SURROGATE MODELING FOR FAILURE PROBABILITY ESTIMATION OF MULTIFUNCTIONAL FLOOD DEFENCES

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ABSTRACT

Reliability of flood defences is one of the main concerns of water managers in low land countries such as the Netherlands. Population growth, economic development and climate change are main drivers for the development of solutions such as multifunctional flood defences (MFFD). This type of structure combines the primary flood defence function with additional ones such as commercial, residential and recreational. Failure mechanisms of mono functional flood defenses have been studied for a long time in the Netherlands, in order to estimate more accurately the reliability of their flood defense system. MFFD's will probably be exposed to these same failure mechanisms but their occurrence might also be triggered by the effect of complementary functions embedded in the defence body. The present study aims to develop a methodology which allows to consider the effect of structure embedment in the occurrence of piping erosion failure mechanism. In particular for the eventual embedment of a sewer pipe underneath a flood defence. The method consisted in modelling via finite element a flood defence with and without a sewer pipe embedded underneath. In order to consider the erosion progression for different water level and hydraulic conductivities, a simplified method for solving the aquifer flow was implemented. From the results obtained, two artificial neural network emulators where trained and validated. As a final step the emulators where used for failure estimation by several Monte Carlo runs. The main results from the study show that the embedment of and additional structure will change the failure probability significantly (by a factor of 8 in the present study) and that emulators are capable of representing the highly nonlinear behavior of complex models without significantly compromising the calculation accuracy.

Keywords: *Multifunctional, Flood defence, Piping, Failure, Surrogate*

1. INTRODUCTION

In the Netherlands, projects like the VNK (Jongejan, et al. 2013) have devoted great attention to determine and improve the probabilistic estimation methods for the safety assessment of their levee systems. One of the main goals of the study was to prioritize which failure mechanisms contributed the most to the total failure probability per flood defence system. Reiteratively backward piping erosion was found to be a major threat for most of the flood defence structures. This type of failure consists in the progressive erosion of the flood defence foundation due to water movement between the impervious flood defence contact area and a granular soil. Several empirical models have been developed since the early 19th century for assessing the piping erosion process. The most common ones for flood defence assessment are Bligh and Lane (Ojha, et al. 2008). More recently, a theoretical and numerical model was developed in the Netherlands by Sellmeijer (1991). It combines the Darcy's porous media flow theory with fluid motion theory solved with the Navier-Stokes equations. This model has proven to be quite accurate for estimating a critical water head for piping to occur when compared to the results of experimental tests. However, the Dutch s methods for flood defence system safety assessment is based on a probabilistic approach. Consequently large amounts of model runs of different parameter combinations are required for the failure probability estimation of such systems. Hence, the complete numerical solution of the theoretical model is not feasible for stochastic purposes. For probabilistic assessments, Limit state functions are developed in order to determine the "safety" state of a structure given a combination of input parameter. Limit state equations have the general form Z = R-S, where (R) denotes the term of resistance to deterioration and (S) refers to the deterioration driving forces. Each failure mechanism has its own LSE which can result in a negative (unsafe) or positive (safe) value, after evaluating function Z. Sellmeijer, also derived a limit state function for the piping erosion failure mechanism. This equation was revised and calibrated later (Sellmeijer, et al. 2011) for different scaled experiments.

In the case of a multi-functional flood defence (MFFD) designs for example, the enlargement of the dimensions and the inclusion of additional functions will most probably require the implementation of sewer and drainage systems. In other words, the embedment of pipes either in the body or the foundation of the flood defence. For the piping erosion case in particular, the effects of such embedment may change the behavior of the water flow inside the aquifer in unknown ways. Such effects are completely dependent on structures characteristics such as location, diameter and orientation with respect to the flood defence. All this considerations cannot be represented by the actual limit state equation previously mentioned. Most of the physical deterioration processes like piping erosion for example, are widely understood and therefore can be modelled by more complex numerical models while considering the embedment of hard structures.

In summary, the actual limit state equations do not consider the effect of embedded additional structures in the flood defence main body and the computational time demand of more complex models make their stochastic implementation non-feasible. The present study aims to apply a different modelling strategy that can help to include the embedment of additional structures such as a sewer pipe inside the foundation. Once the model is constructed, several runs for different water loads are calculated and used to estimate the reliability of the flood defence against piping.

2. SELLMEIJER PIPING EROSION

Flood retaining structures are prone to piping erosion when founded in granular soils. If sufficient pressure is built up underneath the structure, it will eventually be uplifted generating an additional space between the structure and its foundation. This process is also commonly known as heave/uplift. In the case of riverine levees, a clay layer is normally found underneath which helps the maintain the core of the structure isolated from the water inflow of aquifers.

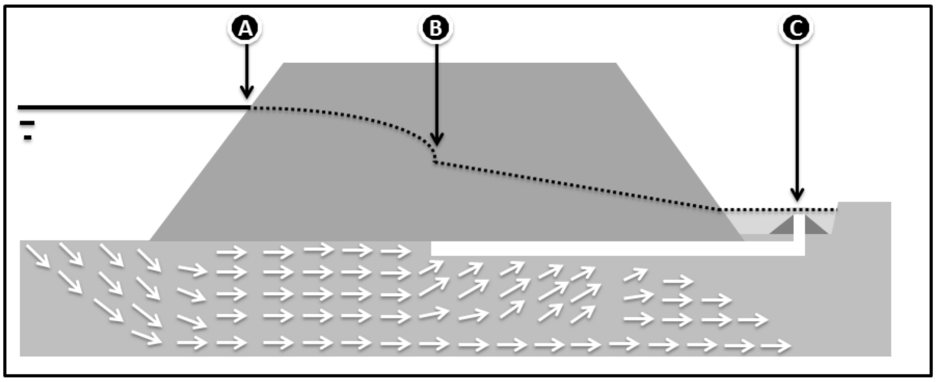

Figure 1. Piping erosion process near the slit and inside the sand boils.

For water to be able to flow through the existent space, an outlet must be also generated in order to ensure the water flow coming from underneath towards the hinter side of the structure. In practice, such outlet (Figure 1.) can be identified with the presence of a “Sand Boil”. Sand boils (location C) are sudden outbursts of water mixed with soil grains which progressively increase until the pressure gradient comes to an equilibrium state. Progressive transport of larger sand grains between the impervious structure and the pervious foundation will increase the length and diameter of the cavity, if sufficient pressure is build up in the outer side of the flood defence. Once the cavity also known as “pipe” has progressed more than half of the width (location B) of the levee, experimental research has shown that the erosion will continue as long as there is any head difference. Therefore, the critical head for this type of failure is estimated as the required head difference (Pressure difference between locations A and C) for the cavity to continue progressing after it has reached the middle of the structure.

2.1 Numerical model

In principle, Sellmeijer model aims to estimate the flow inside the pipe in order to determine if there is sufficient flow momentum so that the particles can be dragged towards the san boil (Figure. 1). Since this flow is a function of the water pressures inside the aquifer from the external hydraulic loads, a porous media model should be solved as well. This model is solved for a steady state of the system where no transient loads are considered inside the porous medium. Consequently no time dependency is required in its numerical solution. Hence, the flow inside the aquifer can be solved by the steady state Darcy’s law (Eq. [1].) :

$$\boldsymbol{u} = -\frac{k}{\mu}(\nabla p + \rho g \nabla z) \quad [1]$$

The equation is used to estimate the flow velocity (u) of the water through the porous medium as a function of the soil permeability (k) and the pressure gradient. For the case of the estimation of the water flow inside the cavity, the

Navier-Stokes equation system is commonly used. Experimental work has also showed that the type of flow encountered inside the cavity is more similar and more often in the laminar Regime (Re<2300) given the small sizes of the cavity's cross section. Thus, this type of flow in longitudinal in pipes is better described by the Hagen-Poiseuille law for horizontal channels presented in Eq. 2 and Eq. 3.

$$\nabla p = \mu \Delta \boldsymbol{u} + \boldsymbol{F} \qquad [2]$$

$$\nabla \cdot (\rho \boldsymbol{u}) = 0 \qquad [3]$$

The first one represents the pressure gradient as a function of the fluid inertial force and F term for other volumetric forces such as gravity and/or external drivers. The second one represents the conservation of mass inside the control volume. These set of equations is obtained by neglecting the inertial terms in the original Navier-Stokes original equations and is also known as the Stokes equations. Despite the fact that both systems can be easily solved via finite element methods (FEM), there are two difficulties which make piping erosion modelling a great challenge for modelers. The first one is derived from the fact that the Darcy's equation and the Stokes equation are not from the same order. However, for finite element modeling there are several methods to overcome this problem such as defining a 'Slip' or a 'No Slip' boundary condition. For the case 'No slip' case where viscous effects along the wall are negligible, an imposed value of 0 can be assumed for the fluid velocity in the interface between the wall and the fluid. In the case of 'Slip' condition, an empirical relationship is imposed to the model so that the interface wall velocity can be estimated as a function of it velocity gradient. Yet the second challenge is originated from the fact that the boundary conditions of both models are dependent on each other. On one hand, the flow inside the cavity depends on the pore pressures inside the bounding aquifer. On the other hand, the pore pressures are determined by the bounding conditions which for the Darcy equation can be either pressures or normal inflow/outflow velocities. In order to overcome this challenge, the results obtained by Bersan, et al. (2013) show that the model can be solved by modeling the cavity flow as a porous media continuum as well. The flow resistance is achieved by including an "fictitious permeability" value which can be derived from Darcy's flow equation (Eq. [1]) and the head loss estimation in pipes of Hagen Poiseuille.

$$\boldsymbol{k}^* = \frac{g {D_h}^2}{32\nu} \qquad [4]$$

The fictitious permeability for a conduit of a general hydraulic diameter (D_h) can be expressed as a function (eq. 4) of the characteristics of the fluid which flows inside it. This follows the same principle for the fracture flow. When replacing, this equivalent value in the Darcy model in the area that represents the cavity, is possible to solve the whole system as a total continuous porous medium. This assumption only works for simple geometries where the cavity doesn't present sudden changes as they could make the flow turbulent. Note that, these methods relays in the assumption that flow inside the cavity is always laminar and that the cavity is prescribed as a straight line. More detailed forms of estimating the correct interaction of the porous media and the fluid flow such as the Darcy-Brinkman and the Forchheimer-Darcy (Zarghami, et al. 2014) can help to reduce the error for the pressure gradient estimation inside the cavity. Nevertheless, the Sellmeijer model assumes that the flow is always laminar inside the cavity and therefore the inertial effects can be neglected. As a final remark, is acknowledged by the authors of this study that the Darcy-Brinkman approach is a more suitable solution for modeling transitional flows between boundary's. Yet, the computational burden is higher in comparison with the fictitious permeability approach which makes it less attractive for stochastic modeling.

2.2 Sellmeijer sand particles limit state

After solving the numerical model presented before, is possible to estimate the pressure head gradient inside the cavity and consequently the force exerted to the grains along the bottom of the channel. The limit state condition of the Sellmeijer model implies that the progression of the erosion cavity exactly in the middle of the structure will continue if the pressure gradient of the water inside the cavity is capable of surpassing the rolling resistance of the superficial grains (Figure 2.). In the latest version of this model, the limit state was derived from a two force equilibrium.

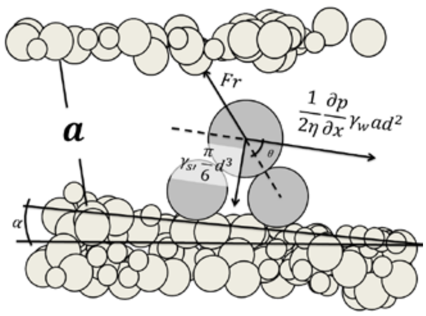

Figure 2. Two forces equilibrium state of sand grain inside the cavity with no slope.

With the summation of forces in x and y assumed in equilibrium, the relation presented in eq. 7 can be used for deriving the critical pressure gradient limit threshold. Note that for the present study, the channel is assumed as horizontal and therefore the inclination angle alpha is equal to 0.

$$\sum \boldsymbol{Fy} = \mathbf{sin}(\boldsymbol{\theta})\,\boldsymbol{Fr} - \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\eta}}\frac{\boldsymbol{\partial p}}{\boldsymbol{\partial x}}\boldsymbol{\gamma_w a d^2} = \mathbf{0} \qquad [5]$$

$$\sum \boldsymbol{Fx} = \mathbf{cos}(\boldsymbol{\theta})\,\boldsymbol{Fr} - \frac{\boldsymbol{\pi}}{\mathbf{6}}\boldsymbol{\gamma_{s'} d^3} = \mathbf{0} \qquad [6]$$

$$\frac{\boldsymbol{\partial p}}{\boldsymbol{\partial x}} = \frac{\boldsymbol{\pi}}{\mathbf{3}}\frac{\boldsymbol{\gamma_{s'}}}{\boldsymbol{\gamma_w}}\frac{\boldsymbol{d\eta}\,\mathbf{tan}(\boldsymbol{\theta})}{\boldsymbol{a}} \qquad [7]$$

When the pressure gradient from the flow inside the cavity exceeds this value the equilibrium condition is surpassed and grains will start flowing towards the sand boil (location c in figure 1). For the present study, the limit state equation (eq. 8) is estimated by assuming the water loads in the riverside and the hydraulic conductivity, as the random variables to evaluate. The rest of the involved variables are assumed as constant.

$$\boldsymbol{G(WL,K)} = \frac{\boldsymbol{\pi}}{\mathbf{3}}\frac{\boldsymbol{\gamma_{s'}}}{\boldsymbol{\gamma_w}}\frac{\boldsymbol{d\eta}\,\mathbf{tan}(\boldsymbol{\theta})}{\boldsymbol{a}} - \frac{\boldsymbol{\partial p}}{\boldsymbol{\partial x}} \qquad [8]$$

3. SURROGATE MODELS FOR PROBABILISTIC ASSESSMENT OF MFFD

In order to quantify the impact in the piping failure mechanisms of the structural embedment, the present study proposed building two different models of the same structure (Figure 3.). Afterwards, surrogate models can be built for each of them in order to calculate the flood defences reliability for each case. Surrogate models are simpler approximations of the original models by the use of algorithms capable or representing the highly nonlinear behavior while being time efficient. Surrogates are constructed from input/output data bases produced from the original model runs. The first step is to be able to construct stable models which are capable of running the different input combinations without crashing. In the case of piping erosion, the simplification proposed by Bersan, allows to solve the model without having the difficulties of the complete Darcy and Navier Stokes mentioned in section 2.1. Before building the surrogates it is required to define the original models from where the input and output relations are going to be produced. The first one consists in a single function flood defence located over a sandy aquifer and a second one with the same geotechnical characteristics but also including a sewer conduct as shown in Figure 2. Both models are two dimensional porous medium models built for steady condition in the Finite element COMSOL package.

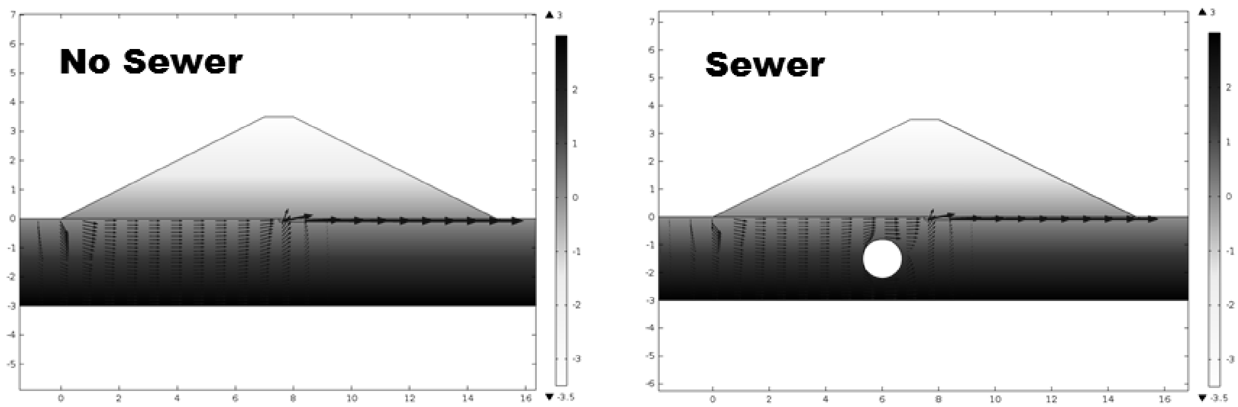

Figure 3. Piping erosion process near the slit and inside the sand boils.

Both models are built with the same type of boundary conditions, however the second one includes an no flow boundary condition inside the aquifer which represents the future sewer pipe. For the present configuration of the sewer model, the addition of a sewer pipe will also represent an additional resistance measure for the flow when moving towards the cavity. Note that there is also a cross section reduction which might also accelerate the flow depending on the sewer pipe position. Therefore, these effects should be reflected in the hydraulic gradient inside the pipe cavity. In this order of ideas, the same combination of parameters for the two models will produce a different hydraulic gradient. This two gradients can be compared with the Sellmeijer equilibrium condition in order to define the safety state of the structure.

In order to express the safety state of the structure in terms of probabilities, a Monte Carlo probabilistic approach is implemented. This method consists in generating a large number of simulations of a system by changing all input parameters involved in the model in such a way that their intrinsic probability density functions are sufficiently explored until desired accuracy is achieved. He parameters involved in the models are sampled by efficient methods such a "The Latin Hyper Cube" or "Sobol Quasi Random Sequences" (Sobol 1998). These methods allow to reduce the number of required runs while maximizing the amount of information for the surrogate building. This procedure is also known as the experiment design.

The most commonly method used in reliability is the Crude Monte Carlo method. The main challenge of this method is the calculation time from the original model as several thousand of runs are required to reduce the failure probability estimation error. Some other methods are also applied in practice which are basically implemented for time efficiency improvement of the reliability method (Hohenbichler and Rackwitz 1982). In our case, instead of improving the reliability estimation method, the surrogate model approach allows to improve the model efficiency for implementing it in the crude Monte Carlo estimation.

3.1 Finite element models

The simulation corresponds to a levee of 3.5 meters high with a maximum seepage length of 15 meters. Two different finite element models are calculated for each of the input combinations. One which considers the model without any structure embedded in the aquifer and a second one which considers a 20" diameter pipe located 10 centimeters underneath the ground level. Each model is programed to predict the water pressure along the cavity in 10 different points. With this information is possible to calculate the different pressure gradients for each K and WL combinations, which later used to evaluate the limit state equation (eq. 8). The boundary conditions are defined as presented in figure 4. Both models have the same external boundary conditions. Still, for the consideration of the embedded sewer in the aquifer, a circular no flow boundary condition is defined. The main assumptions of the model are that the pipe is not deformable and that the energy losses due to friction between the sewer pipe and the flow are negligible.

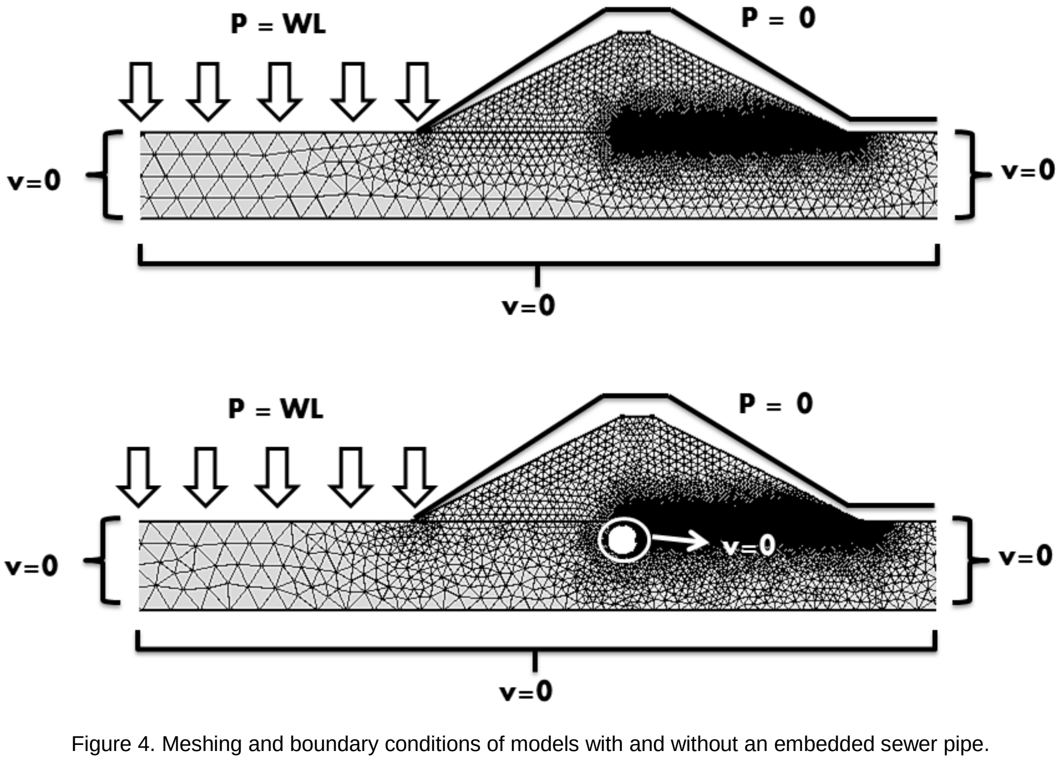

Figure 4. Meshing and boundary conditions of models with and without an embedded sewer pipe.

The diameter of the cavity is selected 9 times the mean grain size value (d70) and is assumed to have a square cross section. Its location is also deterministic (In the middle of the seepage length and one meter below the structure) for all the combinations. The same parameters combination is used to generate data from the two different situations. The flood defence is founded over a granular soil (medium sand). The total calculation takes almost one hour for all predefined combinations in each model. The calculated pressures are used for training an artificial intelligence based emulator able to predict the average pressure gradient inside the cavity. This type of algorithms are very fast and powerful for prediction of highly nonlinear processes. Due to the fact of the emulators being data driven, the calculation times are significantly lower which makes them suitable for probabilistic analysis.

3.2 Surrogate model training data

The experiment design for surrogate model building consists in identifying which parameters are more sensitive to the original model's output variance. As it was shown in section 2.2, the pressure gradient inside the pipe is required to evaluate the limit state equation. Hence, the water load is one of the inputs that influences the pressure gradient the most. For the Sellmeijer model, the hydraulic conductivity is the parameter that affects most the output variance. Once the "important parameters" are identified, it is not smart to start sampling them in an unbounded range. Therefore it is also useful to reduce the sampling space either by the model feasibility and/or by the type of marginal distribution of the parameters themselves. For example, if a model is not applicable above a certain parameter threshold, there is no point in including values above this range in the sampling design. Or, if a certain variable is distributed with a marginal density function which is highly skewed, greater attention should be given to the sampling of tail events.

The model parameter information for the sampling design is presented in table 1. Besides the selection and bounding of the experiment design parameters is also important to define an "Intelligent sampling scheme. The implementation of such a sampling scheme ensures a better coverage of the probabilistic space by ensuring sufficient distance between sampling points in order to have better coverage of the output space as well. In this case, a 2 variable "Latin Hypercube" sampling method is chosen for generating the surrogate training data. 1000 combinations of values of WL and K where generated.

Table 1. Parameter marginal distribution for surrogate input sampling.

Variable	Unit	Dist. Type	Mean	C.V.
η	[-]	Constant	0.25	-
γ'_{sand}	[kN/m^3]	Constant	16	-
γ_w	[N/m^3]	Constant	9.81	-
θ	[deg.]	Constant	37	-
d_{70}	[m]	Constant	1.8e-4	-
K	[m/s]	Log-normal	9.00e-5	1.5
WL	[m]	Gumbel	a=1.117	b=0.491
a	[m]	Constant	1.8e-3	

Emulators are highly accurate inside their training variable space. Therefore they should be trained for situations (parameter combinations) which ensure that the most coverage of the feasible space is achieved. For the present study, both flood defence cases are assessed for the random parameters presented in table 2 which have a smaller spreading in the feasible space. In other words, the random variables for training have intentionally a smaller variance in order to avoid extrapolation as much as possible.

Table 2. Random parameters used for safety assessment.

Variable	Unit	Dist. Type	Mean	C.V.
K	[m/s]	Log-normal	9.00e-4	1.0
WL	[m]	Gumbel	a=1.019	b=0.14

Another important consideration in emulator training is to ensure that the input variables are completely uncorrelated for increasing the coverage of the output space. This characteristic can be ensured during the latin hypercube generation routine. For the two dimensional space of the present emulators the correlation degree of K vs WL is -0.054 and their spatial distribution is presented in figure 5.

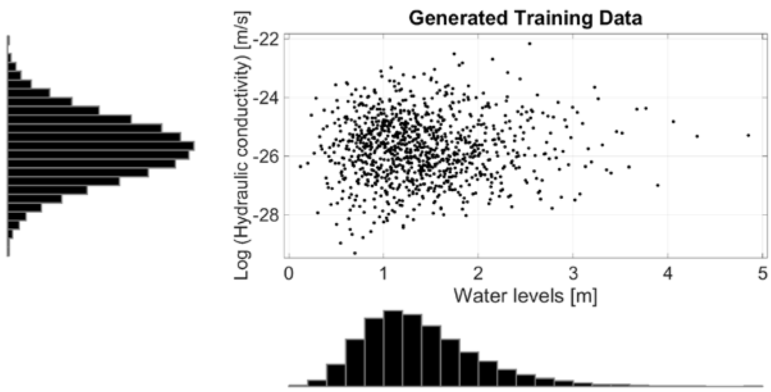

Figure 5. Spatial distribution of generated random values for emulator training

3.3 Surrogate model algorithm

Several machine learning algorithms such as Kriging or Polynomial chaos expansions are commonly used for "imitating" highly nonlinear models such as the FEM models. Yet, for simplicity and because algorithm performance is out of the scope of the present study, an Artificial Neural Network (ANN) algorithm is chosen as the approximation tool. Zobel and Keeling (2008) showed in their study how ANN's are a suitable tool for emulating models for producing probabilistic distributions. This method also presented good accuracy for tail located percentiles which is required in probabilistic studies such as this one. Both neural networks where built with 1 hidden layer of 10 nodes. The 70% of the original FEM output data is used for training and the other 30% is used for validation.

4. RESULTS

The results are divided in two main sections. The first one shows the training and validation performance of the neural networks trained for the emulation process. The second part shows the results of the implementation of the neural networks for the failure probability estimation.

4.1 Neural Network training and validation

Both training and validation plots are presented for both emulation models in figure 6. It can be observed that both models achieve quite high correlation degrees. In normal modeling practice, this can be assumed as an indicator of model "over fitting" of the training data. Nevertheless, in emulation practice this order of magnitude is highly desired as training data is not finite. Note that the input used for calculating the output in the original FEM model is ensured to be uncorrelated which makes the neural network training more difficult when the processes to emulate are highly nonlinear. It can also be observed that the model has less data for training in extreme values which in the present study represent high pressure gradients. In cases the such extreme values have high probability of occurrence during the safety assessment, new data can be generated for obtaining more data around that output data coverage. In that case, the already emulated model can be used to identify the input parameter values that can produce such output and then refine the initial training data set by estimating the output from the original models.

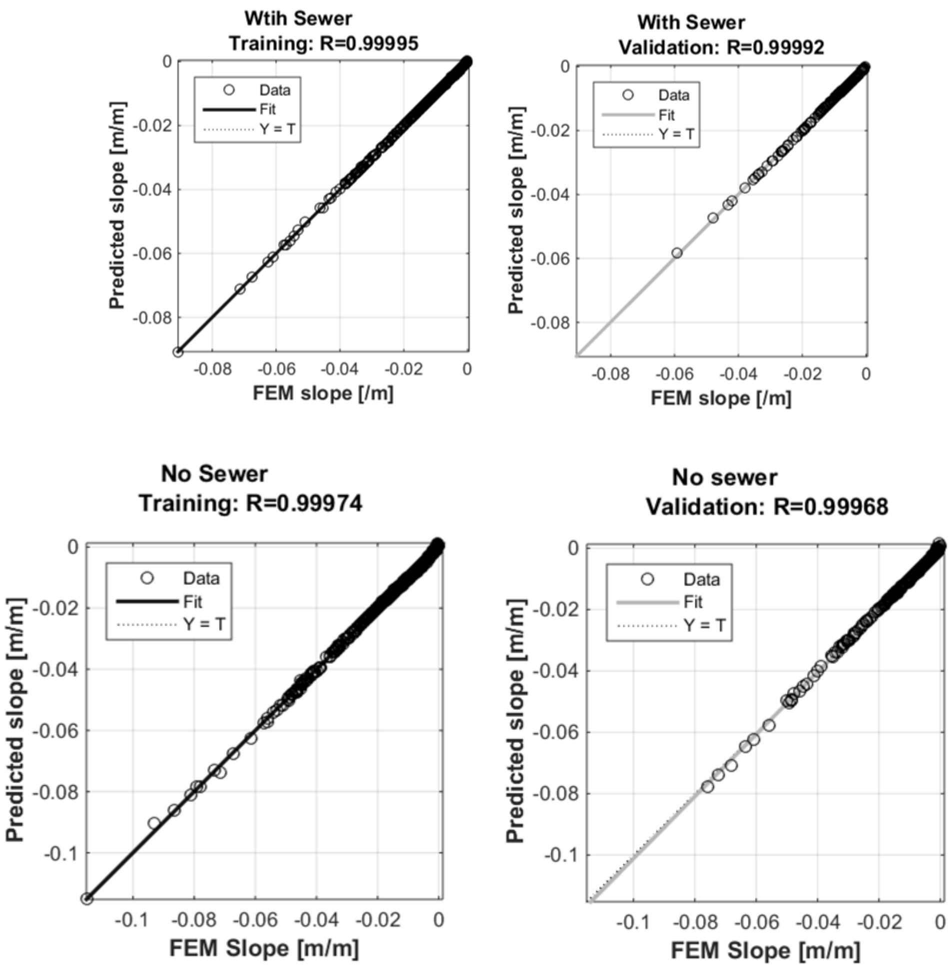

Figure 6. training and validation plots for the two neural networks trained for predicting the pressure gradient inside the cavity

4.2 Failure probability estimation

A Monte Carlo estimation of 1e6 samples was performed for both emulators. The probability marginal distributions of the pressure gradients inside the cavity is presented in figure 7. The inclusion of the pipe not only affects the value of the mean expected value of the pressure gradient but also its standard deviation. The change in both statistical moments affect the failure probability significantly as the area under the curve is redistributed in a nonlinear way. Note that all values with higher pressure gradients than the critical value (-0.0374) estimated with equation 7 are considered in a failure state.

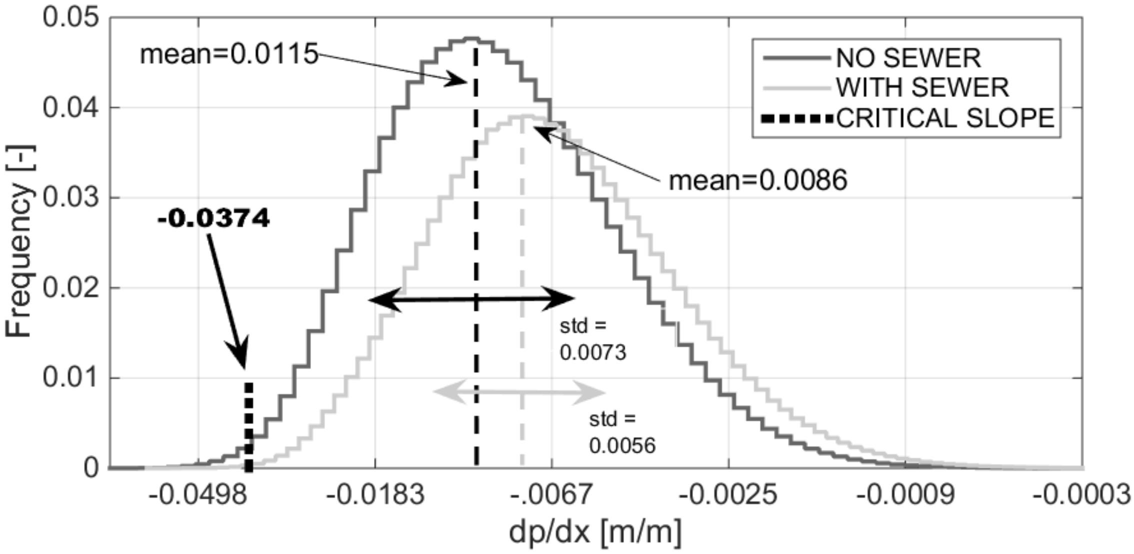

Figure 7. Pressure gradient probability density functions for models with and without structural embedment

In terms of failure probability, after integrating the probability density function it can be observed that for the case of embedment of a sewer pipe inside the aquifer, the failure probability is reduced by a factor of 8 (Figure 8.). For high pressure gradients (dp/dx < -0.05), the function becomes less smooth. This can be attributed either to the difficulty of sampling input combinations of higher resultant higher pressure gradients and the lower information available for training the emulators around the zone of this combination of input variables. This is one of the main challenges for emulator training as low frequency tail located values are more difficult to sample.

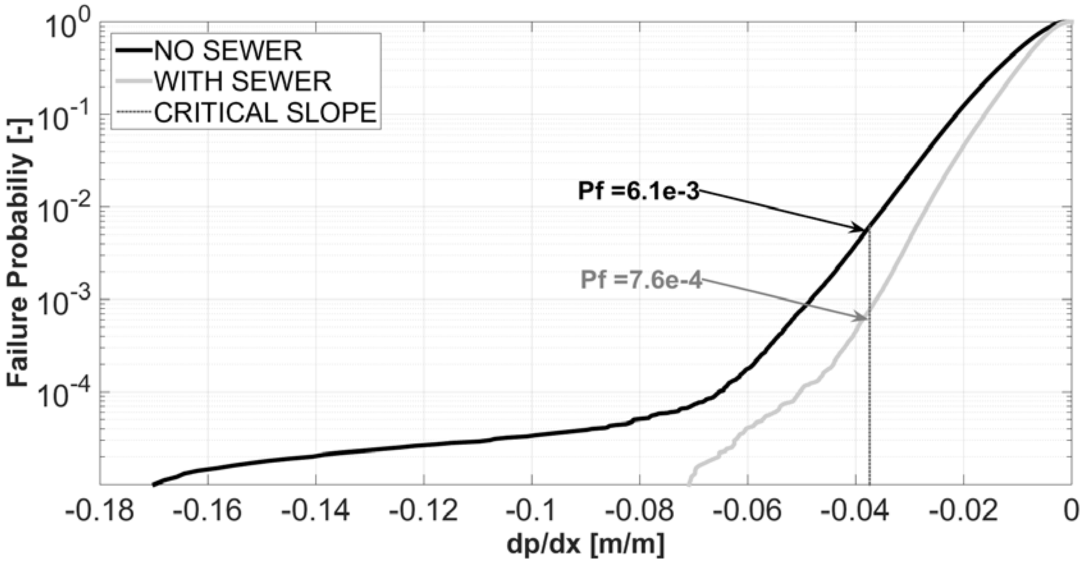

Figure 8. Pressure gradient cumulative density functions for models with and without structural embedment

5. DISCUSSION

When piping erosion needs to be avoided and no enlargement of the levee width (slope modification or berm addition) is possible, a common practice is to include "Sheet piles" in order to increase the water flow resistance and hence the erosion progression. The obstacle obliges the flow lines to diverge around the surface of the embedded structure (Figure 3.). The flow lines that are directed underneath the structure will have longer paths to travel and therefore an additional energy loss is expected. The same principle applies to the embedded structures, in this case the sewer pipe. Yet, the pipes are normally covered by soil on top to protect them. This will mean that there is going to be a space between the body of the embedded structure and the flood defence which allows water to flow in between. Consequently, the energy loss is expected to be less in comparison with a sheet pile. The size of the sewer pipe will also influence this energy loss as the trajectories are also dependent on the size of the obstacle. The case modeled in the present study allows to test the methodology for further study of the influence of characteristics such as size, cover depth and location. For the present study configuration, the failure probability was reduced by almost an order of magnitude (from 6.1e-3 to 7.6 e-4) which probes that the inclusion of embed structures in the soil have a potential use for reduction of piping erosion processes.

The obtained marginal distributions are a good source of information for identifying the weak spots where the emulators can be refined. For the present study it can be observed that the probability failure values presented in figure 8 reflect higher uncertainty for pressure gradients lower than 0.05. This can be easily explained by the fact that the training data is scarce for combinations around that output area (Figure 7.). In any case, the critical slope value is located inside the training space where sufficient data is available. This can also be corroborated by the fast error convergence of the Monte Carlo method for estimating the failure probability.

The methodology tested for simplifying the calculation by including a fictitious hydraulic conductivity simplifies enormously the computational burden when solving the aquifer FEM models. Each model has to solve 243650 elements per run for predicting pressure in all the nodes for each water level and hydraulic conductivity combination. Nevertheless, the original tested method considers the cavity to have an infinite with which coincides with the fracture flow approach commonly used for solving this type of models. For the present study, the cavity was assumed to have a squared cross section with an equivalent dimension of 10 times the representative grain size. This value was taken as a reference value form the conclusions of experimental and filed observations of the piping erosion process. Yet, the fictitious hydraulic conductivity is highly influenced by the selection of this value. In fact, during the safety assessment process of piping erosion the representative particle diameter is also assumed as a random variable which consequently will convert the cross section as a random variable as well. Nevertheless, both models where assessed under the same conditions which means that the uncertainty of the cross section area is included equally in both models. When geometrical uncertainties are required to be included in the finite element models, new stability challenges arouse as the meshing has to be changed as well. This will also affect the calculation times as every model will have to be re-meshed every time a new geometrical parameter is modeled. For the present type of failure, little has been studied about the size of the cavity with respect to the parameters included in its modelling. The obtained failure probabilities represent reasonable values in their order of magnitude when compared to structures (No structural embedment) with similar conditions and parameter uncertainties. However is important to note that piping erosion failure in most of the cases for flood defences, cannot occur unless a heave process occurs as well. The structural embedment will also affect the occurrence of heave which is not taken into account in the present study. In order to include this effects more complex models are required in order to estimate the deformation of the soil because of the additional pressures transmitted from the embedded structures to the flood defence.

6. CONCLUSIONS

The present methodology allows to successfully capture the effect of failure probability with the emulators while improving the time of calculation significantly. The effect of embedded structures in the aquifer cannot be captured by the available limit state equations. Bersan's et al approach of fracture flow modeling proves also to be a powerful simplification of the model for reliability calculations. The most important conclusion from the present study is that the inclusion of embedded structures in the foundation of the flood defence can be beneficiary, as it will include an additional resistance to the water flow against piping erosion of the structure. In the present configuration, the failure probability is reduced from 6.1e-3 to 7.6e-4.

The approximation of solving the cavity as a Darcy's continuum porous medium reduces the time of calculation significantly making it feasible for stochastic modeling. Nonetheless, for FEM models the optimization of the calculation mesh is another important part for computational burden optimization which is out of the scope of this study. Therefore is recommended complement this modeling study with the FEM mesh refinement for an optimal computational time of as single case. For the present study, the size of the eroded cavity plays an important role in the calibration of the model. The importance of this value has not been studied in the actual literature and only reference values where found as qualitative data obtained from the experimental research. This methodology proves to be a useful tool for the assessment of the impact of additional structures in the safety assessment of multifunctional flood defences. The situation studied is not as effective as the inclusion of a sheet pile but for the case of design of multifunctional flood defences, it can be considered as an additional safety margin during the design optimization process.

7. ACKNOWLEDGEMENTS

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