Identification of modal parameters based on moving load excitation

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Abstract

It is often desired to measure the model parameters of bridges, but normally the forcing function cannot be measured so it is necessary to use output-only system identification methods. A moving vehicle provides a force on a bridge that could be used for input-output analysis, which could be superior, but the force varies in space as well as time, so existing system identification methods are not directly applicable. To address this issue, this paper proposes a new strategy to use the moving load response to identify the modal parameters. A numerically simulated simply supported Euler-Bernoulli beam of known parameters is used as an example case to test the validity of the approach. Frequency Domain Decomposition is first implemented to extract the mode shapes from the simulated acceleration responses. Then, by applying the mode superposition method, the simulated force and acceleration responses are transformed into modal space. The accelerances of five modes are then processed using the Levenberg-Marquardt method such that the three unknown parameters, namely natural frequency, damping ratio, and modal mass, of each mode are determined simultaneously. The effectiveness of the proposed method is validated by the low percentage errors (less than 1.7\%) for all three identified modal parameters compared to their actual values for four of the five modes.

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Peer-review under responsibility of the organizing committee of EURODYN 2017.

Keywords: System identification; input-output; moving vehicle; mode superposition; modal mass

1. Introduction

For large structures such as bridges, their large mass makes controlled dynamic loading difficult so the modal parameters are usually estimated using ambient vibrations and output-only system identification methods. Inevitably,
less accurate estimates of damping ratios are found by such methods than can be achieved by input-output methods, where they are applicable. Also, for unknown loading, generalized masses cannot be directly estimated from vibration measurements. Meanwhile, in recent decades there has been much research on vehicle-bridge interaction, normally aimed at calculating the response of a known structure to the passage of vehicles [1]. This paper, in contrast, aims to estimate the modal parameters of the structure based on the measured response to the passage of a vehicle.

The identification of the loading, simultaneously with measurements of the structural response, is made possible by advances in monitoring technology, using an instrumented vehicle and a wireless sensor network [2].

That the loading is varying in space as well as time is the key challenge, which precludes the direct use of existing system identification methods [3, 4]. Therefore, a two stage identification procedure is proposed to modify existing methods to the moving load problem. The whole process is applied to a simply supported Euler-Bernoulli beam of known parameters as an example. Its response to a moving load is calculated numerically, yielding simulated measured accelerations at a series of fixed locations on the structure. In the first stage, the mode shapes are identified using the Frequency Domain Decomposition method. In the second stage the load and responses are transformed into modal space then the modal parameters are extracted for each mode by using a least squares method, yielding the natural frequency, damping ratio, and modal mass simultaneously. The results obtained by the system identification method from the simulated measurements are then compared with the actual values for the original beam.

2. Numerical generation of simulated force and acceleration time histories

2.1. Theoretical generation of a simply supported beam

The proposed system identification approach is demonstrated using the example of a simply supported Euler-Bernoulli beam, as shown in Figure 1. The beam is of length L and is assumed to be uniform, with bending stiffness $EI$ and mass per unit length $\bar{m}$. Ignoring damping and the vertical load, the displacement $v(x, t)$ is then governed by the following partial differential equation of motion [5]:

$$EI \frac{\partial^4 v}{\partial x^4} + \bar{m} \frac{\partial^2 v}{\partial t^2} = 0$$

(1)

The physical displacement $v(x, t)$ can be expressed as a sum of modal contributions with $n$ modes:

$$v(x, t) = \sum_{i=1}^{n} \phi_i(x) y_i(t) \quad n \to \infty$$

(2)

where $\phi_i(x)$ is the mode shape $i$ at location $x$; $y_i(t)$ is the generalized coordinate of mode $i$. Substituting Eq. (2) into Eq. (1), the model shape $\phi_i(x)$, circular natural frequency $\omega_i$ and generalized mass $M_i$ for each mode are given by:

$$\phi_i(x) = \sin \frac{i\pi x}{L} \quad \omega_i = \frac{i^2 \pi^2}{L^2} \sqrt{\frac{EI}{\bar{m}}} \quad M_i = \frac{\bar{m}L^2}{2} \quad i = 1, 2, 3, \ldots, n$$

(3)
In general, a vertical load \( P(x, t) \) is applied to the structure. In the case of a moving point load with a given time varying intensity \( p(t) \) and speed \( V \), the load \( P(x, t) \) can be expressed as

\[
P(x, t) = \begin{cases} 
\delta(x-Vt) p(t) & 0 \leq t \leq t_d \\
0 & \text{otherwise}
\end{cases}
\] (4)

where \( t_d = L/V \) is the time required for the load to cross the bridge and \( \delta \) is the Dirac delta function, which defines the physical location of the moving force. Introducing modal damping ratio \( \xi_i \) for mode \( i \) to the undamped system and applying the orthogonality properties of the mode shapes to Eq.(1), the following system of uncoupled ordinary differential equations is obtained for the modal coordinates

\[
\ddot{y}_i(t) + 2\xi_i\omega_i\dot{y}_i(t) + \omega_i^2 y_i(t) = \frac{P_i(t)}{M_i} \quad (a) \\
P_i(t) = \int_0^L \phi_i(x) P(x, t) \, dx = \begin{cases} 
\phi_i(Vt) p(t) & 0 \leq t \leq t_d \\
0 & \text{otherwise}
\end{cases} \quad (b) \] (5)

Now, along with the modal parameters given by Eq. (3) and the assumed damping ratios, the simulated modal displacements can be obtained by using the ode45 function in Matlab. Then from Eq. (5), the modal accelerations can be easily calculated. The time histories of simulated acceleration measured at discrete locations on the bridge can then be computed using Eq. (2).

2.2. Simulated response of a simply supported beam

Table 1. Actual natural frequencies and damping ratios

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Actual natural frequency ( f_{ai} ) (Hz)</th>
<th>Actual damping ratios ( \xi_{ai} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.181</td>
<td>0.020</td>
</tr>
<tr>
<td>2</td>
<td>12.725</td>
<td>0.150</td>
</tr>
<tr>
<td>3</td>
<td>28.631</td>
<td>0.023</td>
</tr>
<tr>
<td>4</td>
<td>50.900</td>
<td>0.031</td>
</tr>
<tr>
<td>5</td>
<td>79.531</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Fig. 2. Simulated noise free acceleration response of the bridge \( (t_d = 7.2s) \). (a) at L/2; (b) at L/4; (c) at L/8.

The numerical simulation of this study is based on a 40 m long simply supported beam with measurement locations at 2m spacing. The bending stiffness and the mass per unit length of the bridge are set as \( EI = 1.26e+11 \) (N.m²) and \( \bar{m} = 1.20e+04 \) (kg/m) respectively. A moving load traverses the bridge at a speed of 20 km/h (Fig. 1) in 7.2s. The natural frequencies are computed from Eq. (3), but the damping ratios are randomly assigned, and only the first five modes are considered, see Table 1. Both the force and the acceleration time histories are sampled at 800 Hz, the total measurement time length \( t \) equals 20s.

The load on the bridge from the moving vehicle is given by

\[
P(t) = m_v(g + a_v)
\] (6)

where \( m_v \) is the mass of the vehicle, \( g \) the gravitational acceleration, and \( a_v \) the vertical acceleration of the vehicle, which could be measured. In the present example, \( m_v g \) is set as 400 kN and \( m_v a_v \) is characterized by white noise in the range \( \pm 0.05 \) kN to simulate the effect of vehicle motion due to road roughness [6]. The acceleration response of bridge at three different sensor locations due to the passage of a moving load are illustrated in Fig. 2. The red vertical
line shows the time that the vehicle leaves the bridge. It hence separates the forced- and free-vibration parts of the signals.

3. Identification stage 1 – Mode shapes

3.1. Mode shape identification algorithm—Frequency domain decomposition

Having generated simulated time histories of the moving point load and the measured accelerations at a series of points along the bridge for known modal parameters, they are used as input to the system identification problem to re-extract the modal properties. A two stage process is used. Firstly the mode shapes are estimated using Frequency Domain Decomposition (FDD) and subsequently the measured accelerations are decomposed into modal accelerations, which are used along with the estimated modal forces to estimate the natural frequencies damping ratios and modal masses of the modes.

As an extension of the classical frequency domain approach often referred to as the peak-picking technique, the FDD method removes the disadvantages associated with the classical approach, but keeps the important features of user-friendliness and even improves physical understanding by dealing directly with the power spectral density function [7]. As for other output-only identification methods, it makes assumptions of white noise input, light damping and that mode shapes of close modes are geometrically orthogonal. Although the actual scenario does not strictly satisfy these assumptions, it approximately decomposes the system into Single Degree of Freedom (SDOF) systems. The results are significantly more accurate than from the classical approach.

In the FDD identification, the decomposition procedure is done by taking the Singular Value Decomposition (SVD) of the estimated output Power Spectral Density (PSD) \( \hat{G}_{vv}(\omega) \) known at discrete frequencies \( \omega = \omega_i \) [7]

\[
\hat{G}_{vv}(\omega_i) = U_i S_i U_i^H
\]

(7)

where \( S_i \) is a diagonal matrix holding the scalar singular values \( s_i \). If only the \( i \)th mode is dominating, the first singular value \( s_{i1} \) will become the representative of the auto power spectral density function of the corresponding single degree of freedom, and its counterpart located in the unitary matrix \( U_i = (u_{i1}, u_{i2}, \ldots, u_{ir}) \), namely, \( u_{i1} \), is the estimate of the mode shape \( \hat{\phi}_i \), \( r \) and \( \{ \}^H \) denotes the number of transducers and Hermitian transpose respectively.

Instead of using the Modal Assurance Criterion (MAC) [8] or modal discrimination method [9] to perform the peak selection procedure, in this case, the simple manual peak picking method is adopted, whereby potential disadvantages [9] regarding the peak searching procedure based on the comparison of the modal coherence value with a certain limit value is avoided.

3.2. Mode shape identification results

![Fig. 3. PSD of the measured acceleration responses (over 20s).](Image)

![Fig. 4. Identified mode shapes against the target mode shapes (over 20s)](Image)

The SVD results of the entire measured acceleration time history (over 20s) are shown in Fig. 3. Five peaks are visible around the resonances. Their corresponding identified mode shapes are presented in Fig. 4. The frequencies shown at the top of each subgraph are the selected frequencies which are initial estimates of the natural frequencies. By comparing the red circles estimated from the simulated data and the blue continuous lines, representing the actual
mode shapes, for each mode, one can conclude that, although the assumptions of the algorithm are not fully satisfied, the identification results are still good estimates of the real mode shapes. The identified second mode shape is less accurate than the others because of the higher damping ratio and the corresponding smaller amplitude of vibration in the mode.

4. Identification stage 2 – Input-output system

4.1. Accelerance calculation

Having found the estimated mode shapes, they are used along with the measured load and accelerations to identify the other modal properties of the system.

One of the merits of the proposed approach is that the often discarded input data for the large scale structure is utilized. This provides information to identify the modal mass. However, since the forcing is from a single vehicle, the inputs to all modes are correlated, so Multiple-Input-Multiple-Output methods based on the assumption of uncorrelated cases are inapplicable. Nevertheless, modal decomposition offers a useful means to tackle the problem. Applying it to the measured responses, the signals become a series of uncoupled time histories in modal space. Also, knowing the mode shapes and forcing function, $P(t)$, the generalized force on each mode is found from Eq. 5 (b). Hence the acceleration for each mode can be found, whereby the natural frequencies, damping ratios and modal masses can be identified using the Levenberg-Marquardt method [10], a nonlinear least-square technique which uses the trust-region approach, which is superior to the line search based Gaussian-Newton method.

To uncouple the force signal by using the identified mode shapes, interpolation has to be performed first, since in general the vehicle load is not applied directly at one of the measurement locations. Furthermore, the unequal numbers of the identified modes and transducers formed a rectangular mode shape matrix, which necessitate the application of the pseudo inverse so as to convert the measured acceleration responses into modal space. Having done this, the system is decoupled into $k$ SDOF, for each degree of freedom, the acceleration is given by [3]

$$A(\omega) = \frac{Y(\omega)}{P(\omega)} = \frac{-\omega^2 \gamma_i^2}{\omega_i^2 - \omega^2 + i2\omega\omega_i \xi_i} \quad M_i = \frac{1}{\gamma_i^2}$$

(8)

where $Y(\omega)$ and $P(\omega)$ are the Fourier Transforms of the generalized acceleration and generalized force respectively; $\omega_i$ and $\xi_i$ represent the natural frequency and damping ratio of the $i$th mode. The identification process fails if the order of magnitude of the parameters differs a lot. Because of this, the mass-normalized coefficient $\gamma_i$ is introduced in Eq. (8) instead of the modal mass $M_i$.

4.2. Identification of the natural frequencies, damping ratios and modal masses

The results shown in Fig. 5 and Table 2 show the capability of the proposed method, which is able to identify the three parameters of each mode simultaneously. The low percentage errors of the identified parameters in Table 2 demonstrate the good performance of the proposed method. The natural frequencies are identified within 0.11% and the damping ratios, which are normally difficult to identify accurately on full scale structures, within 6.1%. Furthermore, the modal masses have been identified, which is not possible directly identified from output-only methods, with errors less than 14%. The mode with the poorest estimates is mode 2, which has an unrealistically high

![Fig. 5. Measured vs. fitted FRF accelerance plots](image-url-56x173-to-489x230)
damping ratio of 15%, leading to very low amplitude responses. Excluding this mode, the errors in the results are less than 0.2%, 0.8% and 1.7% for natural frequencies, damping ratios and modal masses, respectively.

Table 2. Identification results of modal parameters

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural frequencies (Hz)</th>
<th>Damping ratios</th>
<th>Modal masses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>Identified</td>
<td>Percentage error (%)</td>
</tr>
<tr>
<td>(%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>(2)</td>
<td></td>
<td>(3)</td>
</tr>
<tr>
<td>1</td>
<td>3.181</td>
<td>3.182</td>
<td>0.012</td>
</tr>
<tr>
<td>2</td>
<td>12.725</td>
<td>12.711</td>
<td>-0.108</td>
</tr>
<tr>
<td>3</td>
<td>28.631</td>
<td>28.629</td>
<td>-0.007</td>
</tr>
<tr>
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<td>50.900</td>
<td>50.940</td>
<td>0.080</td>
</tr>
<tr>
<td>5</td>
<td>79.531</td>
<td>79.531</td>
<td>0.000</td>
</tr>
</tbody>
</table>

5. Conclusion

Although there are many existing system identification techniques for modal parameters of structures, they are not directly applicable to the situation of a moving load, such as for a vehicle crossing a bridge. However, by using simultaneous measurements from an instrumented vehicle and on the bridge and by first estimating the mode shapes using Frequency Domain Decomposition, the problem is decomposed into uncoupled modes, for each of which both the input and output are known. Using the Levenberg-Marquardt method, the modal parameters can then be extracted. For the example structure, they are found with very low percentage errors for most cases.

The results show that the proposed method is very promising. Further work is investigating the effects of noise and using more realistic models for the vehicle dynamic loading, considering road roughness.

References