Compliance feedforward of flexible structures

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1 Introduction

Integrated circuits (ICs) which are made with wafer scanners are the key element in the semiconductor industry. These motion lightweight systems confront control problem in terms of position accuracy and speed. To overcome this, a compliance feedforward controller is designed while a simple mass feedforward controller can't compensate for the lightweight systems due to the flexibilities [1].

2 Modeling Framework

The aim of this research is to provide a feedforward scheme which can account for flexible structures with time varying performance location. To this end, we designed a compliance compensation feedforward controller based on the state space presentation rather than on the frequency response. The model is validated for a single structural mode and will be generalized for the infinite dimensional system. For the design the feedforward control, the compliance must be calculated first. Consider a system with a constant input force, and the state being the position. We assume that the position has the following expression.

$$x(t) = x_2 \frac{t^2}{2} + x_1 t + x_0 + x_{st}(t),$$

x_0 is the compliance and $x_{st}(t)$ converges to zero as $t \to \infty$. The following theorem adds additional conditions to find the unique solution of the compliance.

Theorem 2.1 Assume additionally that $x(t) \in X$ satisfies $\dot{x}(t) = Ax(t) + B$, and that the state space X can be written as

$$X = \text{span}\{x_1, x_2, \varphi_1, \varphi_2, ..., \varphi_{n-2}, \cdots\},$$

where $\varphi_1, \varphi_2, ..., \varphi_{n-2}$ span the stable subspace. Then

$$x_0 \in \text{span}\{\varphi_1, \varphi_2, ...\} \iff Z_1^T x_0 = 0 \;\&\; Z_2^T x_0 = 0$$

where

$$A^T Z_2 = 0, \quad A^T Z_1 = Z_2.$$

The proof is omitted due to the lack of space.

The above theorem can be used to derive the expression for the compliance. For this, the spatially continuous dynamics of wafer stage is simplified by Euler-Bernoulli beam exhibiting a position dependent dynamics.The beam moves vertically at one end and the other end has a cantilever support. An actuating force cause the beam to deflect. We can described it with state space representation:

$$\dot{\omega}(t) = \mathscr{A}\omega$$

with

$$\omega = \begin{pmatrix} y \\ \rho A \frac{\partial y}{\partial t} \\ \frac{\partial^2 y}{\partial r^2} \end{pmatrix}$$

and

$$\mathscr{A} = \begin{pmatrix} 0 & (\rho A)^{-1} & 0 \\ 0 & -c_d(\rho A)^{-1}\frac{\partial^4}{\partial r^4} & -EI\frac{\partial^2}{\partial r^2} \\ 0 & (\rho A)^{-1}\frac{\partial^2}{\partial r^2} & 0 \end{pmatrix}.$$

Here $y(t,r)$ is the deflection of the beam at position $r \in [0, L]$ and time t, L is the length of beam, I is the second moment of inertia, A is the area, E denotes Young's modulus, and ρ is the linear mass density. Furthermore, we assume an actuator force at the boundary. Assuming the following solution for the above system

$$\omega_{assumed} = \begin{pmatrix} V_2(r)\frac{t^2}{2} + V_1(r)t + V_0(r) + V_{st}(r,t) \\ \rho A\big(V_2(r)t + V_1(r) + \dot{V}_{st}(r,t)\big) \\ \frac{\partial^2 V_2}{\partial r^2}(r)\frac{t^2}{2} + \frac{\partial^2 V_1}{\partial r^2}(r)t + \frac{\partial^2 V_0}{\partial r^2}(r) + \frac{\partial^2 V_{st}}{\partial r^2}(r,t) \end{pmatrix}$$

and using the differential equation, boundary conditions, and the theorem, results in the unique compliance solution

$$x_0(r) = -\frac{1}{24EIL}r^4 + \frac{1}{6EL}r^3 - \frac{L}{4EI}r^2 + \frac{L^3}{20EI}.$$

This 4th order compliance function is in accordance with the research on the frequency response of the beam [2]. The compliance based on frequency response is validated by simulation results and the performance of the system is satisfactory in terms of control parameters.

References

[1] M.J.C. Ronde, Feedforward control for lightweight motion systems, *Eindhoven University of Technology*, (2014)

[2] N. Kontaras, M.F. Heertjes, and H. Zwart, Continuous compliance compensation of position dependent flexible structures. *IFAC-PapersOnLine*, (2016), 49(13):76 81.