Scale-free Protocol Design for Output Synchronization of Heterogeneous Multi-agent subject to Unknown, Non-uniform and Arbitrarily Large Input Delays

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Abstract: This paper studies output synchronization problems for heterogeneous networks of continuous- or discrete-time right-invertible linear agents in presence of unknown, non-uniform and arbitrarily large input delay based on localized information exchange. It is assumed that all the agents are introspective, meaning that they have access to their own local measurements. Universal linear protocols are proposed for each agent to achieve output synchronizations. Proposed protocols are designed solely based on the agent models using no information about communication graph and the number of agents or other agent models information. Moreover, the protocols can tolerate arbitrarily large input delays.

1 Introduction

Synchronization problem of multi-agent systems (MAS) has become a hot topic among researchers in recent years. Cooperative control of MAS is used in practical application such as robot network, autonomous vehicles, distributed sensor network, and others. The objective of synchronization is to secure an asymptotic agreement on a common state or output trajectory by local interaction among agents (see [1, 18, 33, 44] and references therein).

Most of the attention has been focused on state synchronization of MAS, where each agent has access to a linear combination of its own state relative to that of the neighboring agents, called full-state coupling [24, 25, 26, 31, 32, 37]. A more realistic scenario which is partial-state coupling (i.e. agents share part of their information over the network) is studied in [38, 14, 29, 39].

We identify two classes of MAS: homogeneous and heterogeneous. The agents dynamics can be different in heterogeneous networks. For heterogeneous network it is more reasonable to consider output synchronization since the dimensions of states and their physical interpretation may be different. Meanwhile a common assumption, especially for heterogeneous MAS is that agents are introspective; that is, agents possess some knowledge about their own states. There exist many results about this type of agents, for instance, homogenization based synchronization via local feedback [11, 47], H_∞ design [13], HJB based optimal synchronization [19], adaptation based event trigger regulated synchronization [30], and a feed forward design for nonlinear agents [4]. Recently, scale-free collaborative protocol designs are developed for continuous-time heterogeneous MAS [22] and for homogeneous continues-time MAS subject to actuator saturation [17].

On the other hand, for non-introspective agents, regulated output synchronization for a heterogeneous network is studied in [27, 28]. Other designs can also be found, such as an internal model principle based design [43], distributed high-gain observer based design [10], low-and-high gain based, purely distributed, linear time invariant protocol design [9].

In practical applications, the network dynamics are not perfect and may be subject to delays. Time delays may afflict systems performance or even lead to instability. As it has been discussed in [3], two kinds of delay have been considered in the literature: input delay and communication delay. Input delay is the processing time to execute an input for each agent whereas communication delay can be considered as the time for transmitting information from origin agent to its destination. Some research have been done in the case of communication delay [36, 45, 20, 21, 16, 5, 6, 35, 7, 12]. Regarding input delay, many efforts have been done (see [2, 16, 15, 36, 46, 26]) where they are mostly restricted to simple agent models such as first and second-order dynamics for both linear and nonlinear agents dynamics. Authors of [41, 40] studied state synchronization problems in the presence of unknown, uniform constant input delay for both continuous- and discrete-time networks with higher-order linear agents. [48] has studied synchronization in homogeneous networks of both continuous- and discrete-time agents with unknown, non-uniform, and constant input delays.

In this paper, we deal with output synchronization problem for heterogeneous MAS with continuous- or discrete-time introspective right-invertible agents in presence of unknown, non-uniform and arbitrarily large input delays. Scale-free protocols are designed based on localized information exchange which do not require any knowledge of the communication network except connectivity. In particular, output synchronization of heterogeneous networks are achieved for an arbitrary number of agents. The protocol design is scale-free, namely,

- The design is independent of information about communication networks such as spectrum of the associated Laplacian matrix. That is to say, the universal dynamical protocols work for any communication network as long as it is connected.

- The dynamic protocols are designed solely based on agent models and do not depend on communication network and the number of agents.
- The proposed protocols archive output synchronization for heterogeneous continuous- or discrete-time MAS with any number of agents, any unknown, non-uniform, input delays, and any communication network.

Due to space limitation, we have omitted proofs of Theorem 1 and 2 and numerical example which is available in the extended version of this paper [23].

Notations and definitions

Given a matrix $A \in \mathbb{R}^{m \times n}$, A^{T} denotes its conjugate transpose and $\|A\|$ is the induced 2-norm while $\sigma_{\min}(A)$ denotes the smallest singular value of A. Let j indicate $\sqrt{-1}$. A square matrix A is said to be Hurwitz stable if all its eigenvalues are in the open left half plane and is Schur stable if all its eigenvalues are inside the open unit disc. We denote by $\text{diag}\{A_1, \ldots, A_N\}$, a block-diagonal matrix with $A_1, \ldots, A_N$ as the diagonal elements. $A \otimes B$ depicts the Kronecker product between A and B. I_n denotes the n-dimensional identity matrix and 0_n denotes $n \times n$ zero matrix; sometimes we drop the subscript if the dimension is clear from the context. Notation $\overline{[t_1, t_2]}$ means $\overline{[t_1, t_2]} = \{t \in \mathbb{Z} : t_1 \leqslant t \leqslant t_2\}$.

To describe the information flow among the agents we associate a *weighted graph* $\mathcal{G}$ to the communication network. The weighted graph $\mathcal{G}$ is defined by a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$ where $\mathcal{V} = \{1, \ldots, N\}$ is a node set, $\mathcal{E}$ is a set of pairs of nodes indicating connections among nodes, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the weighted adjacency matrix with non negative elements a_{ij}. Each pair in $\mathcal{E}$ is called an *edge*, where $a_{ij} > 0$ denotes an edge $(j, i) \in \mathcal{E}$ from node j to node i with weight a_{ij}. Moreover, $a_{ij} = 0$ if there is no edge from node j to node i. We assume there are no self-loops, i.e. we have $a_{ii} = 0$. A *path* from node i_1 to i_k is a sequence of nodes $\{i_1, \ldots, i_k\}$ such that $(i_j, i_{j+1}) \in \mathcal{E}$ for $j = 1, \ldots, k-1$. A *directed tree* is a sub-graph (subset of nodes and edges) in which every node has exactly one parent node except for one node, called the *root*, which has no parent node. The *root set* is the set of root nodes. A *directed spanning tree* is a sub-graph which is a directed tree containing all the nodes of the original graph. If a directed spanning tree exists, the root has a directed path to every other node in the tree.

For a weighted graph $\mathcal{G}$, the matrix $L = [\ell_{ij}]$ with

$$\ell_{ij} = \begin{cases} \sum_{k=1}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j, \end{cases}$$

is called the *Laplacian matrix* associated with the graph $\mathcal{G}$. The Laplacian matrix L has all its eigenvalues in the closed right half plane and at least one eigenvalue at zero associated with right eigenvector $\mathbf{1}$ [8]. Moreover, if the graph contains a directed spanning tree, the Laplacian matrix L has a single eigenvalue at the origin and all other eigenvalues are located in the open right-half complex plane [33].

2 Problem Formulation

Consider a MAS consisting of N non-identical linear agents:

$$\begin{cases} s x_i(t) = A_i x_i(t) + B_i u_i(t), \\ y_i(t) = C_i x_i(t), \end{cases} \tag{1}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^p$ are the state, input, output of agent i for $i = 1, \ldots, N$. In the aforementioned presentation, for continuous-time systems, s denotes the time derivative, i.e., $s x_i(t) = \dot{x}_i(t)$ for $t \in \mathbb{R}$; while for discrete-time systems, s denotes the time shift, i.e., $s x_i(t) = x_i(t+1)$ for $t \in \mathbb{Z}$.

The agents are introspective, meaning that each agent collects a local measurement $z_i \in \mathbb{R}^{q_i}$ of its internal dynamics. In other words, each agent has access to the quantity

$$z_i(t) = C_i^m x_i(t). \tag{2}$$

where $z_i(t) \in \mathbb{R}^{q_i}$.

We define the set of graphs $\mathbb{G}^N$ for the network communication topology as following.

Definition 1 *Let $\mathbb{G}^N$ denote the set of directed graphs of N agents which contain a directed spanning tree.*

The communication network provides agent i with the following information which is a linear combination of its own output relative to that of other agents:

$$\zeta_i(t) = \sum_{j=1}^{N} a_{ij}(y_i(t) - y_j(t)) \tag{3}$$

where $a_{ij} > 0$ and $a_{ii} = 0$, and $\mathcal{A} = [a_{ij}]$ is the weighting matrix associated to the network graph $\mathcal{G}$. We refer to this case as partial-state coupling since only part of the states is exchanged among the agents. When $C = I$, it means all states are communicated over the network and we call it full-state coupling. Basically, output synchronization is considered for heterogeneous MAS, therefore, we focus on partial-state coupling.

In this paper, we also introduce a localized information exchange among protocols. In particular, each agent $i = 1, \ldots, N$ has access to the localized information, denoted by $\hat{\zeta}_i(t)$, of the form

$$\hat{\zeta}_i(t) = \sum_{j=1}^{N} a_{ij}(\eta_i(t) - \eta_j(t)) \tag{4}$$

where $\eta_j(t) \in \mathbb{R}^n$ is a variable produced internally by agent j and to be defined in next sections.

In the case of networks with discrete-time agents, for any graph $\mathcal{G} \in \mathbb{G}^N$, with the associated Laplacian matrix L, we define

$$D = I - (I + D_{\text{in}})^{-1} L \tag{5}$$

where

$$D_{\text{in}} = \text{diag}\{d_{in}(i)\} \tag{6}$$

with $d_{in}(i) = \sum_{j=1}^{N} a_{ij}$. The weight matrix $D = [d_{ij}]$ is a so-called, row stochastic matrix, where $d_{ij} \geqslant 0$, and we choose $d_{ii} = 1 - \sum_{j=1, j \neq i}^{N} d_{ij}$ such that $\sum_{j=1}^{N} d_{ij} = 1$ for $i, j \in \{1, \ldots, N\}$. Note that d_{ii} satisfies $d_{ii} > 0$.

Therefore, for discrete-time networks we can obtain the information exchange as

$$\zeta_i(t) = \sum_{j=1}^{N} d_{ij}(y_i(t) - y_j(t)) \tag{7}$$

and we can rewrite $\hat{\zeta}_i(t)$ as

$$\hat{\zeta}_i(t) = \sum_{j=1}^{N} d_{ij}(\eta_i(t) - \eta_j(t)) \tag{8}$$

The heterogeneous MAS is said to achieve output synchronization if

$$\lim_{t \to \infty}(y_i(t) - y_j(t)) = 0, \quad \text{for } i, j \in \{1, \ldots, N\}. \tag{9}$$

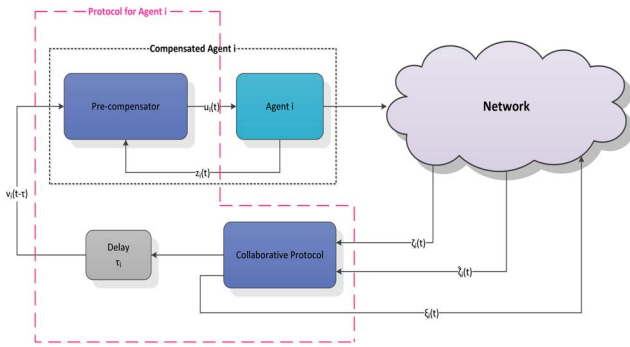In this paper, we introduce a protocol architecture as shown below in Figure 1.

Figure 1: Architecture of the protocol

As seen in the above figure our protocol has two main modules.

1) precompensator
2) collaborative protocol

we also assume that agents are experiencing input delay τ_i represented by the delay block in the figure. Note that as discussed in [3] input delay can be considered as the summation of computation time and execution time. We formulate output synchronization problem for continuous-or discrete-time heterogeneous networks as follows.

Problem 1 *Consider a heterogeneous network of N agents* (1) *with local information* (2) *and a given $\bar{\tau}$, where $\tau_i < \bar{\tau}$ for $i = 1, \ldots, N$. Let the associated network communication be given by* (3) *and* (4) *for continuous-time and by* (7) *and* (8) *for discrete-time networks. Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 1.*

*The **scalable output synchronization problem based on localized information exchange in presence of input delay** is to find, if possible, a linear dynamic controller for each agent $i \in \{1, \ldots, N\}$, as illustrated in Figure 1, using only knowledge of agent models, i.e. (C_i, A_i, B_i), and upper bound on delays $\bar{\tau}$. Then, output synchronization* (9) *is achieved for any N and any graph $\mathcal{G} \in \mathbb{G}^N$.*

Remark 1 *Note that in our problem formulation it is embedded that our linear dynamic protocols are designed only*

based on agent models (C_i, A_i, B_i) and given upper bound on input delays $\bar{\tau}$. Moreover, this universal protocol is scale-free, meaning that it works for any network with any number of agents, as long as the associated communication graph contains a spanning tree.

We make the following assumption for the agents.

Assumption 1 *For agents $i \in \{1, \ldots, N\}$,*
  1) (C_i, A_i, B_i) *is stabilizable and detectable.*
  2) (C_i, A_i, B_i) *is right-invertible*
  3) (C_i^m, A_i) *is detectable.*

Then we have the following solvability condition.

Solvability Condition 1 *Consider a heterogeneous network of N agents* (1) *with local information* (2) *satisfying Assumption 1 and a given $\bar{\tau}$. Let the associated network communication be given by* (3) *and* (7) *for continuous- and discrete-time MAS respectively. Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 1.*

Then, the scalable output synchronization problem based on localized information exchange in presence of input delay as defined in Problem 1 is solvable.

Proof: Proof will be given in next sections. ∎

3 Protocol Design

In this section, we design dynamic protocols to solve scalable output synchronization problem for heterogeneous networks of continuoue- or discrete-time agents. The architecture of the proposed protocol is illustrated in Figure 1.

Architecture of the protocol

Our design methodology consists of two modules as shown in Figure 1.
  1) The first module is to reshape the dynamics of the agents to obtain the target model by designing precompensators following our previous results stated in [47, 42].
  2) The second module is designing collaborate protocols for almost homogenized agents to achieve output synchronization in presence of input delay.

3.1 Designing pre-compensators

In this section, the goal of the design is to reshape the agent models and obtain suitable target model i.e. (C, A, B) such that following conditions are satisfied.
  1) $\text{rank}(C) = p$.
  2) (C, A, B) is invertible of uniform rank $n_q \geq \bar{n}_d$, and has no invariant zeros.
  3) eigenvalues of A are in closed left half plane (closed unit disc for discrete-time systems).
  4) eigenvalues of A satisfy the additional property

$$\bar{\tau}\omega_{\max} < \frac{\pi}{2}, \tag{10}$$

where $\omega_{\max}$ is defined as

$$\omega_{\max} = \begin{cases} 0, & \text{A is Hurwitz stable,} \\ \max\{\omega \in \mathbb{R} \mid \det(j\omega I - A) = 0\}, & \text{otherwise} \end{cases} \tag{11}$$

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for continuous-time systems, and

$$\omega_{\max} = \begin{cases} 0, & A \text{ is Schur stable,} \\ \max\{\omega \in [0,\pi] \mid \det(e^{j\omega}I - A) = 0\}, & \text{otherwise} \end{cases} \quad (12)$$

for discrete-time systems.

Remark 2 *We would like to make several observations:*

1) *The property that the triple (C, A, B) is invertible and has no invariant zero implies that (A, B) is controllable and (C, A) is observable.*

2) *The triple (C, A, B) is arbitrarily assignable as long as the conditions are satisfied. In this paper, A is chosen such that its eigenvalues are in closed left half plane (in closed unit disc for discrete-time systems). For example, one can choose A such that $A^T + A = 0$ for continuous-time systems and $A^T A = I$ for discrete-time systems.*

Next, given chosen (C, A, B), by utilizing the design methodologies from [47, Appendix B] for continuous-time and [42, Appendix A.1] for discrete-time systems, we design a pre-compensator for each agent $i \in \{1, \ldots, N\}$, of the form

$$\begin{cases} s\xi_i(t) = A_{i,h}\xi_i(t) + B_{i,h}z_i(t) + E_{i,h}v_i(t), \\ u_i(t) = C_{i,h}\xi_i(t) + D_{i,h}v_i(t), \end{cases} \quad (13)$$

which yields the compensated agents as

$$\begin{aligned} s\bar{x}_i(t) &= A\bar{x}_i(t) + B(v_i(t) + \psi_i(t)), \\ y_i(t) &= C\bar{x}_i(t), \end{aligned} \quad (14)$$

where $\psi_i(t)$ is given by

$$\begin{aligned} s\omega_i(t) &= A_{i,s}\omega_i(t), \\ \psi_i(t) &= C_{i,s}\omega_i(t), \end{aligned} \quad (15)$$

and $A_{i,s}$ is Hurwitz stable (Schur stable for discrete-time systems). Below figure shows the compensated agents as a component of Figure 1.
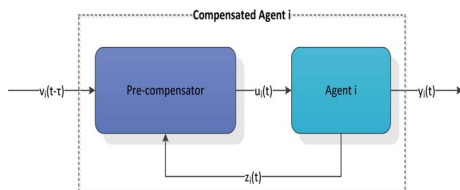

Figure 2: Compensated agent

Note that the compensated agents are homogenized and have the target models (C, A, B).

We consider compensated agents in presence of input delay as

$$\begin{aligned} s\bar{x}_i(t) &= A\bar{x}_i(t) + B(v_i(t - \tau_i) + \psi_i(t)), \\ y_i(t) &= C\bar{x}_i(t), \end{aligned} \quad (16)$$

where $\psi_i(t)$ is given by (15).

3.2 Designing collaborative protocols for compensated agents

In this subsection, to achieve output synchronization, we design collaborative protocols for almost homogenized continuous- or discrete-time agents in presence of unknown, non-uniform and arbitrarily large input delays.

3.2.1 Continuous-time MAS

We design a dynamic protocol based on localized information exchange as

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + B\hat{\zeta}_{i2}(t) + F(\zeta_i(t) - C\hat{x}_i(t)) \\ \dot{\chi}_i(t) = A\chi_i(t) + Bv_i(t - \tau_i) + \hat{x}_i(t) - \hat{\zeta}_{i1}(t) \\ v_i(t) = -\rho B^{\mathsf{T}} P_\varepsilon \chi_i(t), \end{cases} \quad (17)$$

for $i = 1, \ldots, N$ where F is a pre-design matrix such that $A - FC$ is Hurwitz stable and $\rho > 0$. ε is a parameter satisfying $\varepsilon \in (0, 1]$, P_ε satisfies

$$A^{\mathsf{T}} P_\varepsilon + P_\varepsilon A - P_\varepsilon B B^{\mathsf{T}} P_\varepsilon + \varepsilon I = 0 \quad (18)$$

note that for any $\varepsilon > 0$, there exists a unique solution of (18).

Remark 3 (18) *is a special case of the general low-gain H_2 algebraic Riccati equation (H_2-ARE), which is written as follows:*

$$A^{\mathsf{T}} P_\varepsilon + P_\varepsilon A - P_\varepsilon B R_\varepsilon^{-1} B^{\mathsf{T}} P_\varepsilon + Q_\varepsilon = 0 \quad (19)$$

where $R_\varepsilon > 0$, and $Q_\varepsilon > 0$ is such that $Q_\varepsilon \to 0$ as $\varepsilon \to 0$. In our case, we restrict our attention to $Q_\varepsilon = \varepsilon I$ and $R_\varepsilon = I$. However, as shown in [34], when A is neutrally stable, there exists a suitable (nontrivial) choice of Q_ε and R_ε which yields an explicit solution of (19), of the form

$$P_\varepsilon = \varepsilon P \quad (20)$$

where P is a positive definite matrix satisfies $A^T P + PA \leqslant 0$.

The agents communicate $\eta_i = (\eta_{i1}^{\mathsf{T}}, \eta_{i2}^{\mathsf{T}})^{\mathsf{T}}$ where $\eta_{i1}(t) = \chi_i(t)$ and $\eta_{i2}(t) = v_i(t - \tau_i)$, therefore each agent has access to the localized information $\hat{\zeta}_i = (\hat{\zeta}_{i1}^{\mathsf{T}}, \hat{\zeta}_{i2}^{\mathsf{T}})^{\mathsf{T}}$:

$$\hat{\zeta}_{i1}(t) = \sum_{j=1}^{N} a_{ij}(\chi_i(t) - \chi_j(t)), \quad (21)$$

and

$$\hat{\zeta}_{i2}(t) = \sum_{j=1}^{N} a_{ij}(v_i(t - \tau_i) - v_j(t - \tau_j)). \quad (22)$$

while $\zeta_i(t)$ is defined by (3). Finally, we combine the designed protocol for homogenized network with pre-compensators and present our protocols as:

$$\begin{cases} \dot{\xi}_i(t) = A_{i,h}\xi_i(t) + B_{i,h}z_i(t) - \rho E_{i,h}B^{\mathsf{T}} P_\varepsilon \chi_i(t), \\ \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + B\hat{\zeta}_{i2}(t) + F(\zeta_i(t) - C\hat{x}_i(t)) \\ \dot{\chi}_i(t) = A\chi_i(t) - \rho B B^{\mathsf{T}} P_\varepsilon \chi_i(t - \tau_i) + \hat{x}_i(t) - \hat{\zeta}_{i1}(t) \\ u_i(t) = C_{i,h}\xi_i - \rho D_{i,h}B^{\mathsf{T}} P_\varepsilon \chi_i, \end{cases} \quad (23)$$

Then, we have the following theorem for *scalable* output synchronization of heterogeneous continuous-time MAS in presence of input saturation.

Theorem 1 *Consider a heterogeneous network of N agents (1) with local information (2) satisfying Assumption 1 and a given $\bar{\tau}$. Let the associated network communication be given by (3). Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 1.*

Then the scalable output synchronization problem as stated in Problem 1 is solvable. In particular, there exist

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a $\rho^* > 0.5$ and for any fixed $\rho > \rho^*$, there exists an ε^* such that for any $\varepsilon \in (0, \varepsilon^*]$, dynamic protocol given by (23) and (17) solves the scalable output synchronization problem for any N and any graph $\mathcal{G} \in \mathbb{G}^N$.

Proof: Due to space limitation, we have omitted the proof. See [23]. ∎

3.2.2 Discrete-time MAS

In this subsection, we design dynamic protocols with localized information exchange for (14) and (15) as

$$\begin{cases} \hat{x}_i(t+1) = A\hat{x}_i(t) + B\hat{\zeta}_{i2}(t) + F(\zeta_i(t) - C\hat{x}_i(t)) \\ \chi_i(t+1) = A\chi_i(t) + Bv_i(t - \tau_i) + A\hat{x}_i(t) - A\hat{\zeta}_{i1}(t) \\ v_i(t) = -\rho K_\varepsilon \chi_i(t), \end{cases}$$
(24)

for $i = 1, \ldots, N$ where F is a pre-design matrix such that $A - FC$ is Schur stable, and

$$K_\varepsilon = (I + B^{\mathrm{T}} P_\varepsilon B)^{-1} B^{\mathrm{T}} P_\varepsilon A,$$

and $\rho > 0$. ε is a parameter satisfying $\varepsilon \in (0, 1]$, where for any $\varepsilon > 0$, P_ε is the unique solution of

$$A^{\mathrm{T}} P_\varepsilon A - P_\varepsilon - A^{\mathrm{T}} P_\varepsilon B(I + B^{\mathrm{T}} P_\varepsilon B)^{-1} B^{\mathrm{T}} P_\varepsilon A + \varepsilon I = 0 \quad (25)$$

Remark 4 (25) *is a special case of the general low-gain H_2 discrete algebraic Riccati equation (H_2-DARE), which is written as follows:*

$$A^T P_\varepsilon A - P_\varepsilon - A^T P_\varepsilon B(R_\varepsilon + B^T P_\varepsilon B)^{-1} B^T P_\varepsilon A + Q_\varepsilon = 0 \quad (26)$$

where $R_\varepsilon > 0$, and $Q_\varepsilon > 0$ is such that $Q_\varepsilon \to 0$ as $\varepsilon \to 0$. In our case, we restrict our attention to $Q_\varepsilon = \varepsilon I$ and $R_\varepsilon = I$. However, as shown in [34], when A is neutrally stable, there exists a suitable (nontrivial) choice of Q_ε and R_ε which yields an explicit solution of (26), of the form

$$P_\varepsilon = \varepsilon P \quad (27)$$

where P is a positive definite matrix that satisfies $A^T PA \leqslant P$.

In this protocol, the agents communicate $\eta_i = (\eta_{i1}^{\mathrm{T}}, \eta_{i2}^{\mathrm{T}})^{\mathrm{T}}$ where $\eta_{i1}(t) = \chi_i(t)$ and $\eta_{i2}(t) = v_i(t - \tau_i)$, therefore each agent has access to the localized information $\hat{\zeta}_i = (\hat{\zeta}_{i1}^{\mathrm{T}}, \hat{\zeta}_{i2}^{\mathrm{T}})^{\mathrm{T}}$:

$$\hat{\zeta}_{i1}(t) = \sum_{j=1}^{N} d_{ij}(\chi_i(t) - \chi_j(t)), \quad (28)$$

and

$$\hat{\zeta}_{i2}(t) = \sum_{j=1}^{N} d_{ij}(v_i(t - \tau_i) - v_j(t - \tau_j)). \quad (29)$$

while $\zeta_i(t)$ is defined by (7). Finally, we combine the designed protocol for homogenized network with pre-compensators designed in step 1 to get our protocol as:

$$\begin{cases} \xi_i(t+1) = A_{i,h}\xi_i(t) + B_{i,h}z_i(t) - \rho E_{i,h} K_\varepsilon \chi_i(t), \\ \hat{x}_i(t+1) = A\hat{x}_i(t) + B\hat{\zeta}_{i2}(t) + F(\zeta_i(t) - C\hat{x}_i(t)) \\ \chi_i(t+1) = A\chi_i(t) - \rho BK_\varepsilon \chi_i(t - \tau_i) + A\hat{x}_i(t) - A\hat{\zeta}_{i1}(t) \\ u_i(t) = C_{i,h}\xi_i(t) - \rho D_{i,h} K_\varepsilon \chi_i(t), \end{cases}$$
(30)

Then, we have the following theorem for *scalable* output synchronization of heterogeneous discrete-time MAS in presence of input saturation.

Theorem 2 *Consider a heterogeneous network of N agents (1) with local information (2) satisfying Assumption 1 and a given $\bar{\tau}$. Let the associated network communication be given by (7). Let $\mathbb{G}^N$ be the set of network graphs as defined in Definition 1.*

Then the scalable output synchronization problem as stated in Problem 1 is solvable. In particular, there exist a $\rho^* > 0.5$ and for any fixed $\rho > \rho^*$, there exists an ε^* such that for any $\varepsilon \in (0, \varepsilon^*]$, dynamic protocol given by (30) and (24) solves the scalable output synchronization problem for any N and any graph $\mathcal{G} \in \mathbb{G}^N$.

Proof: Due to space limitation, we have omitted the proof. See [23]. ∎

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