Model order reduction of large stroke flexure hinges using modal derivatives

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Abstract

In this work, a strategy for the model order reduction of large stroke flexure mechanisms is presented. In this, geometrically nonlinear finite element models of each flexible body within the mechanism are projected onto a reduction basis. This basis consists of the body's Craig-Bampton modes and corresponding modal derivatives. Validation simulations that are performed on commonly used flexure mechanisms show good accuracy of the reduced order models. Therefore, the proposed reduction strategy can be efficiently applied to reduce computational costs of structural optimizations that are required to improve the design of flexure mechanisms.

1 Introduction

Flexure hinges are commonly used in precision engineering because of the absence of friction, hysteresis and backlash [1]. Typically, these hinges are designed such that they have high compliance in their driving directions and high stiffness in supporting directions. Many modern applications require the flexure hinges to be well-behaved in a large range of motion. In particular, this means that the hinge should maintain its support stiffness for deformations outside of the driving direction's linear range. Because the support stiffness of standard hinges dramatically drops with deflection, more sophisticated flexure hinges are required to meet the stiffness requirement.

Due to the geometrical nonlinear nature of large stroke flexure hinges, the use of nonlinear finite element models is typically required for their design, as expressions for stiffness as a function of deflection are difficult to obtain analytically. In order to further increase the performance of flexure hinges, shape and topology optimization is used, which requires many finite element calculations. Moreover, in order to study the dynamic behavior or stability of an entire mechanism, larger finite element models or flexible multibody models are required. The high computational costs of such advanced analyses, ask for model order reduction techniques that can deal with geometric nonlinearities.

In this work, a model order reduction technique based on modal derivatives is proposed. The modal derivatives can be interpreted as the sensitivity of a structure's vibration modes to a deflection in the generalized direction of any vibration mode. The success of adding modal derivatives to a reduction basis of vibration modes in order to reduce geometrical nonlinear flexible multibody problems was demonstrated before [2]. In previous work, the authors have demonstrated that this technique can also be used to compute so-called frequency derivatives [3]. It was explained that in this way, a flexure hinge's first parasitic frequency can be determined for the large range of motion with great computational efficiency. Because the first parasitic frequency is an often-used measure for the performance of a flexure hinge, this has the potential to significantly speed-up optimization procedures.

In this work, the efficient reduction of the geometrical nonlinear models of large stroke flexure hinges will be explained. To this end, a reduction basis consisting of Craig-Bampton modes and their corresponding modal derivatives is used to reduce the model of a single flexible body. In contrast to vibration modes, the use of Craig-Bampton modes does not require any a priori knowledge of how the body is constrained, which makes the method generally applicable. This strategy is validated with the equilibrium analysis of a number of compound flexure mechanisms.

The remainder of this work is organized as follows: Chapter 2 describes how the modal derivatives of vibration modes can be determined by differentiation of the appropriate eigenvalue problem. This summarizes the essentials from the method as it is reported in literature. Chapter 3 describes how the modal derivatives of Craig-Bampton modes can be determined by differentiation of the appropriate equation of equilibrium. This summarizes the general strategy for the model order reduction of a flexible body that may undergo large deformations. Chapter 4 describes the model order reduction of a flexible multibody system based on Craig-Bampton modes and their modal derivatives. Relevant simulation results are presented and discussed. The paper is finalized by the most important conclusions.

2 Modal derivatives of vibration modes

Consider a structure that is constrained to the fixed world. It is assumed that a nonlinear finite element model is available from which the mass matrix $\mathbf{M}$ and a configuration dependent tangent stiffness matrix $\mathbf{K}(\mathbf{q})$ can be extracted. When the geometric nonlinearities are described using the nonlinear Green-Lagrange strain definition, the sensitivities of the tangent stiffness matrix with respect to the generalized coordinates can be determined analytically. Alternatively, when for instance a corotational finite element formulation is used, these sensitivities can be determined numerically.

Free vibrations about the system’s equilibrium configuration are considered. From the relevant eigenvalue problem, the natural frequencies ω_i and corresponding vibrations modes $\boldsymbol{\phi}_i$ can be determined, which satisfy:

$$\left(\mathbf{K} - \omega_i^2 \mathbf{M}\right)\boldsymbol{\phi}_i = \mathbf{0} \tag{1}$$

In linear modal expansion, the generalized coordinates are expressed as a linear combination of the system’s vibration modes. The response of the system is expressed in terms of the modal coordinates η_i that describe the contribution of vibration mode $\boldsymbol{\phi}_i$ to the response:

$$\mathbf{q} = \sum \boldsymbol{\phi}_i \eta_i \tag{2}$$

The modal derivatives $\boldsymbol{\theta}_{ij}$ are defined as the sensitivity of vibration mode $\boldsymbol{\phi}_i$ with respect to modal coordinate η_j [2]. To this end, the eigenvalue problem (1) is differentiated with respect to the modal coordinates η_j:

$$\left(\frac{\partial \mathbf{K}}{\partial \eta_j} - \frac{\partial \omega_i^2}{\partial \eta_j}\mathbf{M}\right)\boldsymbol{\phi}_i + \left(\mathbf{K} - \omega_i^2 \mathbf{M}\right)\boldsymbol{\theta}_{ij} = \mathbf{0} \tag{3}$$

From literature, it is known that the influence of the inertia terms on this sensitivity can in general be neglected [4]. Therefore, the following expression for the modal derivatives is obtained:

$$\boldsymbol{\theta}_{ij} = -\mathbf{K}^{-1}\frac{\partial \mathbf{K}}{\partial \eta_j}\boldsymbol{\phi}_i \tag{4}$$

In this way, it is possible to determine the modal derivatives of any structure of which a nonlinear finite element model is available. As an example, Figure 1 shows a cantilever beam's lowest 2 vibration modes (ϕ_1, ϕ_2) and the related modal derivatives $(\theta_{11}, \theta_{12}, \theta_{22})$. Note that the vibration modes describe a deformation in the transverse direction $v(x)$, whereas the modal derivatives describe a deformation in the axial direction $u(x)$. In fact the modal derivatives describe the shortening of the beam, that occurs when it is subjected to bending.

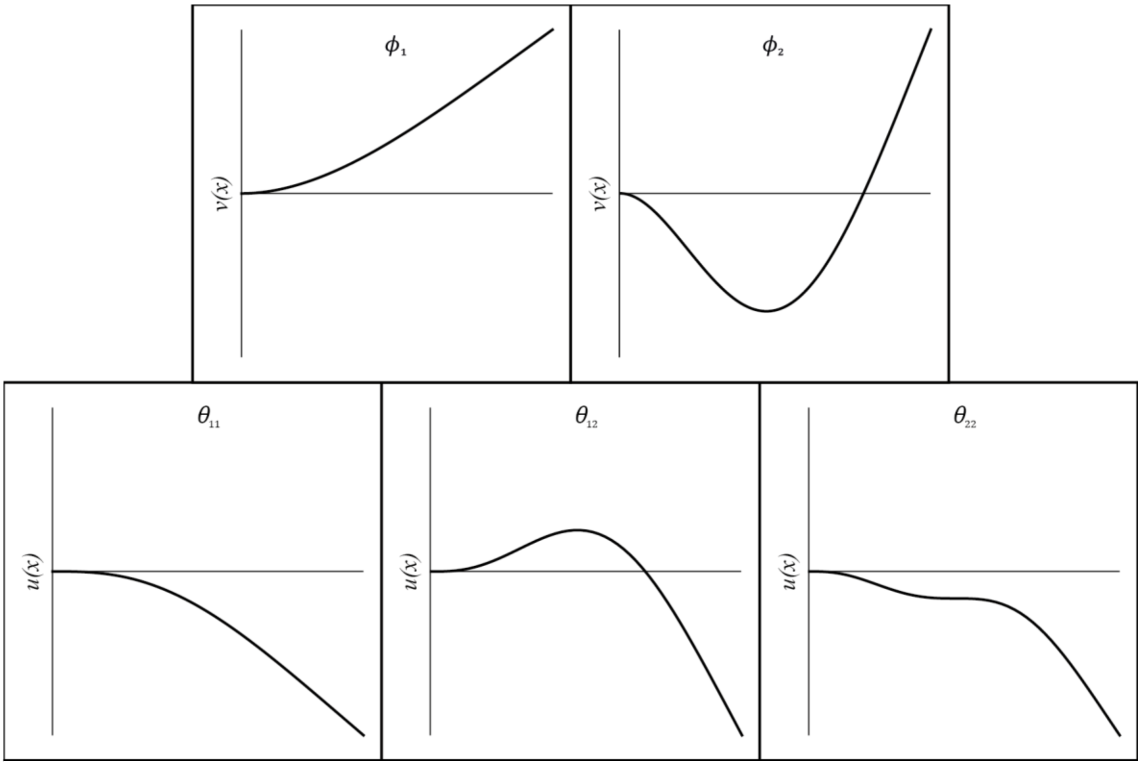

Figure 1: Vibration modes and modal derivatives of a cantilever beam

As soon as the modal derivatives are determined, it is possible to express the generalized coordinates as a combination of the vibration modes and the modal derivatives, which spans a quadratic manifold [5]:

$$\mathbf{q} = \sum \boldsymbol{\phi}_i \eta_i + \sum \sum \boldsymbol{\theta}_{ij}\, \eta_i \eta_j \tag{5}$$

3 Modal derivatives of Craig-Bampton modes

Consider a flexible multibody system in which multiple flexible bodies are coupled together or to the fixed world by joints located at the bodies' interface points. In general these joints may allow for large rigid body motions in between bodies. Unfortunately, the possibility for the system to move in these rigid body motions is the reason that modal derivatives cannot be computed for the multibody system as a whole. To explain this, note that for determining the modal derivatives, the inverse of the stiffness matrix is required in Eq. (4). Because systems that allow for rigid body motions have a singular stiffness matrix, this inverse does not exist. Hence, modal derivatives can only be computed for systems that do not allow for rigid body motions.

In order to allow for the efficient study of such flexible multibody systems as well, it is proposed to apply model order reduction to each individual flexible body within the multibody system. To keep this solution strategy as general as possible, no constraints may be imposed on the interface points of individual bodies. Hence, a strategy is developed that works without any pre-knowledge about how a certain body is constrained to the rest.

For this purpose, Craig-Bampton modes [6] are used for the model order reduction of a body's linear elastic behavior. The Craig-Bampton modes consist of so-called boundary modes and internal vibration modes. A Craig-Bampton boundary mode is determined by fixing all-but-one boundary degrees of freedom and prescribing the remaining interface degree of freedom a unit displacement. The Craig-Bampton internal vibration modes are the vibration modes of the structure with all boundary degrees of freedom fixed. Without loss of generalization, only the boundary modes are considered in this work. Let $\mathbf{q}_i$ and $\mathbf{q}_b$ denote the internal and boundary degrees of freedom, respectively. Then the body's equation of equilibrium can be partitioned as follows:

$$\begin{bmatrix} \mathbf{K}_{ii} & \mathbf{K}_{ib} \\ \mathbf{K}_{bi} & \mathbf{K}_{bb} \end{bmatrix} \begin{bmatrix} \mathbf{q}_i \\ \mathbf{q}_b \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{F}_b \end{bmatrix} \tag{6}$$

The Craig-Bampton modes $\mathbf{\Phi}_{CB}$ are obtained from static condensation to the boundary degrees of freedom:

$$\begin{bmatrix} \mathbf{q}_i \\ \mathbf{q}_b \end{bmatrix} = \begin{bmatrix} -\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib} \\ \mathbf{1} \end{bmatrix} \mathbf{q}_b = \mathbf{\Phi}_{CB}\mathbf{q}_b, \qquad \mathbf{\Phi}_{CB} \equiv \begin{bmatrix} -\mathbf{K}_{ii}^{-1}\mathbf{K}_{ib} \\ \mathbf{1} \end{bmatrix} \tag{7}$$

Next, the modal derivative $\boldsymbol{\theta}_{ij}$ related to a Craig-Bampton mode $\boldsymbol{\phi}_i$ is determined from a modified version of Eq. (4) as follows:

$$\boldsymbol{\theta}_{ij} = -\mathbf{K}^{-1}\frac{\partial \mathbf{K}}{\partial q_j}\boldsymbol{\phi}_i \tag{8}$$

where $\mathbf{K}$ should be interpreted as the stiffness matrix of the body with all boundary degrees of freedom fixed, except for the degrees of freedom of the boundary node of which q_i is a degree of freedom. As an example, Figure 2 shows the Craig-Bampton bending modes of a beam's right boundary node and the corresponding modal derivatives.

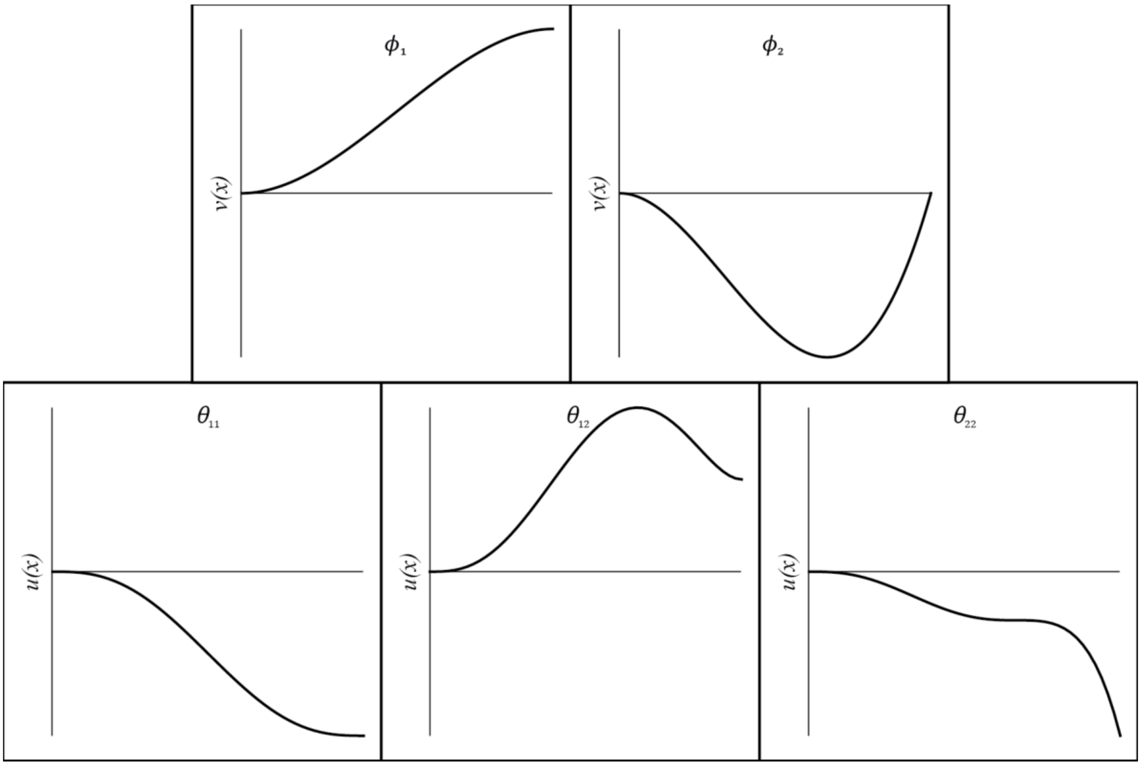

Figure 2: Craig-Bampton modes and modal derivatives related to a beam's right boundary point

For the comparison between a reduction basis of vibration modes and their modal derivatives and a reduction basis of Craig-Bampton modes and their modal derivatives, the equilibrium analysis of a cantilever beam is considered as an example. The nonlinear equations of equilibrium can be formulated in the following form:

$$\mathbf{Kq} = \mathbf{F} \tag{9}$$

where it should be understood that $\mathbf{K} = \mathbf{K}(\mathbf{q})$ is the nonlinear stiffness matrix and $\mathbf{F}$ are the applied external forces. Let $\mathbf{R}$ denote the reduction basis consisting of a set of mode shapes and modal derivatives, then the reduced form of (9) can be written as:

$$\mathbf{R}^T\mathbf{K}\mathbf{R}\overline{\mathbf{q}} = \mathbf{R}^T\mathbf{F}, \qquad \mathbf{R} \equiv [\boldsymbol{\Phi} \quad \boldsymbol{\Theta}] \tag{10}$$

where $\overline{\mathbf{q}}$ is the set of reduced generalized coordinates. Figure 3 shows the deflected shape of a cantilever beam that is subjected to a vertical tip load. The load is such that the beam is deformed slightly beyond its linear range, such that a geometrical nonlinear analysis is required. It can be seen that the solution of the unreduced model is the same as the solution by nonlinear finite element package Ansys. Model order reduction is applied using 2 vibration modes and the 3 corresponding modal derivatives or using 2 Craig-Bampton bending modes and the 3 corresponding modal derivatives. It is observed that both reduced models produce similar results, but that they are slightly stiffer than the exact solution.

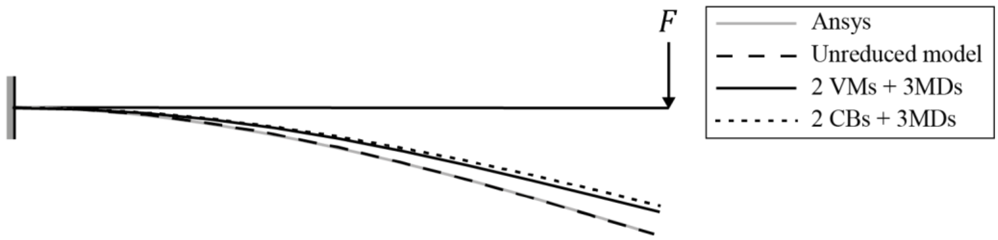

Figure 3: Equilibrium analysis of a cantilever beam subjected to a tip load

4 Model order reduction of a flexible multibody system

For simulating flexible multibody dynamics, the floating frame formulation is a commonly used and well-developed formulation. In the floating frame formulation, the rigid body motion is described by the position and orientation of the body's floating frame relative to the inertial frame. Elastic deformation is described locally, relative to the floating frame. When this elastic deformation is small, linear theory can be applied and model order reduction techniques, such as the Craig-Bampton method, can be applied locally. For our present purposes, a nonlinear finite element model is used for describing local elastic deformations and it is not demanded that these remain small. The local mass and configuration dependent stiffness matrices of each body are reduced using the Craig-Bampton boundary modes and associated modal derivatives.

For equilibrium analysis of a multibody system, the equations of equilibrium for each flexible body need to be solved simultaneously with the kinematic constraint equations that connect different bodies together. This forms a set of combined differential-algebraic equations of the following form:

$$\begin{bmatrix} \mathbf{K}_{sys} & \boldsymbol{\Psi}_{\mathbf{q}}^T \\ \boldsymbol{\Psi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_a \\ \mathbf{0} \end{bmatrix} \tag{11}$$

In this, $\mathbf{K}_{sys}$ is the assembly of the reduced stiffness matrices of all flexible bodies, $\boldsymbol{\Psi}_{\mathbf{q}}$ is the Jacobian of the kinematic constraint equations $\boldsymbol{\Psi} = \mathbf{0}$, $\mathbf{Q}_a$ is the vector of externally applied generalized forces and $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers used to enforce the constraints.

For geometrically large deformations, the constrained equations of equilibrium (11) need to be solved numerically with an incremental method. To this end, the tangent stiffness matrix is derived, which consists of both the material stiffness matrix and geometrical stiffness matrix. The externally applied generalized forces are applied in increments. For each load increment, Newton-Raphson iterations are applied until the solution is converged with sufficient accuracy.

As validation of this solution strategy, an equilibrium analysis is performed on two flexure mechanisms that are commonly used in precision engineering: a parallel leaf spring mechanism and a cross flexure mechanism. These mechanisms are subjected to a static load that deforms the mechanisms beyond the linear range. A nonlinear finite element model of each leaf spring is created using 10 beam elements and is reduced subsequently using Craig-Bampton modes and their model derivatives. As a comparison, an equilibrium analysis of both flexure hinges is performed using the nonlinear finite element formulation of Ansys. Figures 4 and 5 show the deformed configuration of the parallel leaf spring mechanism and cross flexure, respectively. It can be seen that the proposed reduction basis again produces results that are slightly stiffer than the exact solution.

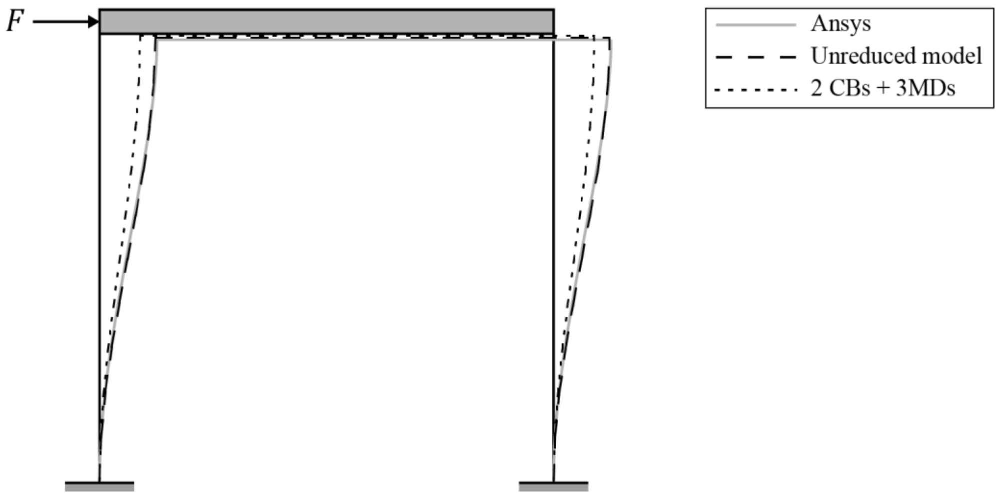

Figure 4: Equilibrium analysis of a parallel leaf spring mechanism subjected to a horizontal load

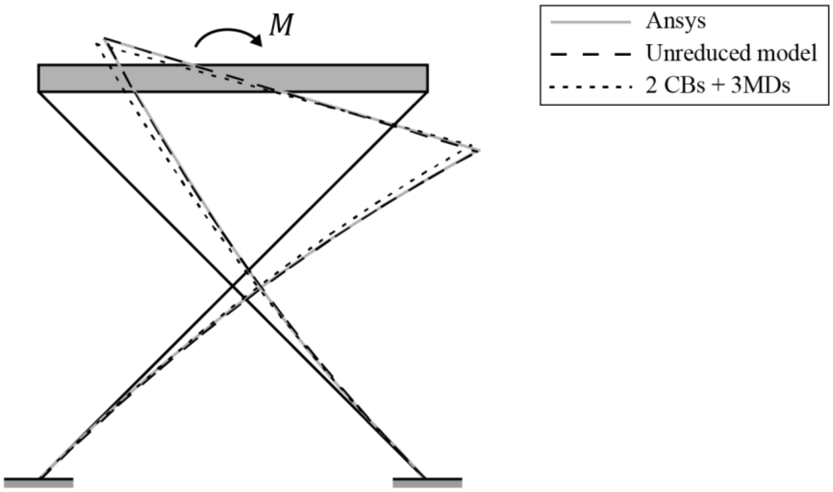

Figure 5: Equilibrium analysis of a cross flexure mechanism subjected to a bending moment

It is observed that when the applied external load increases such that the mechanism’s deformation becomes exceedingly large, the reduction basis is no longer successful. To demonstrate this, Figure 6 shows the results of the equilibrium analysis of the parallel leaf spring mechanism for increased loading. It is concluded that for such large deformations, additional higher order terms should be included. Taking only first order changes of the linear basis into account is clearly not sufficient.

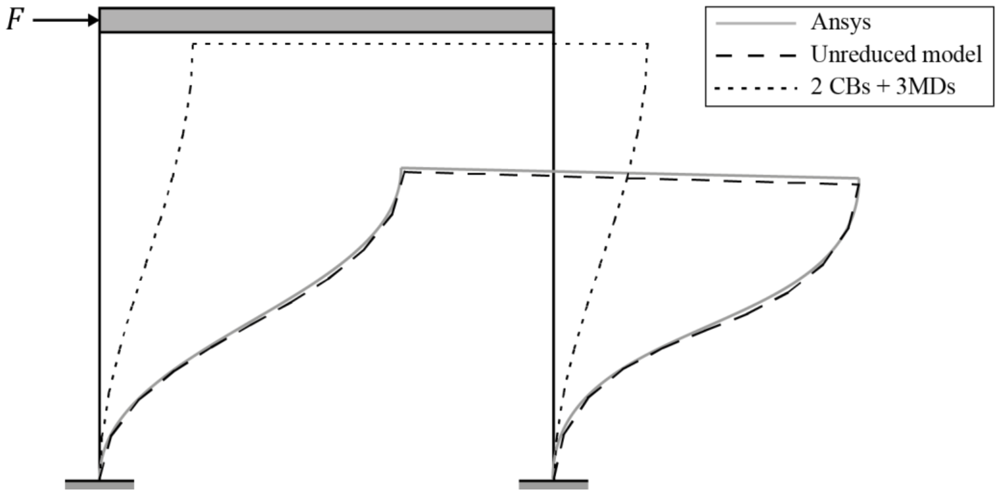

Figure 6: Equilibrium analysis of a parallel leaf spring mechanism subjected to an increased load

5 Conclusions

It is found that the equilibrium analysis of flexible multibody models of large stroke flexure hinges can be reduced successfully by the proposed model order reduction strategy. For each flexible body of which the elastic deformations may become large, its nonlinear finite element model is projected onto a reduction basis consisting of a limited number of Craig-Bampton modes and related modal derivatives. Comparison between the full and reduced simulations of a number of standard flexure hinges shows the high potential of this strategy. In future work, the method will be applied to reduce computational costs of shape and topology optimization of large stroke mechanisms. This potentially leads to more sophisticated designs with improved performance. Additional research is required for determining a reduction basis for situations in which the deformations become exceedingly large.

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