math. Comput. Model. Dyn.*, **25**, 447–462.

Jacob, B. & Zwart, H. (2012) Linear Port-Hamiltonian Systems on Infinite-dimensional Spaces. *Vol. 223 of Operator Theory: Advances and Applications*. Basel: Birkhäuser.

Jacob, B. & Zwart, H. (2018) An operator theoretic approach to infinite-dimensional control systems. *GAMM Mitteilungen*, **41**, 1–14.

Kosaraju, K. C., Pasumarthy, R., Singh, N. M. & Fradkov, A. L. (2017) Control using new passivity property with differentiation at both ports. *Proceedings of the 2017 Indian Control Conference, ICC 2017*. Guwahati, India: Birkhäuser.

KOTYCZKA, P. (2014) On the feedforward control problem for discretized port-Hamiltonian systems. *IFAC Proceedings Volumes*, **47**, 652–658.

KOTYCZKA, P. (2016) Finite volume structure-preserving discretization of 1D distributed-parameter port-Hamiltonian systems. *IFAC-PapersOnLine*, **49**, 298–303.

KOTYCZKA, P. & BLANCATO, A. (2015) Feedforward control of a channel flow based on a discretized port-Hamiltonian model. *IFAC-PapersOnLine*, **28**, 194–199.

KOTYCZKA, P. & BRANDST, S. (2014). Inversion-based feedforward control for discretized port-Hamiltonian systems. *Proceedings of the 21st International Symposium on Mathematical Theory of Networks and Systems*. Groningen, The Netherlands, pp. 729–735.

KOTYCZKA, P., MASCHKE, B. & LEFÈVRE, L. (2018) Weak form of Stokes-Dirac structures and geometric discretization of port-Hamiltonian systems. *J. Comput. Phys.*, **361**, 442–476.

KURULA, M. & ZWART, H. (2015) Linear wave systems on n -D spatial domains. *Internat. J. Control*, **88**, 1063–1077.

LE GORREC, Y., MACCHELLI, A., RAMIREZ, H. & ZWART, H. (2014) Energy shaping of boundary controlled linear port-Hamiltonian systems. *IFAC Proceedings Volumes*, **19**, 1580–1585.

LE GORREC, Y., MASCHKE, B., VILLEGAS, J. A. & ZWART, H. (2006) Dissipative boundary control systems with application to distributed parameters reactors. *Proceedings of the IEEE International Conference on Control Applications*, vol. 3. Munich, Germany: IEEE.

LE GORREC, Y. & MATIGNON, D. (2013) Coupling between hyperbolic and diffusive systems: a port-Hamiltonian formulation. *Eur. J. Control*, **19**, 505–512.

LE GORREC, Y., ZWART, H. & MASCHKE, B. (2005) Dirac structures and boundary control systems associated with skew-symmetric differential operators. *SIAM J. Control Optim.*, **44**, 1864–1892.

LEQUEURRE, J. & TUCSNAK, M. (2015) The piston problem in a port-Hamiltonian formalism. *IFAC-PapersOnLine*, **28**, 212–216.

LOPEZLENA, R. & SCHERPEN, J. M. (2004a) Lumped approximation of transmission line with an alternative geometric discretization. *IFAC Proceedings Volumes*, **37**, 381–386.

LOPEZLENA, R. & SCHERPEN, J. M. (2004b) On distributed port-Hamiltonian process systems. *IFAC Proceedings Volumes*, **37**, 973–978.

MACCHELLI, A. (2011) Energy shaping of distributed parameter port-Hamiltonian systems based on finite element approximation. *SYST CONTROL LETT*, **60**, 579–589.

MACCHELLI, A. (2012a) Asymptotic stability of forced equilibria for distributed port-Hamiltonian systems. *51st IEEE Conference on Decision and Control (CDC)*. Maui, HI, USA: IEEE, pp. 2934–2939.

MACCHELLI, A. (2012b) Energy-based control of spatially-discretized distributed port-Hamiltonian systems. *IFAC Proceedings Volumes*, **45**, 786–791.

MACCHELLI, A. (2013) On the use of Dirac structures on Hilbert spaces in the synthesis of boundary control laws for port-Hamiltonian systems. *Proceedings of the IEEE Conference on Decision and Control*. Florence, Italy: IEEE pp. 3267–3272. IEEE.

MACCHELLI, A. (2014) Dirac structures on Hilbert spaces and boundary control of distributed port-Hamiltonian systems. *SYST CONTROL LETT*, **68**, 43–50.

MACCHELLI, A. (2015) Control by interconnection beyond the dissipation obstacle of finite and infinite dimensional port-Hamiltonian systems. *Proceedings of the IEEE Conference on Decision and Control*. Osaka, Japan: IEEE pp. 2489–2494. IEEE.

MACCHELLI, A. (2016a) Brayton-Moser formulation of infinite dimensional port-Hamiltonian systems with application to boundary control. *2016 IEEE 55th Conference on Decision and Control, CDC 2016*. Las Vegas, NV, USA: IEEE pp. 543–548.

MACCHELLI, A. (2016b) On the control by interconnection and exponential stabilisation of infinite dimensional port-Hamiltonian systems. *2016 IEEE 55th Conference on Decision and Control, CDC 2016*. Las Vegas, NV, USA: IEEE pp. 3137–3142.

Macchelli, A. & Califano, F. (2018) Dissipativity-based boundary control of linear distributed port-Hamiltonian systems. *Automatica J. IFAC*, **95**, 54–62.

Macchelli, A., Le Gorrec, Y. & Ramírez, H. (2015a) Asymptotic stabilisation of distributed port-Hamiltonian systems by boundary energy-shaping control. *IFAC-PapersOnLine*, **28**, 488–493.

Macchelli, A., Le Gorrec, Y. & Ramirez, H. (2015b) Boundary L2-gain stabilisation of a distributed Port-Hamiltonian system with rectangular domain. *Proceedings of the IEEE Conference on Decision and Control*. Osaka, Japan: IEEE, pp. 1236–1241.

Macchelli, A., Le Gorrec, Y., & Ramirez, H. (2017a) Boundary control of distributed port-Hamiltonian systems via generalised canonical transformations. In *2017 IEEE 56th Annual Conference on Decision and Control, CDC 2017*, vol. 2018. Melbourne, VIC, Australia: IEEE pp. 70–75.

Macchelli, A., Le Gorrec, Y., Ramirez, H. & Zwart, H. (2017b) On the synthesis of boundary control laws for distributed port-Hamiltonian systems. *IEEE T Automat Contr*, **62**, 1700–1713.

Macchelli, A. & Melchiorri, C. (2004) Modeling and control of the Timoshenko beam. The distributed port Hamiltonian approach. *SIAM J. Control Optim.*, **43**, 743–767.

Macchelli, A. & Melchiorri, C. (2005) Control by interconnection of mixed port-Hamiltonian systems. *IEEE Trans. Automat. Contr.*, **50**, 1839–1844.

Macchelli, A. & Melchiorri, C. (2009) Control by interconnection of distributed port-Hamiltonian systems based on finite elements approximation. *Proceedings of the 48h IEEE Conference on Decision and Control (CDC), and the 2009 28th Chinese Control Conference*. Shanghai, China: IEEE, pp. 5133–5138.

Macchelli, A. & Melchiorri, C. (2010) Passivity-based control of spatially discretized port-Hamiltonian system. *IFAC Proceedings Volumes*, **43**, 849–854.

Macchelli, A., Melchiorri, C. & Bassi, L. (2005a) Port-based modelling and control of the Mindlin plate. *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference, CDC-ECC '05*, vol. 2005. Seville, Spain: IEEE pp. 5989–5994.

Macchelli, A., Melchiorri, C. & Stramigioli, S. (2007a) Port-based modeling of a flexible link. *IEEE Trans. Robot*, **23**, 650–660.

Macchelli, A., Melchiorri, C. & Stramigioli, S. (2009) Port-based modeling and simulation of mechanical systems with rigid and flexible links. *IEEE Trans Robot*, **25**, 1016–1029.

Macchelli, A., Stramigioli, S. & Melchiorri, C. (2006) Port-based modelling of manipulators with flexible links. *IEEE International Conference on Robotics and Automation*, vol. 2006. Orlando, FL, USA: IEEE pp. 1886–1891.

Macchelli, A., Stramigioli, S. & Melchiorri, C. (2007b) Port-based finite element model of a flexible link. *IFAC Proceedings Volumes*, **40**, 158–163.

Macchelli, A., van der Schaft, A. & Melchiorri, C. (2004a) Multi-variable port-Hamiltonian model of piezoelectric material. *2004 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, vol. 1. Sendai, Japan: IEEE, pp. 897–902.

Macchelli, A., van der Schaft, A. & Melchiorri, C. (2004b) Port Hamiltonian formulation of infinite dimensional systems I. Modeling. *2004 43rd IEEE Conference on Decision and Control (CDC)*, vol. 4, pp. 3762–3767. IEEE.

Macchelli, A., van der Schaft, A. & Melchiorri, C. (2005b) Control by interconnection for distributed port Hamiltonian systems. *IFAC Proceedings Volumes*, **38**, 489–494.

Malzer, T., Rams, H. & Schöberl, M. (2019) Energy-based in-domain control of a piezo-actuated Euler-Bernoulli beam. *IFAC-PapersOnLine*, **52**, 144–149.

Maschke, B. & van der Schaft, A. J. (2013) On alternative Poisson brackets for fluid dynamical systems and their extension to Stokes-Dirac structures. *IFAC Proceedings Volumes*, **46**, 109–114.

Moulla, R., Lefèvre, L. & Maschke, B. (2011) Geometric pseudospectral method for spatial integration of dynamical systems. *MATH COMP MODEL DYN*, **17**, 85–104.

Moulla, R., Leféвre, L. & Maschke, B. (2012) Pseudo-spectral methods for the spatial symplectic reduction of open systems of conservation laws. *J. Comput. Phys.*, **231**, 1272–1292.

NISHIDA, G. & MASCHKE, B. (2018) Topological geometric extension of Stokes-Dirac structures for global energy flows. *18th European Control Conference (ECC)*. Naples, Italy: IEEE, pp. 1878–1883.

NISHIDA, G., MASCHKE, B. & IKEURA, R. (2015) Boundary Integrability of multiple Stokes–Dirac structures. *SIAM J. Control Optim.*, **53**, 800–815.

NISHIDA, G., MASCHKE, B., & YAMAKITA, M. (2008a) Topological geometry and control for distributed port-Hamiltonian systems with non-integrable structures. In *Proceedings of the IEEE Conference on Decision and Control*. Cancun, Mexico: IEEE, pp. 1291–1297.

NISHIDA, G. & SAKAMOTO, N. (2010) Port-based modeling of magnetohydrodynamics equations for tokamaks. *Proceedings of the IEEE International Conference on Control Applications*. Yokohama, Japan: IEEE, pp. 842–847.

NISHIDA, G., SUGIURA, M., YAMAKITA, M., MASCHKE, B. & IKEURA, R. (2009) Boundary detection of variational symmetry breaking using port-representation of conservation laws. *Proceedings of the IEEE Conference on Decision and Control*. Shanghai, China: IEEE, pp. 2861–2868.

NISHIDA, G., SUGIURA, M., YAMAKITA, M., MASCHKE, B. & IKEURA, R. (2012) Multi-input multi-output integrated ionic polymer-metal composite for energy controls. *Micromachines*, **3**, 126–136.

NISHIDA, G., TAKAGI, K., MASCHKE, B., & LUO, Z. (2008b) Multi-scale distributed port-Hamiltonian representation of ionic polymer-metal composite. *IFAC Proceedings Volumes, 17(1 PART 1)*, pp. 2300–2305.

NISHIDA, G., TAKAGI, K., MASCHKE, B. & OSADA, T. (2011) Multi-scale distributed parameter modeling of ionic polymer-metal composite soft actuator. *Control Eng. Pract.*, **19**, 321–334.

NISHIDA, G., YAMAGUCHI, K. & SAKAMOTO, N. (2013) Optimality of passivity-based controls for distributed port-Hamiltonian systems. *IFAC Proceedings Volumes*, **46**, 146–151.

NISHIDA, G. & YAMAKITA, M. (2004) A higher order Stokes-Dirac structure for distributed-parameter port-Hamiltonian systems. *Proceedings of the 2004 American Control Conference*, vol. 6. Boston, MA, USA: IEEE, pp. 5004–5009.

NISHIDA, G. & YAMAKITA, M. (2005) Formal distributed port-Hamiltonian representation of field equations. *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference*, vol. 2005. Seville, Spain, Spain: IEEE, pp. 6009–6015.

NISHIDA, G., YAMAKITA, M. & LUO, Z. (2006) Field Port-Lagrangian Representation of Conservation Laws for Variational Symmetries. *Proceedings of the 45th IEEE Conference on Decision and Control*. Location: San Diego, CA, USA: IEEE, pp. 5875–5881.

NISHIDA, G., YAMAKITA, M., & LUO, Z. W. (2007) Port-representation of bi-Hamiltonian structure for infinite-dimensional symmetry. *Proceedings of the IEEE Conference on Decision and Control*. New Orleans, LA, USA: IEEE, pp. 5588–5593.

PASUMARTHY, R., AMBATI, V. R. & VAN DER SCHAFT, A. J. (2012) Port-Hamiltonian discretization for open channel flows. *Syst. Control Lett.*, **61**, 950–958.

PASUMARTHY, R. & VAN DER SCHAFT, A. (2005) Stability and stabilization of mixed lumped and distributed parameter systems. In *Int. Symp. on Nonlinear Theory and its Applications (NOLTA2005)*.

PASUMARTHY, R. & VAN DER SCHAFT, A. (2006a) A finite dimensional approximation of the shallow water equations: The port-Hamiltonian approach. *Proceedings of the 45th IEEE Conference on Decision and Control*. San Diego, CA, USA: IEEE, pp. 3984–3989.

PASUMARTHY, R. & VAN DER SCHAFT, A. J. (2006b) A port-Hamiltonian approach to modeling and interconnections of canal systems. *16th International Symposium on Mathematical*, 2, pp. 1436–1443.

PASUMARTHY, R. & VAN DER SCHAFT, A. J. (2007) Achievable Casimirs and its implications on control of port-Hamiltonian systems. *Internat. J. Control*, **80**, 1421–1438.

POLNER, M. & VAN DER VEGT, J. J. (2014) A Hamiltonian vorticity-dilatation formulation of the compressible Euler equations. *Nonlinear Anal. Theor.*, **109**, 113–135.

RAMIREZ, H., LE GORREC, Y., MACCHELLI, A. & ZWART, H. (2014) Exponential stabilization of boundary controlled port-Hamiltonian systems with dynamic feedback. *IEEE Trans. Automat. Contr.*, **59**, 2849–2855.

RAMIREZ, H., LE GORREC, Y. & ZWART, H. (2013) Exponential stabilization of a class of flexible microgrippers using dynamic boundary port-Hamiltonian control) *Proceedings of the IEEE Conference on Decision and Control*. Florence, Italy: IEEE, pp. 460–465.

Ramirez, H., Zwart, H. & Le Gorrec, Y. (2017a) Stabilization of infinite dimensional port-Hamiltonian systems by nonlinear dynamic boundary control. *Automatica J. IFAC*, **85**, 61–69.

Ramirez, H., Zwart, H., Le Gorrec, Y., & Macchelli, A. (2017b) On backstepping boundary control for a class of linear port-Hamiltonian systems. In *2017 IEEE 56th Annual Conference on Decision and Control, CDC 2017*. Melbourne, VIC, Australia: IEEE, vol. 2018, pp. 658–663.

Rams, H. & Schöberl, M. (2017) On structural invariants in the energy based control of port-Hamiltonian systems with second-order Hamiltonian. *Proceedings of the American Control Conference*, pp. 1139–1144.

Rodriguez. Seattle, WA, USA: IEEE, H., van der Schaft, A. J., & Ortega, R. (2001) On stabilization of nonlinear distributed parameter port-controlled Hamiltonian systems via energy shaping. *Proceedings of the 40th IEEE Conference on Decision and Control*, vol. 1. Orlando, FL, USA, USA: IEEE, pp. 131–136.

Schlacher, K. (2007) Distributed PCHD-systems, from the lumped to the distributed parameter case. *Advances in Control Theory and Applications*. Berlin, Heidelberg: Springer, pp. 239–255.

Schöberl, M., Ennsbrunner, H. & Schlacher, K. (2008) Modelling of piezoelectric structures - a Hamiltonian approach. *Math. Comput. Model. Dyn.*, **14**, 179–193.

Schöberl, M. & Schlacher, K. (2015a) Lagrangian and port-Hamiltonian formulation for distributed-parameter systems. *IFAC-PapersOnLine*, **28**, 610–615.

Schöberl, M. & Schlacher, K. (2015b) Port-Hamiltonian formulation for higher-order PDEs. *IFAC-PapersOnLine*, **28**, 244–249.

Schöberl, M. & Schlacher, K. (2018) On the extraction of the boundary conditions and the boundary ports in second-order field theories. *J. Math. Phys.*, **59**.

Schöberl, M. & Siuka, A. (2011) On Casimir functionals for field theories in Port-Hamiltonian description for control purposes. *Proceedings of the IEEE Conference on Decision and Control*. Orlando, FL, USA: IEEE, pp. 7759–7764.

Schöberl, M. & Siuka, A. (2013a) Analysis and comparison of port-Hamiltonian formulations for field theories - demonstrated by means of the Mindlin plate. *2013 European Control Conference (ECC)*, vol. 1. Zurich, Switzerland: IEEE, pp. 548–553.

Schöberl, M. & Siuka, A. (2013b) On Casimir functionals for infinite-dimensional port-Hamiltonian control systems. *IEEE T Automat. Contr.*, **58**, 1823–1828.

Schöberl, M. & Siuka, A. (2014) Jet bundle formulation of infinite-dimensional port-Hamiltonian systems using differential operators. *Automatica J. IFAC*, **50**, 607–613.

Serhani, A., Haine, G. & Matignon, D. (2019a) Anisotropic heterogeneous nd heat equation with boundary control and observation: ii. Structure-preserving discretization. *IFAC-PapersOnLine*, **52**, 57–62.

Serhani, A., Matignon, D. & Haine, G. (2018) Structure-preserving finite volume method for 2D linear and non-linear port-Hamiltonian systems. *IFAC-PapersOnLine*, **51**, 131–136.

Serhani, A., Matignon, D. & Haine, G. (2019b) Partitioned finite element method for port-Hamiltonian systems with boundary damping: anisotropic heterogeneous 2d wave equations. *IFAC-PapersOnLine*, **52**, 96–101.

Serhani, A., Matignon, D. & Haine, G. (2019c) A partitioned finite element method for the structure-preserving discretization of damped infinite-dimensional port-Hamiltonian systems with boundary control) *Geometric Science of Information* (F. Nielsen & F. Barbaresco eds). Cham: Springer International Publishing pp. 549–558.

Šešlija, M., Scherpen, J. M. & van der Schaft, A. (2011) A discrete exterior approach to structure-preserving discretization of distributed-parameter port-Hamiltonian systems. *Proceedings of the IEEE Conference on Decision and Control*. Orlando, FL, USA: IEEE, pp. 7003–7008.

Šešlija, M., Scherpen, J. M. & van der Schaft, A. (2014a) Explicit simplicial discretization of distributed-parameter port-Hamiltonian systems. *Automatica J. IFAC*, **50**, 369–377.

Šešlija, M., van der Schaft, A. & Scherpen, J. M. (2010) Reaction-diffusion systems in the port-Hamiltonian framework. *IFAC Proceedings Volumes*, **43**, 837–842.

Šešlija, M., van der Schaft, A. & Scherpen, J. M. (2012) Discrete exterior geometry approach to structure-preserving discretization of distributed-parameter port-Hamiltonian systems. *J. Geom. Phys.*, **62**, 1509–1531.

Šešlija, M., van der Schaft, A. & Scherpen, J. M. A. (2014b) Hamiltonian perspective on compartmental reaction-diffusion networks. *Automatica J. IFAC*, **50**, 737–746.

Siuka, A., Schöberl, M., & Schlacher, K. (2010) Hamiltonian evolution equations of inductionless magneto-hydrodynamics. *Proceedings of the 19th International Symposium on Mathematical Theory of Networks and Systems–MTNS*, vol. 5. Budapest, Hungary.

Siuka, A., Schöberl, M. & Schlacher, K. (2011) Port-Hamiltonian modelling and energy-based control of the Timoshenko beam. *Acta Mech.*, **222**, 69–89.

Talasila, V., Golo, G., & van der Schaft, A. J. (2002) The wave equation as a port-Hamiltonian system and a finite-dimensional approximation. *Proceedings of the 15th International Symposium on Mathematical Theory of Networks and Systems*, pp. 1–15.

Toledo, J., Wu, Y., Ramirez, H. & Le Gorrec, Y. (2019) Observer-based state feedback controller for a class of distributed parameter systems. *IFAC-PapersOnLine*, **52**, 114–119. 3rd IFAC Workshop on Control of Systems Governed by Partial Differential Equations CPDE 2019.

Trang Vu, N. M., Trenchant, V., Ramirez, H., Lefèvre, L. & Le Gorrec, Y. (2017) Parabolic matching of hyperbolic system using control by interconnection. *IFAC-PapersOnLine*, **50**, 5574–5579.

Trenchant, V., Fares, Y., Ramirez, H., Le Gorrec, Y. & Ouisse, M. (2015) A port-Hamiltonian formulation of a 2D boundary controlled acoustic system. *IFAC-PapersOnLine*, **28**, 235–240.

Trenchant, V., Hu, W., Ramirez, H. & Le Gorrec, Y. (2018a) Structure preserving finite differences in polar coordinates for heat and wave equations. *IFAC-PapersOnLine*, **51**, 571–576.

Trenchant, V., Ramirez, H., Le Gorrec, Y. & Kotyczka, P. (2017a) Structure preserving spatial discretization of 2D hyperbolic systems using staggered grids finite difference. *Proceedings of the American Control Conference*, vol. 5. Seattle, WA, USA: IEEE, pp. 2491–2496.

Trenchant, V., Ramirez, H., Le Gorrec, Y. & Kotyczka, P. (2018b) Finite differences on staggered grids preserving the port-Hamiltonian structure with application to an acoustic duct. *J. Comput. Phys.*, **373**, 673–697.

Trenchant, V., Vu, T., Ramirez, H., Lefevre, L., & Le Gorrec, Y. (2017b) On the use of structural invariants for the distributed control of infinite dimensional port-Hamitonian systems. *2017 IEEE 56th Annual Conference on Decision and Control, CDC 2017*. Melbourne, VIC, Australia: IEEE, pp. 47–52.

Trivedi, M. V., Banavar, R. N. & Kotyczka, P. (2016) Hamiltonian modelling and buckling analysis of a nonlinear flexible beam with actuation at the bottom. *Math. Comput. Model. Dyn.*, **22**, 475–492.

Trivedi, M. V., Banavar, R. N. & Maschke, B. (2011) Modeling of hybrid lumped-distributed parameter mechanical systems with multiple equilibria. *IFAC Proceedings Volumes*, **44**, 7696–7701.

van der Schaft, A. & Maschke, B. (2001) Fluid dynamical systems as Hamiltonian boundary control systems. *Proceedings of the 40th IEEE Conference on Decision and Control (Cat. No. 01CH37228)*, vol. 5, pp. 4497–4502. IEEE.

van der Schaft, A. J. & Maschke, B. M. (2002) Hamiltonian formulation of distributed-parameter systems with boundary energy flow. *J. Geom. Phys.*, **42**, 166–194.

Vankerschaver, J., Yoshimura, H., Leok, M. & Marsden, J. E. (2010) Stokes-Dirac structures through reduction of infinite-dimensional Dirac structures. *Proceedings of the IEEE Conference on Decision and Control*, pp. 6265–6270. IEEE.

Villegas, J. A. (2007) A Port-Hamiltonian Approach to Distributed Parameter Systems. *Ph.D. Thesis*, University of Twente.

Villegas, J. A., Zwart, H., Le Gorrec, Y. & Maschke, B. (2009) Exponential stability of a class of boundary control systems. *IEEE Trans. Automat. Contr.*, **54**, 142–147.

Villegas, J. A., Zwart, H., Le Gorrec, Y., Maschke, B. & van der Schaft, A. J. (2005) Stability and stabilization of a class of boundary control systems. *Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference, CDC-ECC '05*, vol. 2005, pp. 3850–3855. IEEE.

Vincent, B., Hudon, N., Lefèvre, L. & Dochain, D. (2017) Burning magneto-hydrodynamics plasmas model: a port-based modelling approach. *IFAC-PapersOnLine*, **50**, 13038–13043.

Voß, T. & Scherpen, J. M. A. (2014) Port-Hamiltonian modeling of a nonlinear Timoshenko beam with Piezo actuation. *SIAM J. Control Optim.*, **52**, 493–519.

Voß, T. & Weiland, S. (2011) Structure preserving spatial discretization of 1D convection-diffusion port-Hamiltonian systems. *Proceedings of the IEEE Conference on Decision and Control*, pp. 6979–6984. IEEE.

Vu, N. M. T. & Lefèvre, L. (2013) Material balance and closure equations for plasmas in Tokamaks. *IFAC Proceedings Volumes*, **1**, 60–65.

Vu, N. M. T., Lefevre, L. & Maschke, B. (2012) Port-hamiltonian formulation for systems of conservation laws: application to plasma dynamics in tokamak reactors. *IFAC Proceedings Volumes*, **45**, 108–113.

Vu, N. M. T., Lefèvre, L. & Maschke, B. (2016) A structured control model for the thermo-magneto-hydrodynamics of plasmas in tokamaks. *Math. Comp. Model. Dyn.*, **22**, 181–206.

Vu, N. M. T., Lefèvre, L. & Nouailletas, R. (2017a) Distributed and backstepping boundary controls for port-Hamiltonian systems with symmetries. *Math. Comp. Model. Dyn.*, **23**, 55–76.

Vu, N. M. T., Lefevre, L., Nouailletas, R. & Bremond, S. (2013a) Geometric discretization for a plasma control model. *IFAC Proceedings Volumes*, **46**, 755–760.

Vu, N. M. T., Lefèvre, L., Nouailletas, R. & Brémond, S. (2017b) Symplectic spatial integration schemes for systems of balance equations. *J. Process Control*, **51**, 1–17.

Vu, N. M. T., Nouailletas, R., Lefevre, L. & Bremond, S. (2013b) An IDA -PBC approach for the control of ID plasma profile in tokamaks. *Proceedings of the IEEE Conference on Decision and Control*. Florence, Italy: IEEE, pp. 4176–4181.

Wu, Y., Hamroun, B., Le Gorrec, Y. & Maschke, B. (2015) Power preserving model reduction of 2D vibro-acoustic system: a port-Hamiltonian approach. *IFAC-PapersOnLine*, **28**, 206–211.

Zhou, W., Hamroun, B., Couenne, F. & Le Gorrec, Y. (2017) Distributed port-Hamiltonian modelling for irreversible processes. *Math. Comput. Model. Dyn.*, **23**, 3–22.

Zhou, W., Hamroun, B., Le Gorrec, Y. & Couenne, F. (2012) Infinite dimensional port-Hamiltonian representation of chemical reactors. *IFAC Proceedings Volumes*, **45**, 248–253.

Zhou, W., Hamroun, B., Le Gorrec, Y. & Couenne, F. (2015) Infinite dimensional port-Hamiltonian representation of reaction diffusion processes. *IFAC-PapersOnLine*, **28**, 476–481.

Zwart, H., Le Gorrec, Y., Maschke, B. & Villegas, J. (2009) Well-posedness and regularity of hyperbolic boundary control systems on a one-dimensional spatial domain. *ESAIM Control Optim. Calc.*, **16**, 1077–1093.