

Numerical simulation of self-sustained oscillation of a voice-producing element based on Navier–Stokes equations and the finite element method

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Surgical removal of the larynx results in radically reduced production of voice and speech. To improve voice quality a voice-producing element (VPE) is developed, based on the lip principle, called after the lips of a musician while playing a brass instrument. To optimize the VPE, a numerical model is developed. In this model, the finite element method is used to describe the mechanical behavior of the VPE. The flow is described by two-dimensional incompressible Navier–Stokes equations. The interaction between VPE and airflow is modeled by placing the grid of the VPE model in the grid of the aerodynamical model, and requiring continuity of forces and velocities. By applying and increasing pressure to the numerical model, pulses comparable to glottal volume velocity waveforms are obtained. By variation of geometric parameters their influence can be determined. To validate this numerical model, an *in vitro* test with a prototype of the VPE is performed. Experimental and numerical results show an acceptable agreement. © 2003 Acoustical Society of America. [DOI: 10.1121/1.1560163]

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I. INTRODUCTION

Surgical removal of the larynx is sometimes the treatment of last resort for patients with laryngeal cancer. The consequences of this removal are serious, implying among others the loss of the vocal folds. Since the first surgical removal of the larynx, over a hundred years ago, several methods to replace the voice source have been developed (Mahieu, 1988). Currently most patients can achieve a useful substitute voice for speaking. For this, air is brought into the esophagus by an air intake via the mouth or from the lungs via a shunt valve, surgically placed in the wall between trachea and esophagus (Fig. 1). While air escapes from the esophagus into the pharynx, the tissue at the esophagus entrance starts to vibrate, as in the case of belching (Blom *et al.*, 1988). The one-way shunt valve prevents leakage of food and fluids from the pharynx into the lungs (Mahieu, 1988).

The basic sound is (as in laryngeal speech production) acoustically converted to speech sound in the pharynx and oral and nasal cavity. However, the source sound has a low fundamental frequency F_0 (60–80 Hz) (Damsté, 1958; Snidecor and Curry, 1959; Cornut *et al.*, 1968), whereas values for F_0 in laryngeal speech are about 110 Hz for males and 210 Hz for females. Especially for a female laryngectomized person, this low F_0 is disturbing. Moreover, speech is often monotonous.

To overcome these drawbacks, a voice-producing element that produces a source sound with a higher F_0 and frequency variation during speech is under development. The underlying principle is comparable to the oscillating lips of a

musician playing a brass instrument (Sram, 1989; Adachi and Sato, 1996). In the voice-producing element, only one oscillating lip will be placed because a voice-producing element consisting of one lip is easier to manufacture and reproduce (Fig. 2). In the neutral position, the lip is slightly pressed against the opposite wall. When air pressure is applied at the inlet of the voice-producing element, the initially closed lip opens and the airflow makes the lip oscillate as a result of aerodynamic and mechanic forces.

The voice-producing element will be placed in the shunt valve between the trachea and the esophagus and consists of a single lip of silicone rubber. Silicone rubber is used for its very low elasticity that makes low F_0 possible. Since the voice-producing element is to be placed in the shunt valve several geometric requirements have to be met: it has to be placed in a lumen with an inner diameter of 5 mm and a length of 5 to 11 mm.

Other requirements concern the sound produced by the voice-producing element; F_0 , pressure, flow, and sound pressure level (SPL) have to fall within normal human ranges:

- (i) We aim at a voice-producing element with an F_0 of about 110 Hz for males and about 210 Hz for females.
- (ii) The sound pressure level at comfortable effort has to be between 70 and 80 dB, measured at a 30-cm distance from the mouth.
- (iii) The required pressure must fall within the range for normal speech, 0.4 to 3.0 kPa (Schutte, 1980).
- (iv) In normal speaking sentences of a certain length are necessary. Considering a total lung volume of 3.5–5

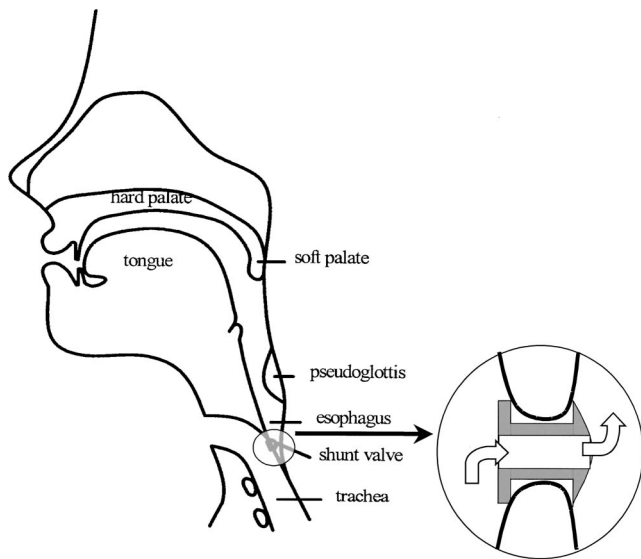


FIG. 1. Situation after laryngectomy with the location of the shunt valve. The arrows indicate the airflow direction during phonation.

L, the air flow range for driving the voice-producing element should fall in a range of 0.1–0.3 L/s as in normal phonation (Schutte, 1980).

- (v) The user of the voice-producing element must be able to vary the intensity and F_0 of the sound. Because no external control mechanisms are desired, both frequency and intensity must increase under influence of an increasing flow through the voice-producing element. While varying the applied aerodynamic forces F_0 must vary between about -10% and $+30\%$ of the values, mentioned above, and the intensity must vary between 70 and 80 dB at a distance of 30 cm (Schutte, 1992).

II. PROBLEM

To find the optimal geometry for the voice-producing element within the constraints of physiological possibilities, numerical modeling is used to avoid a trial-and-error approach. Because no numerical model on voice-producing prostheses exists, numerical models of the vocal folds are

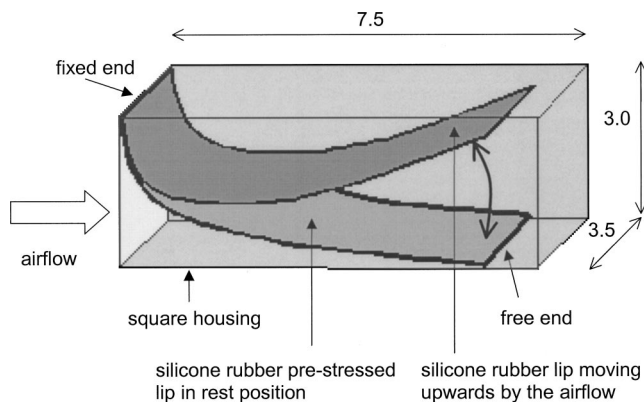


FIG. 2. Schematic representation of the voice-producing element. In rest the lip is slightly pressed against the opposite (lower) wall. The width of the lip is slightly less than the width of the square housing. Typical dimensions are given in mm.

studied. Since the human vocal folds and the new voice-producing element have much resemblance (both sound generators produce glottal waves as a result of interaction between the airflow and the oscillating lip), adapting a numerical model of the vocal folds could be a fast way to obtain a realistic model of the voice-producing element.

In numerical models of the vocal folds, both the vocal folds and the flowing air are simplified considerably, as in the models of several authors (Ishizaka and Flanagan, 1972; Titze, 1973, 1974; Pelorson *et al.*, 1994; Story and Titze, 1995). These models are often referred to as lumped-parameter models, because the properties of the vocal folds are lumped together in a small number of parameters. In most commonly used lumped-parameter models, the vocal folds are described by a number of masses, connected to each other by a number of springs and dampers. In these models, the behavior of the flowing air is approximated by the one-dimensional Bernoulli equation, implying a constant pressure distribution in a cross section. The air around the vocal folds does not behave like a one-dimensional flow, because aerodynamic pressure acts on both sides of the folds. As a consequence, a more detailed description of the aerodynamics is essential for an accurate examination of the behavior of the vocal folds. New numerical models are developed to describe the behavior of the airflow around the lip more accurately using two-dimensional Navier–Stokes equations (Alipour and Titze, 1996; Vries *et al.*, 2002). These models are a good basis for modeling the voice-producing element. Although it is possible to describe the lip by a number of lumped parameters using the numerical method described by de Vries *et al.* (1999), the lip can be represented more accurately with a finite element method (FEM) model.

The aim of this study is to determine the influence of the geometry of the lip and housing using a numerical model. For different configurations of the lip, the pressure where vibration starts, the range of self-sustained oscillation, the flow needed for voice production, and the possibilities to vary F_0 will be determined. From the results of the numerical simulations, an optimal configuration for the lip in the voice-producing element will be derived. To validate the numerical model, *in vitro* experiments are performed.

III. METHOD

A. Numerical model

The developed numerical model consists of two parts: a model of the aerodynamics based on Navier–Stokes equations and a model of the mechanics of the lips based on FEM. First the aerodynamics and mechanics are discussed separately, followed by the description of the interaction between the two models.

B. Aerodynamics

The model of the aerodynamics is a second-order version of the symmetry-preserving method of Verstappen and Veldman (1998) that is used extensively in simulating turbulent flow.

For the computation of the aerodynamic part, incompressible two-dimensional Navier–Stokes equations are

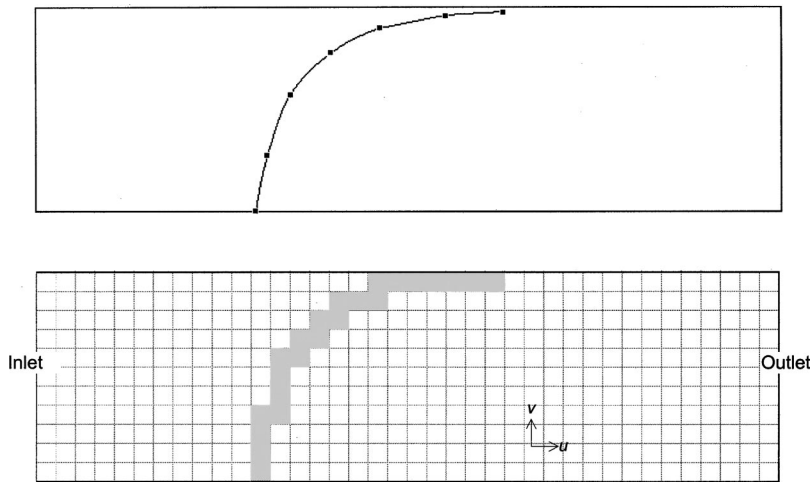


FIG. 3. Mechanical element distribution (above) and schematic representation of the grid of aerodynamic cells (white) in interaction with the mechanic cells (gray, below). Direction of velocities u and v is indicated.

used, as described by de Vries *et al.* (2002). Spatial discretization is performed in a Cartesian grid. This grid can be refined at places of particular interest. The three degrees of freedom of each cell in the grid (Fig. 3) are the horizontal velocity u , the vertical velocity v , and the pressure p .

As boundary conditions, velocities at the solid walls are $u=0$ and $v=0$. This describes an impermeable wall and a no-slip condition, which means that the fluid sticks to the wall due to its viscosity. At the inlet and outlet the pressure is prescribed, with the pressure difference driving the flow. Additionally, at the inlet and outlet the normal derivatives of the velocity components are set to zero, modeling fully developed flow. The latter condition requires the inlet and outlet to lie sufficiently far away from the interesting parts of the flow field.

The boundary conditions at the moving lip will be described below.

C. Mechanics

FEM is used to describe the mechanical behavior of the voice-producing element. Since the numerical model describing the aerodynamics is two-dimensional, the FEM model of the voice-producing element is two-dimensional as well. The thickness of the lip is small compared to its length, therefore it is allowed to approximate the lip using beam elements. In this way, the geometry in a cross-section is fixed, which would not be the fact when a three-dimensional element type, like a shell, would be chosen. The lip is divided in a number of beam elements, connected to each other in nodes. The beam element used has three degrees of freedom in every node: an axial and transverse translation, and a rotation.

The movement of the lip is restricted by the upper and lower wall of the voice-producing element. Therefore, during a cycle of the movement of the lip, contact between the lip and the upper and lower wall is assumed to collide with dissipation of all kinetic energy.

The material of the lip is chosen to be silicone rubber. This material is used for the shunt valves also. In the numerical simulations the silicone rubber lip is assumed to have a density of 1130 kg/m^3 , a Young's modulus of 8.6 MPa , and a proportional damping of 0.01 . The initially straight lip with a

length of 7 mm is bent 90° to fit into the housing that has an inner height of 3 mm . The shape of the deformed lip and the corresponding forces to keep the lip in the deformed shape are calculated with the nonlinear algorithm of the FEM program ANSYS 5.5 (SWANSON Analysis, USA). For all configurations of the lip, this calculation is performed. The forces that are needed to bend the lip are transformed into a pressure distribution along the lip and added to the pressure distribution resulting from the aerodynamic calculations. In this way, the lip is assumed to behave linearly in the final numerical model, moving around an equilibrium state that is calculated using a nonlinear method.

D. Interaction between aerodynamics and mechanics

To study the interaction between aerodynamics and mechanics, the two separate models are integrated by placing the mechanical model (calculated by FEM) in the grid of the aerodynamic model (Fig. 3). The cells of this grid can contain a volume of air or a part of the geometry. The interaction between aerodynamics and mechanics is obtained by exchanging information in the common grid in every time step.

The velocity of the air is equalized to the velocity of the surface of the lip; in this way a continuous velocity field is obtained.

The Navier–Stokes equations compute a pressure field. From this pressure field, forces acting on the lip are calculated by integrating the pressure distribution in the cells adjacent to every beam element and sum the resulting forces in every node between the beam elements. In this way, continuity of the force field is achieved. These forces are used to compute the movement of the lip using FEM. The new positions and velocities of the lips form the input to the next time step of the aerodynamics.

E. Transient numerical simulation

To achieve self-sustained oscillation, a pressure is prescribed at the inlet of the voice-producing element. In the simulations, pressure is increased from 0 up to 3 kPa . During the pressure rise, the lip starts to oscillate at a certain pressure, defined as vibration threshold pressure (Fig. 4), analogous to the phonation threshold pressure defined by Titze

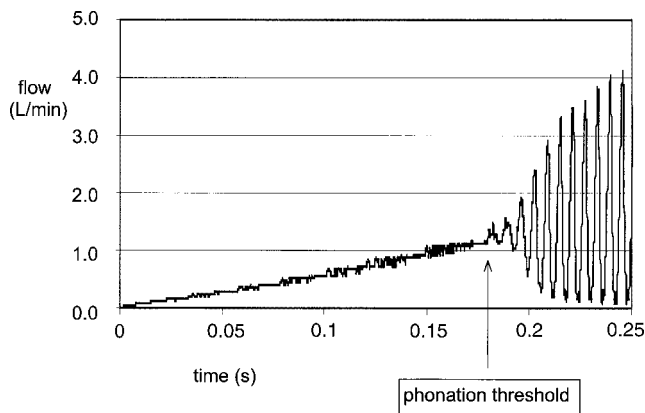


FIG. 4. Determination of vibration threshold pressure: while increasing the tracheal pressure, the moment of the start of self-sustained oscillation is established. The corresponding value of the tracheal pressure at that time is called the vibration threshold pressure for that configuration.

(1992). From this point on to higher pressures, the relation between tracheal pressure, mean flow, and F_0 are investigated, up to the pressure where no regular glottal waves are produced.

Several configurations of the lip are implemented, see Table I. A basic configuration is chosen and relative to this configuration, three parameters are varied: the thickness of the lip, the length of the lip, and the height of the housing that the lip is placed in. From this parameter study, an indication of the desired value of the different parameters will be obtained.

F. Numerical verification

The behavior of the mechanical part of the model is accurate when the behavior of the lip by the linear beam theory in static and dynamic behavior is valid. Because the quotient of thickness and length of the beam is very low, the lip can be considered to behave like the beam theory.

The numerical verification of the complete model, including the interaction between the aerodynamic model and the mechanic model, is done by varying the density of the grid of the aerodynamic model from 80×25 to 150×50 and the number of elements in the mechanic model from 3 to 9. When finer grids and more elements do not result in different results, the grid size and number of elements is considered to be optimal.

TABLE I. Dimensions of the simulated configurations of the voice-producing element. Dimensions that differ from the basic configuration are indicated in bold.

	Thickness lip (mm)	Length lip (mm)	Height element (mm)
Basic configuration	0.25	7	3
Long lip configuration	0.25	9	3
Short lip configuration	0.25	5	3
Higher opening configuration	0.25	7	4
Tapered configuration	0.25 (bottom)– 0.125 (free tip)	7	3
Thin lip configuration	0.125	7	3

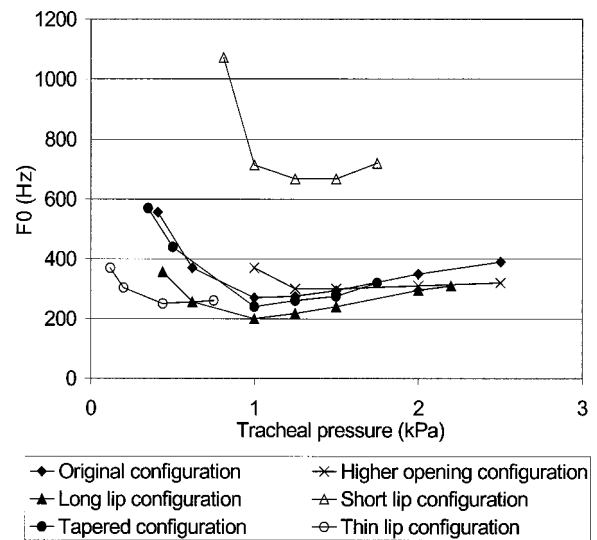


FIG. 5. F_0 as a function of tracheal pressure, as calculated by the simulation model. Curves start at pressures, where oscillation starts, and ends at pressures where oscillations become nonperiodical.

IV. EXPERIMENTS

To validate the numerical model of the lip principle and to obtain insight into the behavior of the lip principle, *in vitro* experiments are performed with a prototype of the voice-producing element in a test setup as described by Van der Plaats *et al.* (submitted). The flow is slowly increased by changing the pressure relief valve of a pressurized air cylinder from 0 to a flow that in this setup relates to a pressure of 3 kPa. The vibration threshold pressure F_0 as a function of tracheal pressure and mean flow rate is registered.

Both pressure and flow values of the *in vitro* experiments mentioned above are decreased by 40% to let the dip of the *in vitro* curve coincide with the dip of the simulated curve. This necessity can be explained by leakage along the lip and leakage caused by torsion of the lip. This rather high amount of leakage is caused by the provisional way of manufacturing and corresponds to other experiences with valve systems (Mihaylov *et al.*, 2000).

The corrected results of the *in vitro* experiments are compared to the values determined with the numerical model. Also a comparison with values obtained in laryngeal phonation is made to examine the correspondence between the voice produced by the voice-producing element and voice produced by the vocal folds.

V. RESULTS

A basic grid of 100 cells in the flow direction and 30 cells in the transverse direction appeared to be optimal because refining the grid did not change the results significantly, it only increased calculation time. For the same reason, a representation of the lip by six elements appeared to be the best.

In Fig. 5, the relation between F_0 and the simulated tracheal pressure is shown. At low tracheal pressure, the lip is in rest and no sound is made. At very high tracheal pressure, the lip is pressed against the opposite wall and no periodic sound is produced. Between these extremes, the lip vibrates

TABLE II. Characteristics of the relation between F_0 and tracheal pressure and between F_0 and mean flow though the voice-producing element, resulting from the numerical simulation model.

	Vibration threshold pressure (kPa)	Minimum F_0 (Hz)	Pressure at minimum F_0 (kPa)	Mean flow at minimum F_0 (L/s)	Increase of F_0 with mean flow (1/L)
Basic configuration	0.41	270	1.00	0.10	822
Long lip configuration	0.44	200	1.00	0.13	688
Short lip configuration	0.81	667	1.30	0.08	3533
Higher opening configuration	1.00	300	1.50	0.26	87
Tapered configuration	0.35	230	1.00	0.10	1067
Thin lip configuration	0.12	250	0.40	0.05	256

periodically. From Fig. 5 it can be seen that F_0 depends on the tracheal pressure, but in a different way for each configuration. All configurations show an initial decrease and slow increase of F_0 with increasing pressure. It is clear that the vibration threshold pressure (Fig. 4) differs for the different configurations, as is presented in Table II. It can be seen that the thickness and length of the lip and the height of the opening in the housing is important for the determination of the vibration threshold pressure: glottal volume velocity waveforms are produced at 0.12 kPa for the thin lip, whereas 0.41 kPa is needed for the basic configuration. The short lip starts oscillating at a pressure of 0.81 kPa, whereas the basic and long lip start oscillating at pressures of 0.41 and 0.44 kPa, respectively. The configuration with the highest opening has the highest vibration threshold pressure.

The range over which self-sustained oscillation occurs depends on the configuration: the thin lip has a phonation range of only 0.6 kPa, whereas the basic configuration has a range of more than 2 kPa.

The relation between F_0 and the mean flow is shown in Fig. 6. The mean flow through the simulated voice-producing element is almost linearly related to the tracheal pressure, therefore a minimum in the F_0 as a function of the mean flow

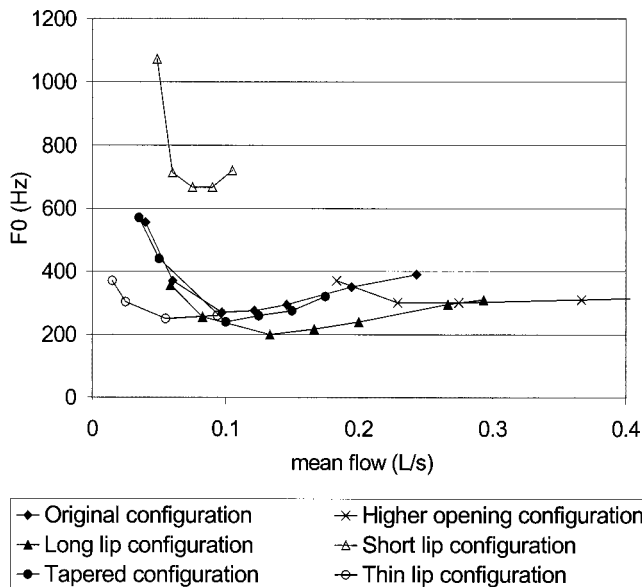


FIG. 6. F_0 as a function of mean flow, as calculated by the simulation model. Curves start at flows, where oscillation starts and ends at flows where oscillations become nonperiodical.

is also present. It must be noted that the higher opening configuration results in a considerable shift towards higher flow values.

Figures 4, 6, and 7 show that the relation between F_0 and tracheal pressure and between F_0 and mean flow all show a U-shape of the graph. A rise of tracheal pressure and flow results in a decrease of F_0 for all configurations until a

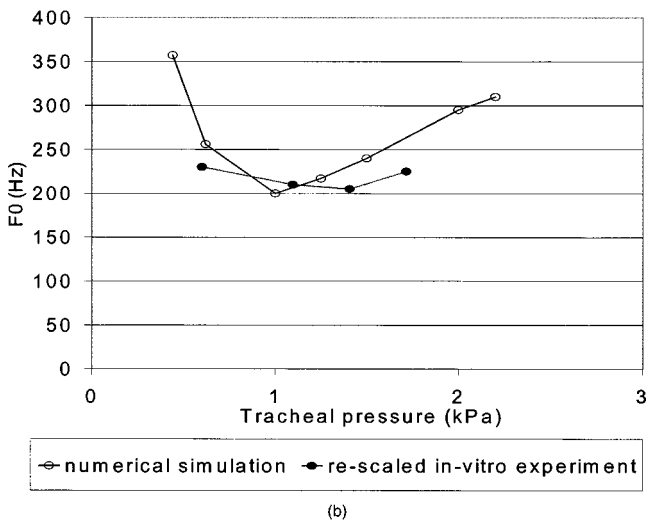
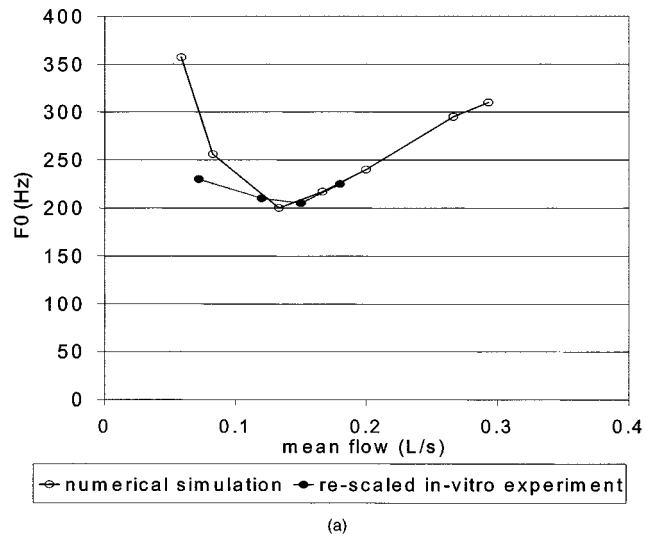


FIG. 7. Relation between F_0 and mean flow (a) and between F_0 and mean pressure (b) of the long lip configuration, resulting from the numerical simulation model and from the *in vitro* experiments. *In vitro* results are re-scaled by 0.6 to account for leakage.

minimum F_0 is reached. The minimum F_0 and the corresponding tracheal pressure and mean flow are presented in Table II. After that point F_0 increases slightly with increasing pressure and flow. In Table II, the rate of increase is represented by the slope of the line connecting the two most extreme F_0 values. The tracheal pressure at which the minimum F_0 occurs appears to be related with the vibration threshold pressure: a low vibration threshold pressure corresponds to a low pressure at which the minimum F_0 occurs.

The relations between F_0 , tracheal pressure, and mean flow of the prototype used in the *in vitro* experiments are shown in Fig. 7, together with the simulation results of the corresponding long lip configuration. In this way, the difference and correspondence between the results of the *in vitro* experiments and of the numerical simulations can be used to judge the validity of the numerical model.

From Fig. 7(a) it can be seen that the two curves of the relation between F_0 and mean flow, resulting from the *in vitro* experiments and from the simulations both have a falling and raising part, so show the same trend. The rising parts of both graphs even coincide almost completely. The two curves of the relation between F_0 and tracheal pressure, resulting from the *in vitro* experiments and from the simulations [Fig. 7(b)], correspond less, but are still acceptable. Both in Figs. 7(a) and (b), it can be seen that the range in mean flow, respectively tracheal pressure, as is simulated is larger than the range found during the *in vitro* experiments.

VI. DISCUSSION

The values of tracheal pressure needed for the production of glottal waves with the voice-producing element should fall in a range comparable to laryngeal voice production. Therefore, these values are compared to those measured in laryngeal phonation (Schutte, 1980). A mean value for the tracheal pressure of 0.44 kPa and a mean airflow rate of about 0.2 L/s were measured. Considering the results of the simulations, it appeared that four of the six configurations oscillate at that pressure. However, the ideal relation between F_0 and pressure is not yet obtained. At low pressures, an increasing pressure causes a decreasing F_0 . The normal intonation pattern is characterized by an increase of F_0 with increasing pressure. Patients who are able to put more effort in speech than the comfortable value of about 1 kPa (Schutte, 1980), by producing pressures up to 3 kPa, can reach that intonation pattern. The other two configurations (short lip and higher opening) start to oscillate only at higher pressures.

In the ideal flow range of 0.1–0.3 L/s, three of the six configurations oscillate according to the requirements: increasing mean flow causes an increasing F_0 . The higher opening configuration requires too much mean flow, and speech duration will be shortened too much; the thin lip and short lip configuration require a nonphysiological low mean flow.

The long lip configuration fulfills frequency requirements for female patients best. The lowest frequency obtained is 200 Hz. The required female frequency increase of

80 Hz per 0.2 L/s = 400 Hz/L/s is realized by the basic, long lip, short lip, and tapered configuration.

The assumption of the flow to be incompressible is valid in our model because in our application, the airflow velocity did not exceed 50 m/s, which is below Mach 0.2, at which compressible effects can be considered negligible. The 2D approach sometimes leads to unrealistic results. The pressure and flow range at which oscillation occurs is larger in the simulations than in the *in vitro* experiments: *In vitro* the vibration threshold pressure appeared to be slightly higher; also, oscillation above the maximum phonation tracheal pressure and mean flow appears to be nonperiodic and not useful for voice production, whereas during simulation much higher pressures and flows still produce periodic oscillations. Most probably, 3D effects like leakage along the lip and torsion of the lip (absent in the 2D numerical model) cause the limited range found during the *in vitro* experiments.

In both Figs. 7(a) and (b), the curves resulting from the simulation and from the *in vitro* experiments show a large similarity, thus providing the validity of the numerical model.

As a continuation, the numerical model has been used to determine the optimal male and female configuration. The resulting prototypes have been tested on aero-acoustics (Van der Torn *et al.*, 2001).

VII. CONCLUSIONS

The numerical model appears to be a valid tool to study the mechanical and aerodynamical behavior of a voice-producing element.

Lip length, lip thickness, and height of the housing influence the performance of the voice-producing element. From the simulated versions, the best female voice-producing element is the “long lip configuration.”

The numerical model can be used for further optimization studies of the voice-producing element, for instance by combining several successful configurations like the long and tapered one. A lower opening configuration could lead to a shift towards a lower flow range.

ACKNOWLEDGMENTS

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