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A Hamiltonian formulation of boundary control systems

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Abstract

A port controlled Hamiltonian formulation of the dynamics of distributed parameter systems is presented, which incorporates the energy flow through the boundary of the domain of the system, and which allows to represent the system as a boundary control Hamiltonian system. This port controlled Hamiltonian system is defined with respect to a Dirac structure associated with the exterior derivative and based on Stokes' theorem. The definition is illustrated on the examples of the telegrapher's equations, Maxwell's equations, the vibrating string and Euler's equations for fluid dynamics.

Keywords

Distributed parameter systems, Hamiltonian systems, Dirac structures, boundary control

Recently, for *finite-dimensional* nonlinear systems we have proposed a generalized Hamiltonian formulation of physical systems' dynamics with external (input and output) variables. This has led to the notions of *port-controlled Hamiltonian (PCH) systems* [7] [6], and *port-controlled Hamiltonian systems with dissipation* (PCHD systems) [5] defined with respect to a geometric structure, called Dirac strucure, which expresses the dynamic invariants and contraints arizing from the balance and continuity equations of physical systems' models. This theory is aimed at applications in the modelling and simulation of complex *interconnected* physical systems, and in the design and *control* of such systems, exploiting the Hamiltonian and passivity structure in a crucial way [5], [3].

In the present paper we present an extension of finite-dimensional PCH and PCHD systems to the distributed parameter (or, infinite-dimensional) case. It extends also the Hamiltonian formulations theory as for instance exposed in [4] to distributed parameter systems *with external variables* (inputs and outputs)by including boundary conditions inducing *energy exchange through the boundary*.

Therefore, in the present paper we the definition *Dirac structure* on certain spaces of differential forms on the spatial domain and its boundary proposed in previous publications [1] [2]. This construction of the Dirac structure is based on the use of Stokes' theorem. Then we employ the definition of a port-controlled Hamiltonian system with respect to a Dirac structure, as already given in previous papers (see e.g. [7]) for the finite-dimensional case, to describe *implicit* PCH systems, in order to formalize distributed parameter systems with boundary external variables as infinite-dimensional PCH systems. This framework is then applied to the port-controlled Hamiltonian formulation of Maxwell's equations on a bounded domain, the telegrapher's equations for an ideal transmission line, and the vibrating string. Furthermore, by modifying the Stokes-Dirac structure with an additional term corresponding to three-dimensional convection we provide an extension of port-controlled Hamiltonian systems suitable for the formulation of the ideal adiabatic fluid (e.g. Euler's equations).

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