# Connecting nonlinearities: damage precursors detection and control methodology

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## Abstract
Fatigue damage precursors identification and control procedure is developed for nonlinear aero-structures and dynamic systems exposed to realistic vibratory conditions. The identification is achieved by connecting nonlinearities between macro- and micro-states, and including those connection in the equations of motion; i.e. the interplay between the local nonlinearities (or materials micro-plasticity) and the global nonlinearities (nonlinear dynamic parameters). Exploiting the nonlinear connections is key in linking the health of a system to its control algorithms, which can manipulate the structural dynamic responses in order to slow the progression of fatigue. This approach allows for continual re-estimation of the dynamics of the macro-states due to degradations in the properties of the micro-states at unhealthy locations by identifying precursors to damage prior to crack formation. Capturing and controlling the interplay between the local and global nonlinearities appear to hold promise in extending the service-life of high-value systems.

## 1 Introduction

Current prognostics health management (PHM), structural health monitoring (SHM), and condition based maintenance (CBM) methods are capable of executing health assessment for high-value military and commercial aerospace platforms, vehicular systems, and infrastructures. However, there are intrinsic and extrinsic factors that limit the accuracy of measured and calculated parameters provided by PHMs/SHMs/CBMs. Some of the intrinsic factors are due to the complexity and diversity of health monitoring and management hardware required for collecting and processing data [1], and due to unavoidable design assumptions and correction factors embedded in the software [2]. On the other hand, the extrinsic factors tend to be application specific. For example, in aerospace platforms, these factors consist of variability in flight conditions, structural geometries, and type of materials [1]-[6]. The intrinsic and extrinsic factors can be mitigated but cannot be eliminated, thus, providing accurate identification and assessment of fatigued components can be extremely challenging. In military aviation, scheduled maintenance inspections are still required in order to reduce uncertainties in health measurements or estimates, which can be expensive and time consuming.

It is important to point out that the intrinsic and extrinsic factors have coupling effects, which often instigate or augment various nonlinearities at the system, subsystem, component, and materials levels. In many high-value systems, current state of the art in PHMs/SHMs/CBMs is based on linear or reduced models to assess the health of systems that are highly nonlinear [2]. In this paper, we focus on simulating the relevant nonlinearities that may ameliorate some of these intrinsic-extrinsic coupling effects, where the extrinsic factor is realistic vibration, and the intrinsic factor is nonlinearities in the outputs signal. Aero-structures are often exposed to high amplitude vibrations, which introduce additional and enhance existing nonlinearities

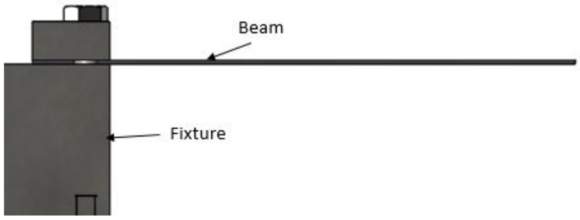


Figure 1: Cantilever beam setup for modeling

in aircraft [3, 6]. Indeed, light wight and flexible aero-structures such as helicopter blades, and aircraft wings are often exposed to fast and high amplitude motions [4, 5], which are the most encountered nonlinearity in aerospace applications. Consequently, the effective structural stiffness is modified based on the variation in periodic deflective motions, which can be exploited to detect fatigue [7]. Cole et al. have shown that nonlinear vibration is a practical method for simulating realistic flight dynamics and have been able to monitor changes in global nonlinear stiffness to assess fatigue severity [8]. Cole et al. have observed that the global nonlinear stiffness parameters were sensitive to fatigue damage precursors at the micro-state level, and proposed exploiting the nonlinear stiffness as an indicator for detecting changes in the materials mechanical properties prior to crack initiation. They defined damage precursor (DP) as any observable degradation of the material microstructural morphology and resulting changes in the physical properties prior to any detectable fatigue crack [8].

In this paper, we simulated the interplay between the micro-state nonlinearity, namely materials micro-plasticity, and the macro-state nonlinear stiffness to estimate changes in the global dynamic response of a beam-like structure. An aerospace alloy aluminum 7075-T6 was selected for this study. The simulation was accomplished analytically, where the material and geometric nonlinearities were contained in the equation of motion. The micro-states properties were obtained from the nano-indentation results performed by Haynes et al. for Al 7075 beams fatigued up to 25%, 50%, and 75% their fatigue-life [7]. The Harmonic-Balance method (HBM) was utilized to approximate a solution for the nonlinear differential model, and to provide a frequency response equation with the relevant nonlinear parameters. The main advantage of deriving a frequency response equation is to provide fast updates of the structural health-state, which may potentially allow for aligning PHM/SHM/CBM, and for integrating the health-state into flight control laws. The idea basic was demonstrated analytically by showing drifts in the structural dynamic response due to the development of damage precursors. Identifying changes in the nonlinear parameters is far more useful for a control system than locating and measure the size of a crack. Based on updates into the linear and nonlinear stiffness matrices, adaptive control algorithms could potentially ensure optimal operations below high stress regimes.

## 2 Approach

The equation of motion is, perhaps, one of the primary links between controls and structural dynamics theories. Thus, our approach consisted of : (i) connecting and amplifying nonlinearities to study the possibility of extracting health indicators sensitive enough to early fatigue build; (ii) demonstrating the concept analytically using thin aluminum 7075-T6 beam-like structures exposed to high amplitude harmonic excitations; and (iii) updating the effective structural stiffness in the equation of motion. The beam was modeled as a cantilever-beam attached to a rigid fixture as shown in Figure 1. The Young elastic modulus, $E$, and density, $\rho$, for aluminum 7075 are $71.7GPa$, and $2810kg/m^3$, respectively. The beam length, $L$, is $127mm$, and its cross-section area is $50 \times 1mm^2$.

### 2.1 Macro-State

In linear systems, Hooke′s law is adequate for approximating the displacement in structures exhibiting linear elastic deformation due to external loads in the form of restoring forces proportional to structural displacement, $x$. For thin and light weight structures, two common assumptions are applied when analyzing their structural dynamics: (i) these structures can endure reversible kinematic deformations before their inherent stress-strain characteristics show significant departure from the linear regime [8, 10]; and (ii) they also encounter kinematic nonlinearities before reaching the materials inherent plasticity [11]. In fact, the kinematic nonlinearity is manifested as a dynamic hardening or softening response, where the restoring force is proportional to the cubic structural displacement, $x^3$ [8]. In this case, the total restoring force is expressed in terms of the linear and nonlinear structural stiffness coefficients, $k_1$ and $k_3$, respectively, as:

$$F_r(x) = k_{eq}x \tag{1}$$

where, the equivalent structural stiffness is:

$$k_{eq} = k_1 + \alpha k_3 x^2 \tag{2}$$

Dynamicists use $\alpha$, which is a dynamic hardening-softening factor, to describe or correct for potential nonlinearity in restoring forces. When $\alpha$ is positive, $k_3$ assists the linear restoring force, $k_1x$, in making the structure stiffer and resistive to applied loads. Subsequently, the maximum dynamic response amplitude occurs at a forcing frequency, $\omega$, higher than the structure's fundamental frequency, $\omega_n$ [12]. This type of response is called dynamic hardening, which is analogous to stiffening in solid mechanics and materials since. The nonlinearity associated with $k_3$ is due to the potential energy stored in periodic bending. A negative $\alpha$, on the other hand, opposes the linear restoring force. The oscillatory behavior of the structure may appear dynamically softer with a drop in its natural frequency [4]. This phenomenon is called dynamic softening. Dynamicists attribute the hardening−softening behavior to the kinematics of the structure and rarely to structural fatigue. For completeness, we mention another cubic nonlinearity associated with the beam inertia, $m_i$, which tends to oppose the restoring forces.

In this study, the global nonlinear equation of motion for the aluminum structure was derived from nonlinear Euler-Bernoulli beam theory [13]. The model includes both inertial and stiffness nonlinearities. For fatigued structures, the dynamic performance was estimated by updating the equivalent stiffness $k_{eq}$ based on the evolution in the materials stiffness at the micro-state. For simplicity, we only focused the beam first flexural mode. Thus, the beam was exposed to simulated base sine-sweep excitation at an amplitude $1g$ near the beam's natural frequency for the following fatigue-life levels: 0% (pristine condition), 25%, 50%, and 75%.

### 2.2 Micro-State

Changes in the micro-state stiffness values were obtained from nano-indentation measurements in a separate study for the same fatigue-life levels [7], as shown in Figure 2. The measurements were conducted near the beam root, $0mm \rightarrow 1mm$ (Figure 2), where high stress concentrations occurred. The point $0mm$ is approximately at the beam-fixed boundary. Figure 2 shows the relevant changes in $E$ for fatigue-life levels 25%, 50%, and 75%, where results are normalized by the average modulus of an un-fatigued Al 7075-T6 specimen. The indentation measurements show that the material experienced softening at 25% of fatigue-life followed by hardening and softening at 50% and 75% of life, respectively (Figure 2). Explanation of the material microscopic behavior after to various fatigue cycles is detailed in [7].

## 3 Modeling Development

The model assumes that the slender cantilever beam is an isotropic structure with uniform cross-sectional area, $A$, length, $L$, and volumetric density $\rho$. The beam is clamped to a rigid fixture, which can be excited

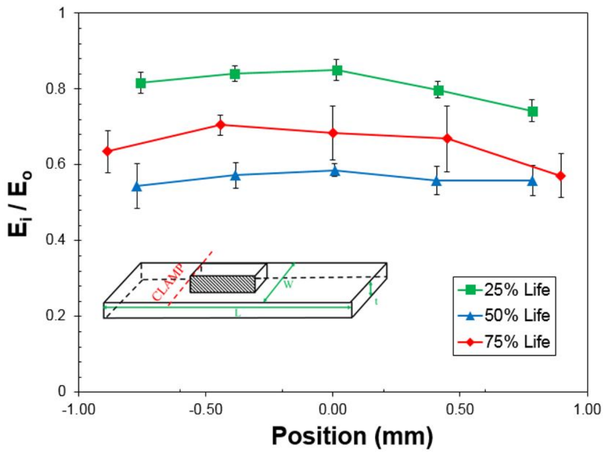


Figure 2: Changes in the elastic modulus obtained from nano-indentation measurements near beam root after various vibration fatigue cycles [7]

periodically in the transverse direction, as Shown in Figure 1. Since the study focuses on the beam first flexural mode, it is reasonable to assume that the transverse vibration is purely planar as long as the cross-section geometry are symmetric with respect to the beams centerline [13]. Stretch of the neutral axis of the beam can be neglected by applying the inextensibility assumption. Subsequently, effects due to warping and shear deformations are ignored. Therefore, nonlinear Euler-Bernoulli beam theory is applicable for modeling the transverse motion of the structure under investigation.

### 3.1 Nondimensional Time Domain

Including the nonlinear inertial and restoring forces into the equation of motion yields the following nonlinear differential equation [8]:

$$m\ddot{x} + c\dot{x} + m_i(x^2\ddot{x} + x\dot{x}^2) + k_1 x + \alpha k_3 x^3 = -mY_o sin(\omega t) \tag{3}$$

The over-dots denote derivatives with respect to time, $t$. The combined distributive mass and rotational inertia of the structure is $m$. The beam tip relative displacement, and the steady state base motion are $x$, and $Y_o$, respectively. The coefficient $c$ is the system viscous damping, which causes an exponential dissipation of the systems natural dynamics over time. Rayleigh-Ritz procedure is applied to equation 3 to remove the spatial dependence. Further simplification is performed by redefining the temporal ordinary differential equation as function of a nondimensional time, $\tau$, where, $t$, is transformed into a dimensionless variable based on the system's inverse natural frequency:

$$\tau = \omega_n t \tag{4}$$

The final dimensionless nonlinear differential equation normalized by the natural frequency and mass of the system is:

$$\ddot{\eta} + 2\zeta\dot{\eta} + m_n(\eta^2\ddot{\eta} + \eta\dot{\eta}^2) + \eta + k_n\eta^3 = -Y sin(\Omega\tau) \tag{5}$$

The dimensionless variables are:

$$\eta = \frac{x}{L}, \quad \Omega = \frac{\omega}{\omega_n}, \quad 2\zeta = \frac{c}{m\omega_n}, \quad m_n = \frac{m_i}{m}L^2, \quad k_n = \alpha\frac{k_3}{k_1}L^2 \tag{6}$$

### 3.2 Frequency Domain

To avoid expensive time-marching numerical techniques for solving the equation of motion, the Harmonic Balance Method (HBM) is utilized to provide direct steady-state periodic solution of the nonlinear differential equation [14]. HBM is a common methods for approximating periodic solutions for oscillatory systems in many applications if transient is not present [9]. However, Nyfeh described the HBM as an analytical approach that is equivalent to performing a stepped-sine or slow sine-sweep testing [12]. In general, periodic solutions can be estimated using truncated Fourier series, where coefficients are determined by constructing series that satisfy the deferential equations [14]. Clearly, the accuracy of these solutions can be improved by increasing the number of terms obtained from Fourier series into trial solutions. In practice, however, reasonable accuracy of the solutions can be achieved by using limited number of terms in Fourier series. For the nonlinear system in this paper, an approximated periodic solution can be expressed with the following terms:

$$\eta = A\cos\phi + B\sin\phi \tag{7}$$

where, $\phi = \Omega\tau$. In addition $\eta$, the dimensionless velocity, acceleration, and cubic components will also have their own truncated terms. The forcing function is also assumed to be periodic, $Y = \Omega^2 Y_o \cos\phi$. Assembling the contribution of every variable into equation 5 produces four harmonics: $\cos\phi$, $\sin\phi$, $\cos 3\phi$, and $\sin 3\phi$, which are matched (or balanced). Splitting $\cos\phi$, and $\sin\phi$, harmonics, and neglecting higher ones yield the following:

$$H_c := -A\Omega^2 + 2\zeta B\Omega - \frac{3}{2}m_n AB^2\Omega^2 + A + \frac{3}{4}k_n\Big(A^3 + AB^2\Big) + \Omega^2 Y_o = 0 \tag{8}$$

$$H_s := -B\Omega^2 - 2\zeta A\Omega - \frac{3}{2}m_n A^2 B\Omega^2 + B + \frac{3}{4}k_n\Big(B^3 + A^2 B\Big) = 0 \tag{9}$$

$H_c$, and $H_s$ are the $\cos\phi$, and $\sin\phi$ components, respectively. The autoperiodic force response amplitudes $A$, and $B$ are unknowns, which can be obtained by varying the angular frequency $\Omega$. To this end, Newton-Raphson method is utilized to numerically calculate $A$, and $B$ for each $\Omega$ [15]. A frequency response approximation that resembles an FRF (frequency-response-function) can be achieved by performing polar transformation in equations 8 and 9. With additional algebraic manipulations, the system motion can be expressed in the following familiar frequency domain form:

$$\frac{X^2}{Y_o^2} = \frac{1}{\Big(\Omega_{1,2}^2 - k_{eq} + 2\mu\Big)^2 - 4\mu(\mu - k_{eq})} \tag{10}$$

where, $X$ is a byproduct of the polar transformation procedure, which contains both $A$ and $B$. The effective damping has a nonlinear inertial contribution:

$$\mu = \frac{2\zeta}{2 + m_n X^2} \tag{11}$$

The effective or equivalent stiffness is:

$$k_{eq} = 1 + \frac{3}{4}k_n X^2 \tag{12}$$

# 4 Results

Prior to numerically solving equations 8 and 9, a static deflection experiment was performed for three cantilever beams. Tip displacement measurements were obtained for each static load applied incrementally to the beam tip. The force-displacement results are shown in Figure 3. The tip displacement values in Figure 3 are normalized by the beam total free length. It can be concluded that Al 7075-T6 structures experience hardening behavior for high displacement. The $k_1$ and $k3$ values were estimated using a cubic polynomial curve fitting procedure, which was performed in Matlab (Figure 3). The experimental results show that the beam linear and nonlinear stiffness coefficients were approximately 10% higher than the theoretical values.

As mentioned above, Newton-Raphson method was applied to calculate the beam tip displacement amplitude due to 1g harmonic base excitation. The established convergence error for 0% fatigue-life was $\leq 10^{-10}$,

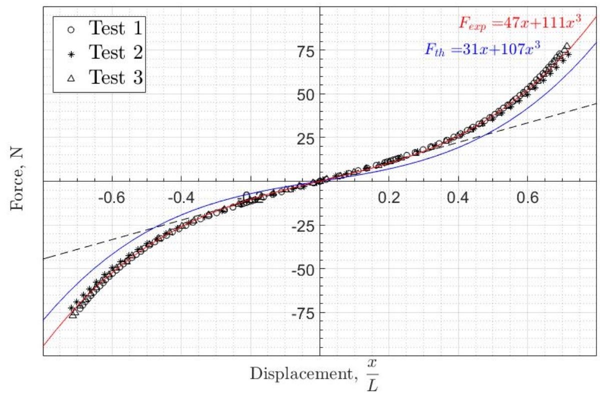


Figure 3: Restoring force due to nonlinear tip deflections of cantilever beams

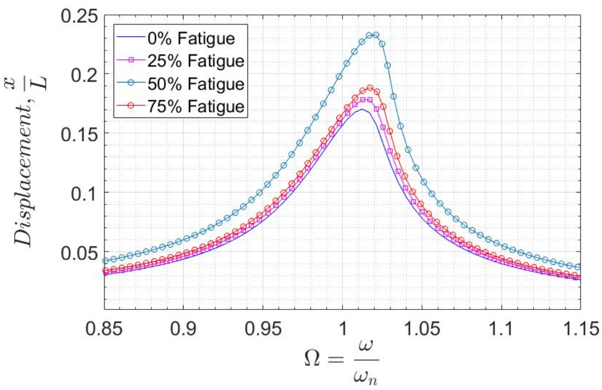


Figure 4: Beam tip maximum amplitudes due to 1g base harmonic excitation at fatigue-life levels: 0%, 25%, 50%, and 75%

which was set to be the accuracy criterion for all fatigue cases as well. The estimated changes in $E$ obtained from the nano-indentation results were used to update the macro-state stiffness values in the beam spatial region $0mm \rightarrow 1mm$ for the following cases: 25%, 50%, and 75% fatigue-life. The dynamic maximum beam tip displacement provided in Figure 4 is normalized by the beam length, $L$, and plotted as a function of $\Omega$.

As expected, the beam compliance increased when the number of fatigue cycles was increased. Compliance is defined as an increase in the beam tip deflection due to fatigue accumulation without an increase in the input force. Interestingly, the maximum compliance occurred at 50% of fatigue-life level. The beam appeared to experience softening-stiffing-softening behavior at 25%, 50%, and 75% fatigue levels, respectively. The softening-stiffing-softening behavior is illustrated in Figure 5, which shows the changes in the equivalent stiffness, $keq$, due to fatigue build up near the beam root. While this is a known behavior for aluminum 6061 subject to medium strain-rate fatigue conditions [16], it is possible, however uncertain, that Al 7075-T6 may have similar reaction to medium strain-rate vibration fatigue. Further studies will be required to gain insights into the mechanics of the softening-stiffing-softening behavior.

# 5 Conclusion

The structural dynamic behavior of Al 7075-T6 beam-like structures exposed to high harmonic base excitation was examined to gain insights into the influence of the nonlinear parameters in equation of motion on the

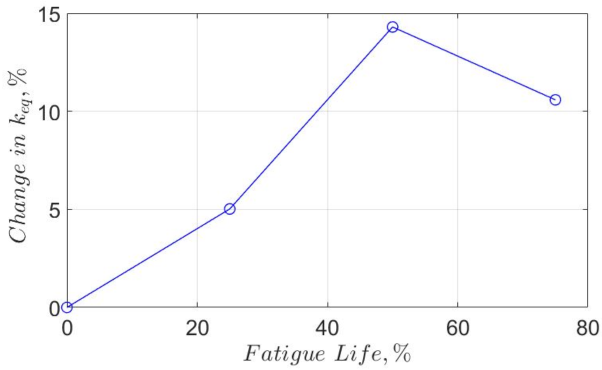


Figure 5: Change in $k_{eq}$ due to 25%, 50%, and 75% fatigue

global dynamic performance. The Harmonic Balance method was utilized to produce a frequency-response equation, and a macro-state equivalent stiffness parameter based on updates in micro-state nonlinearities were acquired from the nonlinear dynamic analysis. The model was utilized to modify the parametric identification scheme based on changes in the materials health-state due to the development of damage precursors. The frequency-response equation is ideal for controls algorithms, thus, the evolution in the materials properties can be identified and controlled by adjusting the macro-states inputs. The parametric study revealed that the nonlinear stiffness coefficient is an important parameter for estimating and controlling the dynamic of aerospace structures. In general, nonlinear parameters are sensitive to damage precursors, thus, changes in those parameters must be included in calculating restoring forces instead of using a dynamic hardening-softening factor for control, which is not adequate. Estimating and controlling the health state of critical components in high-value system will continue to be challenging, but can be improved. Future research will focus on integrating PHM/SHM/CBM into basic adaptive control-system to study whether an integrated approach could ameliorate the effect of extrinsic factors such as aeroelastic instability, vibration fatigue, and thermal and solar loads.

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