PLURALISM IN DISTRIBUTED PARAMETER SYSTEMS,

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Preface

Many of the participants to this workshop know the Netherlands very well from previous visits to Dutch universities and from past conferences such as the MTNS and the ECC. While *Distributed Parameter Systems* has always been a significant component in the numerous conferences on systems and control held around the world, there have not been many which have focused on this aspect alone. Notable exceptions have been the Vorau and Bozeman conferences and the INRIA conference in 1992. After almost 20 years of fruitful collaboration which has produced 7 ph.d. theses, 3 books and many papers on *Distributed Parameter Systems*, we decided that it was high time we organized a workshop focusing on our research area on our own home ground. Of course this is easier said than done and we would like to take this opportunity to acknowledge the support we have had from many sources. First we thank the Faculty of Mathematical Sciences of the University of Twente for providing secretarial support, Carla Hassink for the efficient way in which she carried out her task, and Michel ten Bulte for his professional support with the web pages. Hans would like to express his thanks to Hubert van Mastrigt for his skillful help in designing the logo, poster and cover of this book of abstracts. Next we gratefully acknowledge the financial support we have received from the Dutch Academy of Sciences (KNAW), the Netherlands Organization for Scientific Research, the Division of Physical Sciences (NWO/EW) and from the Faculty of Mathematical Sciences of the University of Twente, and the University of Twente. Last, but not least we are especially grateful to the University of Twente for hosting this workshop.

You may recall that the aim of our workshop is to bring together scientists who are all studying *Distributed Parameter Systems*, but from different points of view and possessing different types of expertise. We have been pleasantly surprised by the number of participants and by the breadth of the topics: from control of thin film growth in chips to a behavioural approach to DPS. We invite you to explore other avenues of studying *Distributed Parameter Systems*, not only by listening to the presentations, but by asking (challenging) questions. We hope that this will be a workshop in the true sense of the word and that it opens up new directions for future research.

We are delighted to welcome you to the University of Twente and we wish you an informative and enjoyable sojourn in the Netherlands.

Ruth Curtain and Hans Zwart.

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Similarity problem for minimal dissipative scattering systems.

D.Z. Arov

1.1 Formulation of problem.

Dissipative scattering linear stationary continuous-time system Σ is defined by the system of the equtions

$$\dot{x} = Ax(t) + Bu(t), \quad y(t) = N(x(t), u(t))$$

with the dissepativness condition

$$\|u(t)\|^2 - \|y(t)\|^2 \geq \frac{d}{dt}\|x(t)\|^2,$$

more details see in [1]. The theory of such systems is closely connected with theory of well possed systems, see[2]. Transfer function (scattering matrix) Θ_Σ of such a system belongs to the Schur class $S(U, Y)$ of holomorphic contractive in the upper half plane $\mathbb{C}_+$ functions with values from $[U, Y]$, where $[U, Y]$ is the space of linear bounded operators, acting from (input) separable Hilbert space U into (output) separable Hilbert space Y. Moreover, a function $\Theta \in S(U, Y)$ is scattering matrix of dissipative scattering system that satisfy the condition of minimality (controlability and observability). But such a system Σ with $\Theta_\Sigma = \Theta$ is defined by Θ only up to weak similarity: weak simisilaty operator may be unbounded or it may have unbounded inverse operator.

We solve the following similarity problem: find a necessary and sufficient condition on $\Theta(\in S(U, Y))$ under which a minimal dissipative scattering system Σ with $\Theta_\Sigma = \Theta$ is defined by Θ up to usual similarity, when a similarity operator could be bounded and it could have a bounded inverse operator .

1.2 Scattering suboperator of the inner scattering.

Let $\Theta \in S(U, Y)$. Then there exist essentually unique a simple conservative scattering system $\hat{\Sigma}$ with $\Theta_{\hat{\Sigma}} = \Theta$, see [1]. Let $\hat{T}(t) = e^{\hat{A}t}$ be the evolution (contractive) semigroup of the system $\hat{\Sigma}$. Let $\overset{0}{D}_+$ ($\overset{0}{D}_-$) be the maximal subspace on which $\hat{T}(t)$ ($\hat{T}^*(t)$, resp.) induce a semigroup $\overset{0}{V}_+ (t)$ ($\overset{0}{V}_- (t)$, resp.) of isometrical operators.

We have $\overset{0}{D}_{+}= \{0\}$ $\quad (\overset{0}{D}_{-}= \{0\})$ iff the factorization inequality problem

$$\varphi^{*}\varphi \leq I - \Theta^{*}\Theta, \quad \varphi \in S(U, Y_{\varphi}) \tag{1.1}$$

$$(\psi\psi^{*} \leq I - \Theta\Theta^{*}, \quad \psi \in S(U_{\psi}, Y), resp) \tag{1.2}$$

have only trivial solution $\varphi = 0(\psi = 0$, resp).

If $\overset{0}{D}_{+}\neq \{0\}$ and $\overset{0}{D}_{+}\neq \{0\}$, then $\hat{T}(t)$ is contractive coupling of $\overset{0}{V}_{-}(t)$ and $\overset{0}{V}_{+}(t)$ and we obtain a generalized dissipative scattering Lax-Phillips scheme. For this scheme is defined the notion of scattering suboperator, see [3]. We call it as scattering suboperator of the inner scattering for the system $\hat{\Sigma}$ and we denote it by $s_{\Theta}(\mu)$. The function $s_{\Theta} \in L_{\infty}(\overset{0}{U}, \overset{0}{Y})$, $\|s_{\Theta}\|_{\infty} \leq 1)$ is defined by Θ up to constant unitary right and left multipliers; s_{Θ} is essentually unique solution of the problem: find $s(\mu)$ such that

$$\tilde{S}(\mu) := \begin{bmatrix} \psi_{\Theta}(\mu) & \Theta(\mu) \\ s(\mu) & \varphi_{\Theta}(\mu) \end{bmatrix} \in L_{\infty}(\overset{0}{U} \oplus U, Y \oplus \overset{0}{Y}), \quad \|\tilde{S}\|_{\infty} \leq 1 \tag{1.3}$$

where φ_{Θ} and ψ_{Θ} are maximal solution of the problem (1) and (2), see [4]. In the case, when the equality problems

$$\varphi^{*}(\mu)\varphi(\mu) = I - \Theta^{*}(\mu)\Theta(\mu) \quad \text{a.e.}, \quad \varphi \in S(U, Y_{\varphi}) \tag{1.4}$$

$$\psi(\mu)\psi^{*}(\mu) = I - \Theta(\mu)\Theta^{*}(\mu) \quad \text{a.e.}, \quad \psi \in S(U_{\psi}, Y) \tag{1.5}$$

are solvable, φ_{Θ} and ψ_{Θ} are outer and $*$ -outer solutions of these problems, resp., and $S_{\Theta}(\mu) \in L_{\infty}(\overset{0}{U}, \overset{0}{Y})$ may be defined by the equivalent relations

$$s_{\theta}(\mu)\psi_{\theta}^{*}(\mu) = -\varphi_{\Theta}(\mu)\theta^{*}(\mu) \quad \text{a.m.}$$

$$\varphi_{\Theta}^{*}(\mu)s_{\Theta}(\mu) = -\Theta^{*}(\mu)\psi_{\Theta}(\mu) \quad \text{a.m.}$$

1.3 Solution of the similarity problem.

In the joint works with Nudelman we obtained the following results.

Theorem 1.3.1. *Let $\Theta \in S(U, Y)$. Then a minimal dissipative scattering system Σ with $\Theta_{\Sigma} = \Theta$ is defined by Θ up to unitary similarity iff at least one of the problems (1.1) or (1.2) have only trivial solution or $s_{\Theta}(\mu)$ is the boundary value of a function $s_{\Theta}(\lambda) \in S(\overset{0}{U}, \overset{0}{Y})$,*

Theorem 1.3.2. *Let $\Theta \in S(U, Y)$. Let both problems (1.1) or (1.2) have nontrivial solutions. then a minimal dissipative scattering systems Σ with $\Theta_{\Sigma} = \Theta$ is defined by Θ up to similarity iff the Hankel operator with the symbol $s_{\Theta}(\mu)$ has a closed range.*

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2

Receding horizon optimal control problems for infinite dimensional systems

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Abstract

In this paper we consider the receding horizon optimal control problems for nonlinear control problems governed by PDEs. We discuss the receding horizon control with the terminal cost chosen as a control Liapunov function and the asymptotic behavior and performance of the receding horizon synthesis is analyzed for regulator as well as disturbance attenuation problems. Both the continuous as well as the discrete–time cases are treated. Further the approximation of the continuous time optimal control problem by the discrete–time receding horizon control is analyzed. Examples illustrating the applicability of the concepts and results of this paper are presented.

Keywords

Receding Horizon Control, Control Liapunov Function, Stability

2.1 Introduction

We consider the optimal control problems in Hilbert spaces X and W

$$\inf \int_0^{T_\infty} f^0(x(t), u(t))\, dt \tag{2.1}$$

subject to

$$\frac{d}{dt}x(t) = f(x(t), u(t)), \quad \text{for } t > 0, x(0) = x_0, \quad \text{and } u(t) \in U. \tag{2.2}$$

We refer to $x(\cdot)$ and $u(\cdot)$ as state and control functions in X and U and U is a closed convex subset of W. We assume the following conditions

- For $x_0 \in X$ and $u \in U_{ad} = \{u \in L^2(0, T; W) : u(t) \in U \; a.e.\}$ there exists a X-valued continuous semi-flow $x(t) = x(t; x_0, u)$ which is a weak solution to (2.2).

- For each $(x, p) \in X \times X$ the functional

  $$u \to H(x, u, p) = f^0(x, u) + (p, f(x, u))$$

  admits a unique minimizer over U denoted by $\Psi(x, p)$, and that Ψ is continuous and f^0 is C^2.

Under appropriate conditions (2.1)–(2.2) admits a solution which satisfies the minimum principle

$$\begin{cases} \frac{d}{dt}x(t) = H_p(x(t), u(t), p(t)), \; x(0) = x_0, \\ \frac{d}{dt}p(t) = -H_x(x(t), u(t), p(t)), \; p(T_\infty) = 0, \\ u(t) = \arg\min_{u \in U} H(x(t), u, p(t)), \end{cases} \tag{2.3}$$

The coupled system of two-point boundary value problems with initial condition for the primal equation and terminal condition for the adjoint equation represents a significant numerical challenge in the case the T_∞ is large and has been the focus of many research efforts.

In view of the difficulties explained above the question of obtaining suboptimal controls arises. One of the possibilities is given by receding horizon formulations [1]. Receding horizon formulations have proved to be effective numerically both for optimal control problems governed by ordinary (e.g. [2, 7, 9]) and for partial differential equations, e.g. in the form of the instantaneous control technique for problems in fluid mechanics [3, 4, 6]. The receding horizon optimal control problem involves the successive finite horizon optimal control on $[T_i, T_i + T]$:

$$\min \int_{T_i}^{T_i+T} f^0(x(t), u(t))\, dt + G(x(T_i + T)),$$

subject to

$$\frac{d}{dt}x(t) = f(x(t), u(t)), \; t \geq T_i, \quad x(T_i) = \bar{x}(T_i),$$

where $\bar{x}$ is the solution to the auxiliary problem on $[T_{i-1}, T_{i-1} + T]$. If $T > T_i - T_{i-1}$, then we have the overlapping horizons. If $x(T_i)$ is observed, then the receding horizon control is a state feedback since the receding horizon optimal control on $[T_i, T_{i+1}]$ is determined as a function of the state $x(T_i)$. The optimal pair $(\bar{x}(t - T_i), \bar{u}(t - T_i)), \; t \in [T_i, T_{i+1}]$ satisfies the two point boundary value problem (2.3) on the interval $[0, T]$ with the terminal condition $p(T) = G_x(x(T))$ and the initial condition $x(0) = \bar{x}$. If $T > 0$ is small, (2.3) is better conditioned and much easier to be solved numerically. In order to establish the asymptotic stability of the receding horizon control we utilize the terminal penalty terms rather than terminal constraints (e.g., [8]).

The functional $G\colon R^n \to R$ will be chosen as an appropriately defined control Liapunov function (see, Definition 2.1.1). It will be shown that the addition of the terminal cost G to the costfunctional provides the asymptotic stability and the suboptimal performance for minimizing (2.1) of the receding horizon control.

Let us now outline the contributions of this paper. We assume throughout that the sampling points are equidistant and that the sampling rate coincides with the time horizon, so that $T = T_i - T_{i-1}$ and $T_i = i\,T$. Let G be a control Liapunov function;

Definition 2.1.1. A nonnegative continuous function G with $G(0) = 0$ is a control Liapunov function for (2.1)–(2.2) if for all $x_0 \in R^n$ and $T > 0$ there exists a control $u = u(\cdot; x_0, T) \in U_{ad}$ such that

$$\int_0^T f^0(x(t), u(t))\,dt + G(x(T)) \le G(x_0),$$

where $x(t)$ is a solution to (2.2).

Then we first establish the monotonicity of the value function $V_T(x_0)$:

$$V_T(x_0) = \inf\left\{\int_0^T f^0(x(t), u(t))\,dt + G(x(T)) \text{ subject to (2.2)}\right\}$$

with respect to T, i.e., $V_T(x_0) \le V_{\hat{T}}(x_0) \le G(x_0)$ for $0 \le \hat{T} \le T$ and $x_0 \in X$. Thus, we have

$$G(x_{i+1}) + \int_{T_i}^{T_{i+1}} f^0(\bar{x}(t), \bar{u}(t))\,dt \le G(x_i)$$

where $x_i = \bar{x}(T_i)$. This implies that x_i confined in the level set $S_\alpha = \{x \in X : G(x) \le G(x_0) = \alpha\}$. Assume that $f(0,0) = 0$ and $G(0) = 0$ and that $f^0(x,u) > 0$ and $G(x) > 0$ except at $(0,0)$. Then, we have $G(x_{i+1}) < V_T(x_i) \le G(x_i)$. If S_α is compact, then we have $G(x_{i+1}) \le \rho\, G(x_i)$ for $x \in S_\alpha$ some $\rho < 1$. Hence $G(x_k) \le \rho^k G(x_0) \to 0$ as $k \to \infty$, which implies the asymptotic stability. Moreover, if $f^0(x,u) \ge \omega G(x)$ for some $\omega > 0$, then $G(x_{i+1}) \le e^{-\omega T} G(x_i)$. We prove that the quadratic $G(x) = \frac{\alpha}{2}|x|^2$, $\alpha > 0$, can be chosen as a control Liapunov function, if the control system (2.2) is closed loop dissipative;

Definition 2.1.2. The control system (2.1)–(2.2) is closed-loop dissipative if there exists a locally Lipschitzian feedback law $u = -K(x) \in U$ such that

$$f(x, -K(x)) \cdot (\alpha\, x) + f^0(x, -K(x)) \le 0$$

for some $\alpha > 0$ and all $x \in R^n$, (which is a useful assumption for certain classes of dissipative equations).

In general the quadratic terminal penalty is not a Liapunov function. But we have $V_T(x) \le \rho_T\, G(x)$ with $\rho_T < 1$ for T sufficiently large (see, Theorem 2.5) provided that the value function $V(x)$ of the infinite time horizon problem satisfies $V(x) \le \frac{\beta}{2}|x|^2$ and the corresponding optimal trajectory satisfies $|x^*(t)| \le M\,e^{-\omega t}|x_0|,\ \omega > 0$.

In our discussion above it is implicitly assumed that the infinite time horizon optimal control problem admits solutions (i.e., regular case), as for example in the case of stabilizable steady states. In general this assumption is not satisfied and we analyze the general case in Section 2.3. It applies to disturbance attenuation problems and to problems with cost functionals of tracking type. We introduce a control λ-Liapunov function (see, Definition 2.3) and the above analysis is extended to the general case. Here a positive constant λ represents the attenuation or tracking rate: $\lim_{T\to\infty} \frac{1}{T}\int_0^T f^0(\bar{x}(t), \bar{u}(t))\,dt$.

We also discuss discrete–time control problems. Our interest is the case when the discrete–time problems arise from a finite difference approximation of (2.1)-(2.2), for example the explicit Euler approximation: $x_j = x_{j-1} + \Delta t\, f(x_{j-1}, u_j)$ or in general the one–step method. The results as well as concepts parallel to the continuous–time case are established.

In addition we investigate the asymptotic behavior of the continuous time systems (2.2) if the controls are determined by means of a discrete–time synthesis, in fact, one–step, receding horizon formulation. This analysis is of practical importance since numerical methods for solutions of (2.3) commonly use a difference approximation of (2.1)–(2.2) and thus the control synthesis $\bar{u}(t)$ is computed in terms of the discrete–time control system.

Examples illustrating the applicability of the concepts and results of this paper will be presented.

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3

A comparison of two balancing methods for low order controller design

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Abstract

In this talk, we will compare two model reduction techniques that can be used to provide low order models for control design. The first, balancing and truncation balances the model based on the controllability and observability Grammians; truncation decisions can be made from the singular values of the Hankel operator. The second approach is termed LQG balancing and balances the model based on the solutions of the control and filter Riccati equations. Model truncation is based the Riccati singular values

Keywords

Reduced Order Controllers, Balanced Truncation, LQG Balancing

3.1 Introduction

The design of low order controllers is a fundamental step for real-time control. Not only is the concept important for large scale systems of ordinary differential equations (ODE), but it is imperative for control of systems modeled by partial differential equations (PDE). A typical approach is to reduce the model and design a control using standard designs such as linear-quadratic-gaussian

(LQG), minmax, or H_∞. For PDE systems, the first step in computation of a control design is typically numerical approximation by a large-scale ODE system, and then the tools for ODE systems can be applied. Two model reduction techniques commonly found in the literature are based on proper orthogonal decomposition (POD) [2, 3, 4, 9, 10, 12, 13] and balanced realization and truncation. The latter technique for infinite dimensional systems can be found in [5, 6]. Much can be found regarding finite dimensional systems; some recent citations are [1, 7, 16].

In this talk, we compare two balancing techniques followed by model truncation. The first is the standard balanced realization which involves model balancing based on the original system matrices. The second technique balances based on the controller for the PDE system. Specifically, a feedback controller is computed which is then used for balancing and truncation; it can be found in the literature for finite dimensional systems in [8, 14, 15].

3.2 The PDE control problem

Linear quadratic Gaussian (LQG) control theory provides a controller which allows for limited state measurement and feeding back an estimate of the state that is based upon that measurement. Such a controller can be computed via the solution of two algebraic Riccati equations, as we outline below.

Given a system in the abstract form

$$\dot{w}(t) = Aw(t) + Bu(t), \qquad w(0) = w_0, \tag{3.1}$$

suppose measurement of the state is given by

$$y(t) = Cw(t). \tag{3.2}$$

We will use the notation $\Sigma(A, B, C)$ to refer to the system defined by (3.1), (3.2). The class of systems considered here are those that are exponentially stabilizable and detectable on Hilbert space X, where A is the infinitesimal generator of a strongly continuous semigroup $S(t)$ on X. The operators B and C are assumed to be finite rank and bounded, specifically, $B \in \mathcal{L}(\mathbb{R}^m, Z)$, $C \in \mathcal{L}(Z, \mathbb{R}^k)$. LQG design as applied to $\Sigma(A, B, C)$ provides a state estimate, $w_c(t)$ and a feedback law. These are given as

$$\dot{w}_c(t) = A_c w_c(t) + Fy(t), \quad w_c(0) = w_{c_0} \tag{3.3}$$

and

$$u(t) = -Kw_c(t), \tag{3.4}$$

respectively. In (3.4), the operator K is called the feedback operator.

Control design means determining K, F and A_c; LQG theory provides these operators as follows. Define a non-negative definite, self-adjoint state weighting operator $Q : W \to W$ and a positive definite, self-adjoint control weighting operator $R : U \to U$. Given $\theta \geq 0$, the algebraic Riccati equation

$$A^*\Pi + \Pi A - \Pi[BR^{-1}B^*]\Pi + Q = 0 \tag{3.5}$$

is solved for the non-negative definite, self-adjoint solution, Π. Then, given a positive definite self-adjoint weighting operator H, P is found as the non-negative definite, self-adjoint solution to

$$AP + PA^* - P[C^*H^{-1}C]P + DD^* = 0. \tag{3.6}$$

If the solutions P and Π exist, we define

$$\begin{aligned} K &= R^{-1}B^*\Pi, \\ F &= PC^*H^{-1}, \qquad \text{and} \\ A_c &= A - BK - FC. \end{aligned} \tag{3.7}$$

Using Equations (3.1), (3.2), (3.3) and (3.4), the closed loop system can be written in matrix form as

$$\begin{bmatrix} \dot{w}(t) \\ \dot{w}_c(t) \end{bmatrix} = \begin{bmatrix} A & -BK \\ FC & A_c \end{bmatrix} \begin{bmatrix} w(t) \\ w_c(t) \end{bmatrix}, \quad \begin{bmatrix} w(0) \\ w_c(0) \end{bmatrix} = \begin{bmatrix} w_0 \\ w_{c_0} \end{bmatrix}. \tag{3.8}$$

Results from distributed parameter control theory can be used to establish the existence of such a control design, and the stability properties of the closed loop system. The challenge for implementation is to design low order controllers that can be used for real-time control and which stabilize the original plant modeled by the PDE.

3.3 Reduced order compensators

Implementation of a controller for a PDE system requires a numerical discretization. For example, a finite element method provides semi-discrete finite dimensional approximations of $\Sigma(A, B, C)$ of order N (where order refers to the number of basis elements), given by

$$\dot{w}^N(t) = A^N w^N(t) + B^N u^N(t), \qquad w^N(0) = w_0^N, \tag{3.9}$$

$$y^N(t) = C^N w^N(t). \tag{3.10}$$

We denote this system as $\Sigma(A^N, B^N, C^N)$. In a full order compensator design, the order N approximations are used to compute K^N, F^N and A_c^N. Finite dimensional approximations of the compensator equation (3.3) and control law (3.4) are given by

$$\dot{w}_c^N(t) = A_c^N w_c^N(t) + F^N y^N(t), \quad w_c^N(0) = w_{c_0}^N, \tag{3.11}$$

$$u^N(t) = -K^N w_c^N(t). \tag{3.12}$$

Convergence of the finite dimensional approximations to the distributed parameter system in (3.8) can be addressed by established theory.

One approach to the design of low order controllers is to reduce $\Sigma(A^N, B^N, C^N)$ and then to formulate a controller based on the low order model. There are several ways to do this. One which has attracted much attention in the recent literature is the proper orthogonal decomposition (POD), as cited in the introduction. In [11], the authors show a connection between POD and the standard balanced realization and truncation.

In this talk, we will compare the results of balancing and truncation with a technique known as LQG balancing followed by truncation [8]. Here we provide a very brief (and rough) explanation of the two methods. In balanced realizations, the balancing procedure involves producing a realization of $\Sigma(A^N, B^N, C^N)$ for which the controllability and observability gramians are equal. Moreover, they are diagonal with the singular values of the Hankel operator on the main diagonal. Truncation is based upon the magnitude of the singular values, and states corresponding to the "small" singular values are neglected. In LQG balancing, a realization of $\Sigma(A^N, B^N, C^N)$ is produced in which the Riccati operators, Π and P are equal, and equal to Riccati singular values. Truncation is based on the magnitude of these values.

The low order models, denoted as $\Sigma(A^M, B^M, C^M)$ where $M << N$, are then used for controller design. In reality, this low order controller would be applied to the plant. To test our control designs, we apply the low order controller to the full order model. That is, to investigate the performance of the design, a simulation of the system consisting of the full order (N) state and reduced order (M) state estimate given by

$$\begin{aligned} \begin{bmatrix} \dot{w}^N(t) \\ \dot{w}_c^M(t) \end{bmatrix} &= \begin{bmatrix} A^N & -B^N K^M \\ F^M C^N & A_c^M \end{bmatrix} \begin{bmatrix} w^N(t) \\ w_c^M(t) \end{bmatrix} \\ \begin{bmatrix} w^N(0) \\ w_c^M(0) \end{bmatrix} &= \begin{bmatrix} w_0^N \\ w_{c_0}^M \end{bmatrix} \end{aligned} \tag{3.13}$$

is typically computed.

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4

Modeling and control issues associated with atomic force microscopy

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Abstract

This paper addresses the modeling and control of hysteresis and nonlinear dynamics inherent to the piezoceramic positioning mechanisms employed in atomic force microscopes (AFM). To quantify these effects, domain wall theory for ferroelectric materials is combined with classical rod or shell theory to provide nonlinear models which characterize the behavior of the piezoceramic elements in an AFM. These models are then discretized through Galerkin techniques to obtain full-order, semidiscrete models which are appropriate for simulation purposes. Reduced-order models based on proper orthogonal decompositions (POD) are subsequently constructed to facilitate real-time implementation. A variety of control techniques based on full or partial inverse compensators will be discussed and illustrated through both numerical and experimental examples.

Keywords

Atomic Force Microscope, Hysteresis, Reduced-order Models, Nonlinear Control Design

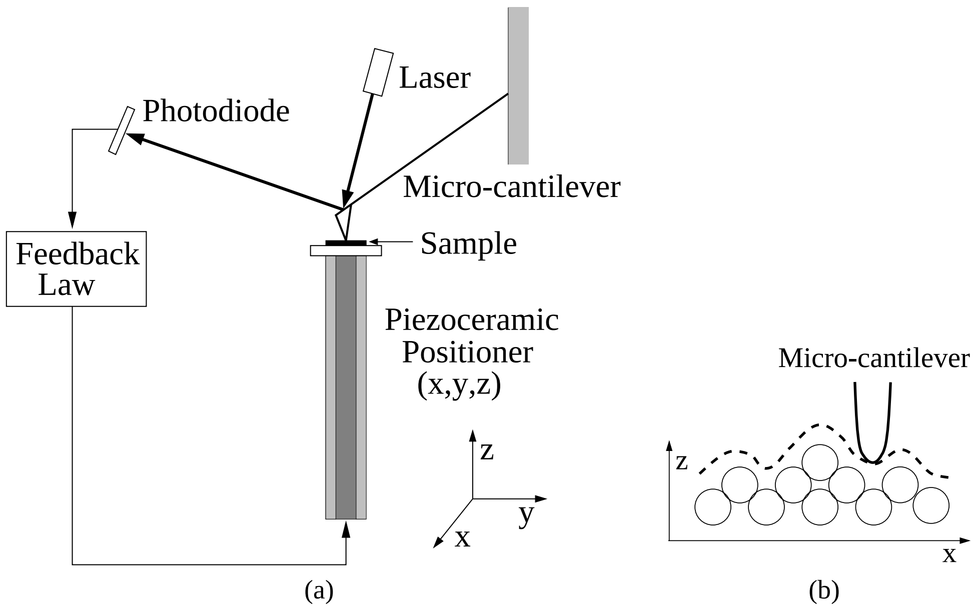

Figure 4.1: (a) Configuration of a prototypical AFM; (b) Surface image determined after one lateral sweep.

4.1 Introduction

Atomic force microscopes (AFM) provide the capability for obtaining angstrom-level measurements of both organic and inorganic compounds by monitoring and regulating the forces between the tip of a cantilever mechanism employed in the microscope and atoms in the sample. To illustrate the principles and components of an AFM, consider the prototypical device depicted in Figure 1a. The sample is moved both laterally and vertically beneath a soft cantilever by either a stacked piezoceramic (PZT) actuator or a PZT shell. To ascertain the 3-D structure of the compound, the sample is moved along an x-y grid using the lateral positioning mechanisms. As the sample moves, displacements in the cantilever tip are monitored using the photodiode and corresponding forces are determined via Hooke's law. The sample is then displaced in the z-direction to maintain constant forces, with the amount of displacement determined through the feedback law. A complete scan in this manner provides a surface image of the compound. Details regarding the construction and applications utilizing atomic force microscopes and scanning tunneling microscopes (STM) can be found in [5].

The degree of accuracy to which the PZT elements can laterally and vertically position the sample is crucial to the accuracy of the final images. Two commonly employed positioning mechanisms include stacked actuators and cylindrical shells. While both constructions provide highly repeatable and accurate set point placement, the relations between the input voltages and displacements produced in the PZT material exhibit inherent hysteresis and constitutive nonlinearities as illustrated in Figure 2 with data collected from an AFM. Feedback mechanisms can reduce these effects to some degree, but ultimately, models quantifying these mechanisms in combination with control laws based on these models are required to achieve the speed and accuracy required for both future AFM applications and nanoconstruction utilizing PZT positioning mechanisms.

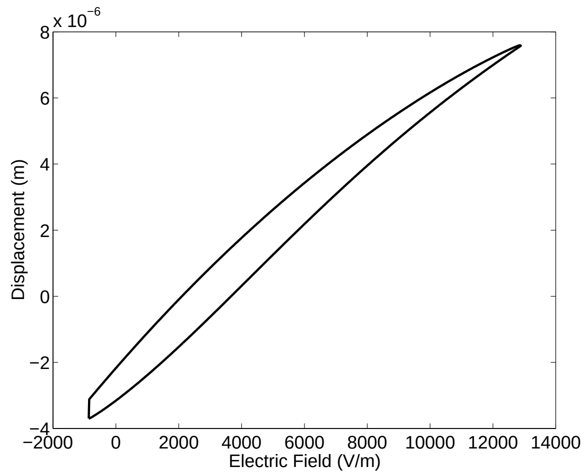

Figure 4.2: Relation between the input field E and displacements generated by the PZT positioning mechanism in an AFM.

4.2 Model development

The displacements generated in either stacked or cylindrical actuators in response to input voltages are quantified in two steps. In the first, domain wall dynamics are employed to provide nonlinear constitutive relations for the material. These are then employed in classical rod or shell theories to provide nonlinear models which quantify the PZT dynamics.

To illustrate, consider a stacked actuator which can be modeled as a rectangular rod generating longitudinal displacements v in response to an input voltage V or field E. From the theory developed in [8, 9], the relation between the input field and generated polarization P can be expressed as

$$\begin{aligned} \frac{dP}{dE} &= F\,(E, P, v) \\ P(E_0) &= P_0 \end{aligned} \tag{4.1}$$

where F depends upon material properties in addition to the polarization, field and displacement. Relation (4.1) quantifies the hysteresis and constitutive nonlinearities illustrated in Figure 2.

Newtonian or Hamiltonian principles can then be employed to obtain modeling PDE which characterize displacements generated at points within the actuator. For the stacked actuator, which is modeled as a rod of length ℓ and cross-sectional area A, this yields the model

$$\int_0^\ell \rho \frac{\partial^2 v}{\partial t^2} \phi \, dx + \int_0^\ell \left[c^P \frac{\partial v}{\partial x} + c_D \frac{\partial^2 v}{\partial x \partial t} \right] \frac{\partial \phi}{\partial x} = - \int_0^\ell c^P \alpha P(E, v) \phi \, dx \tag{4.2}$$

which must be satisfied for all $\phi \in H_L^1(0, \ell) = \{\phi \in H^1(0, \ell) \,|\, \phi(0) = 0\}$. Here, ρ, c_P and c_D respectively denote the density, Young's modulus and Kelvin-Voigt damping parameters for the PZT material. The electromechanical parameter α quantifies the degree of strain produced by changes in the polarization. Analogous, albeit more complicated, models result for cylindrical actuators modeled as thin shells.

The model (4.2) is nonlinear in both the state v and the control input $u = E$. For certain operating regimes, it is reasonable to linearize with respect to v while retaining the nonlinear relation between the control input and resulting polarization. Under this assumption, either full or reduced-order approximations in space yield the semidiscrete system

$$\begin{aligned} \dot{y}(t) &= Ay(t) + [B(u)](t) \\ y(0) &= y_0 \end{aligned} \tag{4.3}$$

where

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad , \quad [B(u)](t) = \begin{bmatrix} 0 \\ M^{-1}[b(u)](t) \end{bmatrix}$$

and M, K, C respectively denote the mass, stiffness and damping matrices. For full-order approximation of the rod model (4.2), linear splines are employed as basis elements whereas linear and cubic splines are employed when discretizing shell models for cylindrical actuators [3, 4].

To obtain reduced-order models, proper orthogonal decomposition (POD) elements constructed using snapshots in time are employed when constructing the basis. As illustrated in [1], the use of POD basis functions can reduce the dimension of the problem from over 700 coefficients to 12 coefficients for thin shell models. This facilitates significantly the construction of cylindrical actuator models for real-time implementation.

4.3 Control design

To obtain vertical displacements which maintain constant interatomic forces as the sample is moved laterally, it is necessary to have a precise characterization of the nonlinear and hysteretic constitutive properties of the PZT and to incorporate this characterization into the control law. This can be accomplished through a variety of control designs [2, 11] including linear control laws which utilize full or partial inverse compensators [6, 7, 10, 12]. The latter technique is illustrated in Figure 3.

To construct a full inverse compensator, the differential equation

$$\begin{aligned} \frac{\partial P^{-1}}{\partial E} &= \frac{1}{F\,(P^{-1}, E)} \\ P^{-1}(E_0) &= P_0^{-1} \end{aligned} \tag{4.4}$$

is used to invert the model (4.1) as a step in constructing an inverse operator B^{-1} which can be employed as a filter before the physical device. In this case, the signal u to the device is that specified by the control law to within the accuracy of the model. While this technique provides highly accurate control results from theoretical and numerical perspectives, it can be difficult to implement experimentally due to robustness issues such as the accuracy of initial conditions.

A second technique which provides less accuracy, but is more efficient to implement, is based on the construction of partial compensators B_p^{-1} using the hysteresis-free, or anhysteretic, component of the models. As illustrated in Figure 3b, the partial inverse compensator incorporates the constitutive nonlinearities but neglects the hysteresis. It is illustrated in [6] that the phase delays typically associated with uncompensated hysteresis can be minimized through the use of appropriate feedback mechanisms. Various features of the two techniques are illustrated through numerical and experimental examples in [6, 7, 10].

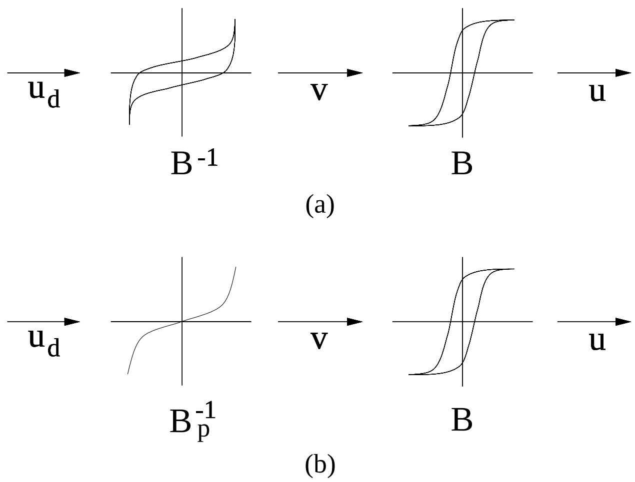

Figure 4.3: (a) Input u_d to a full inverse compensator B^{-1} and output u from the PZT positioning mechanism; (b) Input u_d to a partial inverse compensator B_p^{-1} and output u.

4.4 Concluding remarks

In this note, we have summarized model-based control strategies for accommodating hysteresis in PZT positioning mechanisms employed in atomic force microscopes to provide the capability for improving resolution, scanning speeds, and robustness with regard to changes in operating conditions such as temperature. The extension of these techniques to positioning mechanisms for nanoscale construction is under current investigation. The models and control laws employing model-based inverse compensators have been validated for a variety of ferroelectric and ferromagnetic compounds, and the experimental validation of the models and control laws for the AFM is also under current investigation.

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5

J-energy preserving well-posed linear systems

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Abstract

This is a short overview of the notion of a well-posed linear system. We start by describing the most basic concepts, proceed to discuss dissipative and conservative systems, and finally introduce J-energy preserving systems, i.e., systems that preserve energy with respect to some generalized inner products (possibly semi-definite or indefinite) in the input, state, and output spaces. The class of well-posed linear systems contains most linear time-independent distributed parameter systems: internal or boundary control of PDE:s, integral equations, delay equations, etc. These systems have existed in an implicit form in the mathematics literature for a long time, and they are closedly connected to the scattering theory by Lax and Phillips and to the model theory by Sz.-Nagy and Foiaş. The theory has developed independently in many different schools, and only recently these different approaches have begun to converge. One of the most interesting objects of present study is the Riccati equation theory for this class of infinite-dimensional systems (H^2- and H^∞-theory).

Keywords

Well-posed linear system, system operator, transfer function, Lax–Phillips semigroup, dissipative system, conservative system, model theory, conservative realization, J-energy preserving system, Lyapunov equation, Riccati equation.

5.1 Well-posed linear systems

Many infinite-dimensional linear time-independent continuous-time systems can be described by the equations

$$\begin{aligned} x'(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \quad t \geq 0, \\ x(0) &= x_0, \end{aligned} \tag{5.1}$$

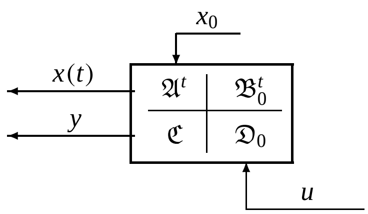

Figure 5.1: Input/state/output diagram of Σ

on a triple of Hilbert spaces, namely, the input space U, the state space X, and the output space Y. We have $u(t) \in U$, $x(t) \in X$ and $y(t) \in Y$. The operator A is supposed to be the generator of a strongly continuous semigroup $t \mapsto \mathfrak{A}^t$. The generating operators A, B and C are usually unbounded, whereas D is always bounded.

It is often convenient to use the integral representation of the system, which consists of the four operators from the initial state x_0 and the input function u to the final state $x(t)$ and the output function y (see Figure 5.1):

$$\begin{aligned} x(t) &= \mathfrak{A}^t x_0 + \mathfrak{B}_0^t u, \\ y &= \mathfrak{C} x_0 + \mathfrak{D}_0 u. \end{aligned} \tag{5.2}$$

Here, $\mathfrak{A}^t = e^{At}$ is the *semigroup* generated by A (which maps the initial state x_0 into the final state $x(t)$), $\mathfrak{B}_0^t$ is the map from the input u (restricted to the interval $[0, t]$) to the final state $x(t)$, $\mathfrak{C}$ is the map from the initial state x_0 to the output y, and $\mathfrak{D}_0$ is the input-output map from u to y. We get formulas for the operators $\mathfrak{B}_0^t$, $\mathfrak{C}$, and $\mathfrak{D}_0$ in (5.2) by using the standard "variation of parameters formula":

$$\begin{aligned} \mathfrak{B}_0^t u &:= \int_0^t \mathfrak{A}^{t-v} B u(v)\, dv, && t \geq 0, \\ (\mathfrak{C} x_0)(t) &:= C \mathfrak{A}^t x_0, && t \geq 0, \\ (\mathfrak{D}_0 u)(t) &:= C \int_0^t \mathfrak{A}^{t-v} B u(v)\, dv + D u(t), && t \geq 0. \end{aligned} \tag{5.3}$$

The standard *well-posedness* assumption is that (5.1)–(5.2) behave well in an L^2_{loc}-setting, i.e., for all $t \geq 0$, it is required that the final state $x(t) \in X$ and the output $y \in L^2_{\mathrm{loc}}(\mathbf{R}^+; Y)$ depend continuously on the initial state $x_0 \in X$ and on the input $u \in L^2_{\mathrm{loc}}(\mathbf{R}^+; U)$. If this is the case, then we call $\left[\begin{array}{c|c} \mathfrak{A}^t & \mathfrak{B}_0^t \\ \hline \mathfrak{C}_0^t & \mathfrak{D}_0^t \end{array}\right]$ a well-posed linear system (with initial time zero and final time t), where $\mathfrak{C}_0^t$ and $\mathfrak{D}_0^t$ are the "restrictions" of $\mathfrak{C}$ and $\mathfrak{D}_0$ to the interval $[0, t)$:

$$\begin{aligned} \mathfrak{C}_0^t &:= \pi_{[0,t]} \mathfrak{C} \text{ (= the output map on } [0,t]), \\ \mathfrak{D}_0^t &:= \pi_{[0,t]} \mathfrak{D}_0 \text{ (= the I/O map on } [0,t]), \\ (\pi_{[0,t)} u)(s) &:= \begin{cases} u(s), & s \in [0,t), \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \tag{5.4}$$

It is often convenient to get rid of the initial time 0 and the final time t by replacing $\mathfrak{B}_0^t$ and $\mathfrak{D}_0$

by two extended operators $\mathfrak{B}$ and $\mathfrak{D}$:

$$\begin{aligned} \mathfrak{B}u &= \int_{-\infty}^{0} \mathfrak{A}^{-v} Bu(v)\,dv, \\ \mathfrak{D}u &= C\int_{-\infty}^{t} \mathfrak{A}^{t-v} Bu(v)\,dv + Du(t), \quad t \in \mathbf{R}. \end{aligned} \tag{5.5}$$

We use the following terminonlogy: $\mathfrak{B}$ is the input, or *reachability map*, or controllability map, with initial time $-\infty$ and final time zero, $\mathfrak{C}$ is the output, or *observability map*, with initial time zero and final time $+\infty$, and $\mathfrak{D}$ is the *input-output map* with initial time $-\infty$ and final time $+\infty$. The maps $\mathfrak{B}_0^t$ and $\mathfrak{D}_0$ can be recovered from $\mathfrak{B}$ and $\mathfrak{D}$ (by shifting and cutting), and therefore we often alternatively denote the system by $\Sigma = \left[\begin{smallmatrix} \mathfrak{A} & \mathfrak{B} \\ \mathfrak{C} & \mathfrak{D} \end{smallmatrix}\right]$ (with no reference to initial and final times).

It is also possible to introduce the notion of an well-posed linear system without any reference to the differential system (5.1). Instead we use a set of algebraic conditions that the operators $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ must satisfy (the continuity assumptions stay the same). The operator $\mathfrak{A}$ is required to be a *strongly continuous semigroup*, and the other three operators should satisfy the following conditions:

1. *the input map* $\mathfrak{B}$ *intertwines the left shift on* $\mathbf{R}^-$ *with* $\mathfrak{A}$, i.e., $\mathfrak{A}^t\mathfrak{B} = \mathfrak{B}\tau_-^t$ for all $t \geq 0$ (where τ_-^t is the left-shift on $\mathbf{R}^-$);
2. *the output map* $\mathfrak{C}$ *intertwines* $\mathfrak{A}$ *with the left shift on* $\mathbf{R}^+$, i.e., $\mathfrak{C}\mathfrak{A}^t = \tau_+^t\mathfrak{C}$ for all $t \geq 0$ (where τ_+^t is the left-shift on $\mathbf{R}^+$);
3. *the input-output map is time-invariant, causal, and the Hankel operator of* $\mathfrak{D}$ *is* $\mathfrak{C}\mathfrak{B}$, i.e., $\pi_+\mathfrak{D}\pi_- = \mathfrak{C}\mathfrak{B}$ (where π_+ is the causal and π_- is the anti-causal projection).

If we use this algebraic definition of the system $\Sigma = \left[\begin{smallmatrix} \mathfrak{A} & \mathfrak{B} \\ \mathfrak{C} & \mathfrak{D} \end{smallmatrix}\right]$ instead of (5.1), then it is still true that (5.1) holds in the following weak sense:

Theorem 5.1.1. *Every well-posed linear system* $\Sigma = \left[\begin{smallmatrix} \mathfrak{A} & \mathfrak{B} \\ \mathfrak{C} & \mathfrak{D} \end{smallmatrix}\right]$ *has a unique closed (unbounded) densely defined system operator*

$$S\colon X \times U \supset \mathcal{D}(S) \to X \times Y$$

with the following properties. If $x_0 \in X$, $u \in W^{1,2}_{\mathrm{loc}}(\mathbf{R}^+; U)$ *and* $\left[\begin{smallmatrix} x_0 \\ u(0) \end{smallmatrix}\right] \in \mathcal{D}(S)$, *then the state* $x(t)$ *and the output* $y(t)$ *of* Σ *with initial state* x_0, *and input function* u *satisfy* $\left[\begin{smallmatrix} x(t) \\ u(t) \end{smallmatrix}\right] \in \mathcal{D}(S)$ *for all* $t \geq 0$, *and*

$$\begin{aligned} \begin{bmatrix} x'(t) \\ y(t) \end{bmatrix} &= S\begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \qquad t \geq 0, \\ x(0) &= x_0. \end{aligned} \tag{5.6}$$

The system operator S can always be split into $S = \left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right]$, where $A\colon X \to X_{-1}$, $B\colon U \to X_{-1}$, $C\colon W \to Y$, and $D\colon U \to Y$. Here

$$\mathcal{D}(A) =: X_1 \subset W \subset X \subset X_{-1} := [\mathcal{D}(A^*)]^*$$

(we identify X with its dual) and $W = (\alpha I - A)^{-1}(X + BU)$ where $\alpha \in \rho(A)$. The domain of S is given by

$$\mathcal{D}(S) = \left\{\left[\begin{smallmatrix} x \\ u \end{smallmatrix}\right] \in X \times U \,\middle|\, Ax + Bu \in X\right\}. \tag{5.7}$$

We warn the reader that although A and B are unique, *the operators C and D are not always unique*. The restriction of C to $X_1 = D(A)$ is unique and the restriction of $\begin{bmatrix} C & D \end{bmatrix}$ to $D(S)$ is unique. We denote the latter operator by $C\&D$. The operators A, B, and C can be unbounded (with respect to the space X), but D is always bounded.

Every well-posed linear system also has a *transfer function*.

Theorem 5.1.2. *Every well-posed linear system* $\Sigma = \left[\begin{array}{c|c} \mathfrak{A} & \mathfrak{B} \\ \hline \mathfrak{C} & \mathfrak{D} \end{array}\right]$ *has a unique analytic* $\mathcal{L}(U;Y)$*-valued transfer function* $\widehat{\mathfrak{D}}$ *defined (at least) on* $\Re z > \omega_{\mathfrak{A}}$ *(where* $\omega_{\mathfrak{A}}$ *is the growth bound of* $\mathfrak{A}$*), determined by the fact that the Laplace transform* $\widehat{\mathfrak{D}_0 u}$ *of the input-output term* $\mathfrak{D}_0 u$ *in* (5.2) *is given by, for all* $u \in L^2_{\omega_{\mathfrak{A}}}(\mathbf{R}^+;U)$,

$$\widehat{\mathfrak{D}_0 u} = \widehat{\mathfrak{D}}(z)\hat{u}(z), \quad \Re z > \omega_{\mathfrak{A}},$$

where $\hat{u}$ *is the Laplace transform of* u*. Moreover, the transfer function* $\widehat{\mathfrak{D}}$ *of* Σ *is given by*

$$\widehat{\mathfrak{D}}(z) = C(zI - A)^{-1}B + D, \quad \Re z > \omega_{\mathfrak{A}}. \tag{5.8}$$

Here the notation $L^2_\omega(\mathbf{R}^+;U)$ stands for the set of functions u for which the function $(t \mapsto e^{-\omega t}u(t))$ belongs to $L^2(\mathbf{R}^+;U)$. Observe that in the finite-dimensional case we have $D = \widehat{\mathfrak{D}}(\infty)$. In the infinite-dimensional case $\widehat{\mathfrak{D}}(\infty)$ *need not be well-defined*. For more details, explanations and examples we refer the reader to [1], [2, 3], [4], [10], [24, 25], [30, 31, 32, 33, 35, 37], [38, 39], [42, 43, 44, 45, 46, 47] and [51] (and the references therein).

There is an alternative set of notations (introduced by George Weiss) which is also in widespread use:

$$\left[\begin{array}{c|c} \mathbb{T}_t & \Phi_t \\ \hline \Psi_t & \mathbb{F}_t \end{array}\right] := \left[\begin{array}{c|c} \mathfrak{A}^t & \mathfrak{B}^t_0 \\ \hline \mathfrak{C}^t_0 & \mathfrak{D}^t_0 \end{array}\right], \qquad \Psi_\infty := \mathfrak{C}, \qquad \mathbb{F}_\infty := \mathfrak{D}_0,$$
$$\mathbf{G}(s) := \widehat{\mathfrak{D}}(s), \qquad \mathbf{P}_t := \pi_{[0,t)}, \qquad \mathbb{S}^*_t := \tau^t_+.$$

5.2 Lax–Phillips scattering

A generalized Lax–Phillips scattering model is a semigroup $\mathfrak{T}$ defined on

$$Y \times X \times U = L^2(\mathbf{R}^-;Y) \times X \times L^2(\mathbf{R}^+;U)$$

with certain additional properties. We call U the incoming subspace, X the central state space, and Y the outgoing subspace. In the classical cases treated in [20, 21] $\mathfrak{T}$ is required to be unitary (the conservative case) or a contraction semigroup (the dissipative case). Below we use the following notation:

$$(\pi_J u)(s) := \begin{cases} u(s), & s \in J, \\ 0, & \text{otherwise.} \end{cases},$$
$$(\pi_+ u)(s) := \begin{cases} u(s), & s \in \mathbf{R}^+, \\ 0, & s \in \mathbf{R}^-, \end{cases},$$
$$(\tau^t u)(s) := u(t+s), \quad s,\ t \in \mathbf{R}.$$
$$\tau^t_+ := \pi_+ \tau^t, \qquad t \geq 0,$$

Theorem 5.2.1. *Let $\mathcal{Y} = L^2(\mathbf{R}^-; Y)$ and $\mathcal{U} = L^2(\mathbf{R}^+; U)$. For all $t \geq 0$ we define on $\mathcal{Y} \times X \times \mathcal{U}$ the operator $\mathfrak{T}^t$ by*

$$\mathfrak{T}^t = \begin{bmatrix} \tau^t & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \tau_+^t \end{bmatrix} \begin{bmatrix} I & \mathfrak{C}_0^t & \mathfrak{D}_0^t \\ 0 & \mathfrak{A}^t & \mathfrak{B}_0^t \\ 0 & 0 & I \end{bmatrix}. \tag{5.9}$$

Then $\mathfrak{T}$ is a strongly continuous semigroup. If x and y are the state trajectory and the output function of Σ with initial state $x_0 \in X$ and input function $u_0 \in \mathcal{U}$, and if we define $y(t) = y_0(t)$ for $t < 0$, then for all $t \geq 0$,

$$\begin{bmatrix} \tau^t & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \tau^t \end{bmatrix} \begin{bmatrix} \pi_{(-\infty,t]} y \\ x(t) \\ \pi_{[t,\infty)} u_0 \end{bmatrix} = \mathfrak{T}^t \begin{bmatrix} y_0 \\ x_0 \\ u_0 \end{bmatrix}. \tag{5.10}$$

Formula (5.10) shows that at any time $t \geq 0$, the first component of $\mathfrak{T}_t \left[\begin{smallmatrix} y_0 \\ x_0 \\ u_0 \end{smallmatrix}\right]$ represents the past output, the second component represents the present state, and the third component represents the future input. The preceding theorem is taken from [38], and it is also found in [37]. Special cases of this result (where either the input or the output is missing) appear in [12] and [17].

5.3 Dissipative and conservative systems

A system $\Sigma = \left[\begin{smallmatrix} \mathfrak{A} & \mathfrak{B} \\ \mathfrak{C} & \mathfrak{D} \end{smallmatrix}\right]$ is *dissipative* if, for every $t \geq 0$, every inital state $x_0 \in X$ and every input $u \in L^2(0, t; U)$, the following energy inequality holds:

$$\|x(t)\|^2 + \int_0^t \|y(s)\|^2 ds \leq \|x_0\|^2 + \int_0^t \|u(s)\|^2 ds. \tag{5.11}$$

Here $x(t)$ is the state at time t and y is the output function of Σ with initial state x_0 and input function u. Thus, a system is dissipative if there are no internal energy sources.

Theorem 5.3.1. *The following conditions are equivalent:*

1. *Σ is dissipative.*
2. *For every $t \geq 0$, every inital state x_0 and every input $u \in L^2(0, t; U)$, the state $x(t)$ and the output y of Σ with initial state x_0 and input function u satisfy* (5.11)*.*
3. *The corresponding Lax–Phillips model is a contraction semigroup.*
4. *For all $t > 0$, the operator $\Sigma_0^t = \left[\begin{smallmatrix} \mathfrak{A}^t & \mathfrak{B}_0^t \\ \mathfrak{C}_0^t & \mathfrak{D}_0^t \end{smallmatrix}\right]$ is a contraction from $X \times L^2([0, t); U)$ to $X \times L^2([0, t); Y)$.*

Dissipativity can also be characterized by some algebraic operator inequalites involving the system operator $S = \left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right]$ (which are **L**inear **M**atrix **I**nequalities in the finite-dimensional case).

A system $\Sigma = \left[\begin{smallmatrix} \mathfrak{A} & \mathfrak{B} \\ \mathfrak{C} & \mathfrak{D} \end{smallmatrix}\right]$ is *energy preserving* if, for every $t \geq 0$, every inital state $x_0 \in X$ and every input $u \in L^2(0, t; U)$, the following energy balance equation holds:

$$\|x(t)\|^2 + \int_0^t \|y(s)\|^2 ds = \|x_0\|^2 + \int_0^t \|u(s)\|^2 ds. \tag{5.12}$$

Again $x(t)$ is the state at time t and y is the output function of Σ with initial state x_0 and input function u. Thus, a system is energy preserving if there are no internal energy sources or sinks.

A system $\Sigma = \left[\begin{smallmatrix}\mathfrak{A} & \mathfrak{B} \\ \mathfrak{C} & \mathfrak{D}\end{smallmatrix}\right]$ is *conservative* if both the original system and the dual system is energy preserving. A finite-dimensional system is conservative if and only if it is energy preserving and $U = Y$. Some related (but more complicated) results are true also in infinite-dimensions.

Theorem 5.3.2. *The following conditions are equivalent:*

1. Σ *is conservative.*
2. *For every* $t \geq 0$, *every inital state* x_0 *and every input* $u \in L^2(0, t; U)$, *the state* $x(t)$ *and the output* y *of* Σ *with initial state* x_0 *and input function* u *satisfy* (5.12), *and the same condition is true for the dual system.*
3. *The corresponding Lax–Phillips model is a unitary semigroup.*
4. *For all* $t > 0$, *the operator* $\Sigma_0^t = \begin{bmatrix}\mathfrak{A}^t & \mathfrak{B}_0^t \\ \mathfrak{C}_0^t & \mathfrak{D}_0^t\end{bmatrix}$ *is a unitary operator from* $X \times L^2([0, t); U)$ *to* $X \times L^2([0, t); Y)$.

Preservation of energy can also be characterized by some algebraic operator equalites involving the system operator $S = \left[\begin{smallmatrix}A & B \\ C & D\end{smallmatrix}\right]$: using Theorem 5.1.1 and differentiating the energy balance equation with respect to t we get, for all $\left[\begin{smallmatrix}x \\ u\end{smallmatrix}\right] \in D(S) = \left\{\left[\begin{smallmatrix}x \\ u\end{smallmatrix}\right] \in X \times U \middle| Ax + Bu \in X\right\}$,

$$\langle Ax + Bu, x\rangle_X + \langle x, Ax + Bu\rangle_X + \langle Cx + Du, Cx + Du\rangle_Y = \langle u, u\rangle_U. \tag{5.13}$$

In the finite-dimensional case this set of equations decouples into the three independent equations

$$\begin{aligned} A + A^* + C^*C &= 0, \\ B + C^*D &= 0, \\ (B^* + D^*C &= 0,) \\ D^*D &= I. \end{aligned} \tag{5.14}$$

(The third equation above is the adjoint of the second.) In the infinite-dimensional case such a decoupling is much more difficult.

The results presented in this section are taken from [4] and [48], and they are also found in [37].

5.4 The universal model of a contraction semigroup

There is a classical problem in mathematics:

Let $\mathfrak{A}$ *be an arbitrary contraction semigroup on some Hilbert space* X. *Is it always possible to find a* unitary dilation $\widetilde{\mathfrak{A}}$ *of* $\mathfrak{A}$ *defined on some larger space* $\widetilde{X}$*?*

By this we meant the following: X is a subset of $\widetilde{X}$, $\widetilde{\mathfrak{A}}$ is a unitary semigroup on $\widetilde{X}$, and for all $t \geq 0$ and $x \in X$,

$$\mathfrak{A}^t x = \pi_X \widetilde{\mathfrak{A}}^t x,$$

where π_X is the orthogonal projection of $\widetilde{X}$ onto X. We also say that $\mathfrak{A}$ a *compression* of $\widetilde{X}$. The answer to this question is:

Theorem 5.4.1. *Given an arbitrary contraction semigroup $\mathfrak{A}$ on a Hilbert space X, it is always possible to find Hilbert spaces U and Y and operators $\mathfrak{B}$, $\mathfrak{C}$, and $\mathfrak{D}$ so that $\Sigma = \left[\begin{smallmatrix} \mathfrak{A} & \mathfrak{B} \\ \mathfrak{C} & \mathfrak{D} \end{smallmatrix}\right]$ is a conservative system with input space U, state space X, and output space Y. The corresponding Lax–Phillips semigroup is a unitary dilation of $\mathfrak{A}$ with state space $\widetilde{X} = L^2(\mathbf{R}^-; U) \oplus X \oplus L^2(\mathbf{R}^+; U)$. The system Σ is unique modulo unitary similarity transformations in the input and output spaces if we require $\widehat{\mathfrak{D}}$ to be strictly contractive.*

Strict contractivity of $\widehat{\mathfrak{D}}$ means that there is no nontrivial subspace of U on which $\widehat{\mathfrak{D}}$ reduces to a unitary constant. George Weiss and Marius Tuchnak have recently proved a result [49] which is closely related to this theorem.

The preceding theorem also has a "converse":

Theorem 5.4.2. *Every contractive causal shift-invariant operator $\mathfrak{D}$ from $L^2(\mathbf{R}^+; U)$ to $L^2(\mathbf{R}^+; Y)$ has a conservative realization, i.e., there exists a conservative system $\Sigma = \left[\begin{smallmatrix} \mathfrak{A} & \mathfrak{B} \\ \mathfrak{C} & \mathfrak{D} \end{smallmatrix}\right]$ with this input/output map. The system Σ is unique modulo unitary similarity transformations in its state space if we require $\mathfrak{A}$ to be completely nonunitary.*

Complete nonunitarity of the semigroup $\mathfrak{A}$ means that there is no nontrivial invariant subspace on which $\mathfrak{A}$ is unitary. This result is found in [4].

By combining the preceding results with a further representation result for the Lax–Phillips semigroup corresponding to a conservative system we get the following universal model of a completely nonunitary contraction semigroup:

Theorem 5.4.3. *Every completely nonunitary contraction semigroup $\mathfrak{A}$ on some Hilbert space X is unitarily equivalent to a compression of the (bilateral) shift operator on some subspace $\widetilde{X}$ of $L^2(\mathbf{R}; Z)$ (and there are formulas for how to find the space Z and the subspace $\widetilde{X}$).*

This is the continuous time version of the functional model established by Béla Sz.-Nagy and Ciprian Foiaş in 1964. See their book [40] or the book [37] for the details of the results presented in this section.

5.5 (J, P, R)-energy preserving systems

The standard energy balance equation (5.12) of a energy preserving system can also be written with the help of inner products:

$$\langle x(t), x(t)\rangle_X + \int_0^t \langle y(s), y(s)\rangle_Y ds = \langle x_0, x_0\rangle_X + \int_0^t \langle u(s), u(s)\rangle_U ds.$$

We can make this equation more general by introducing self-adjoint cost operators: $R\colon U \to U$ is the input cost operator, $P\colon X \to X$ is the state cost operator, and $J\colon Y \to Y$ is the output cost operator. A system is (J, P, R)*-energy preserving* if, for every $t \geq 0$, every inital state x_0 and every input $u \in L^2(0, t; U)$, we have

$$\langle x(t), Px(t)\rangle_X + \int_0^t \langle y(s), Jy(s)\rangle_Y ds = \langle x_0, Px_0\rangle_X + \int_0^t \langle u(s), Ru(s)\rangle_U ds. \tag{5.15}$$

As before $x(t)$ is the state at time t and y is the output function of Σ with initial state x_0 and input function u. Thus, preservation of (J, P, R)-energy means that there is a energy balance equation where we use R to measure the input energy, use P to measure the state energy, and use J to measure the output energy. These systems appear in optimal control (in the form of optimal closed loop systems). There J is usually known, and R and P are to be determined. Typically, P is a Gramian or a Riccati operator.

Differentiating the (J, P, R)-energy balance equation with respect to t we get the following *Lyapunov equation*,

$$\begin{aligned}\langle Ax+Bu, Px\rangle_X + \langle x, P(Ax+Bu)\rangle_X \\ + \langle Cx+Du, J(Cx+Du)\rangle_Y = \langle u, Ru\rangle_U,\end{aligned} \tag{5.16}$$

valid for all $\left[\begin{smallmatrix} x \\ u \end{smallmatrix}\right] \in \mathcal{D}(S)$. In the finite-dimensional case this set of equations decouples into the three independent equations

$$\begin{aligned} A^*P + PA + C^*JC &= 0, \\ PB + C^*JD &= 0, \\ (B^*P + D^*JC &= 0,) \\ D^*JD &= R. \end{aligned} \tag{5.17}$$

In optimal control the original system is typically not (J, P, R)-energy preserving, but the optimal closed loop system is. We use a feedback of the type $u = v + Kx$, where K is the feedback operator and v is the closed loop input. The optimal system is (J, P, R)-energy preserving with respect to v. By replacing u in the Lyapunov equation by $v = u - Kx$ we get the *Riccati equation*

$$\begin{aligned}\langle Ax+Bu, Px\rangle_X + \langle x, P(Ax+Bu)\rangle_X \\ + \langle Cx+Du, J(Cx+Du)\rangle_Y = \langle u-Kx, R(u-Kx)\rangle_U,\end{aligned} \tag{5.18}$$

valid for all $\left[\begin{smallmatrix} x \\ u-Kx \end{smallmatrix}\right] \in \mathcal{D}(S)$. In the finite-dimensional case this set of equations decouples into the three equations

$$\begin{aligned} A^*P + PA + C^*JC &= K^*RK, \\ PB + C^*JD &= -K^*R, \\ (B^*P + D^*JC &= -RK,) \\ D^*JD &= R. \end{aligned} \tag{5.19}$$

In the infinite-dimensional case some additional correction terms pop up (similar to those seen in the discrete time Riccati equation).

By appropriate choice of R, P, and J we get all the standard one-block and two-block "optimal control" results. Here J is given and P and R are unknown, and we work with the extended system of Figure 5.2, where we have added a copy of the input to the output (in addition some of the systems below have two independent inputs):

1. $y = \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$, $D = \left[\begin{smallmatrix} 0 \\ I \end{smallmatrix}\right]$, $C = \left[\begin{smallmatrix} C \\ 0 \end{smallmatrix}\right]$, $J = \left[\begin{smallmatrix} I & 0 \\ 0 & R \end{smallmatrix}\right]$: This is the standard LQR Riccati equation. Here $J \geq 0$, $P \geq 0$, $R \geq 0$.
2. $y = \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$, $D = \left[\begin{smallmatrix} D \\ I \end{smallmatrix}\right]$, $C = \left[\begin{smallmatrix} C \\ 0 \end{smallmatrix}\right]$, $J = \left[\begin{smallmatrix} I & 0 \\ 0 & I \end{smallmatrix}\right]$: This is the normalized coprime factorization problem. Still $J \geq 0$, $P \geq 0$, $R \geq 0$.

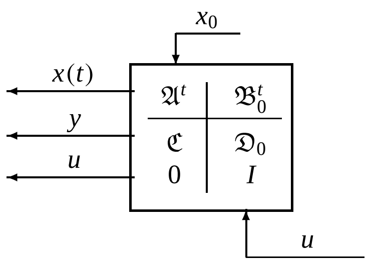

Figure 5.2: Input added to output of Σ

3. $y = \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$, $D = \left[\begin{smallmatrix} D \\ I \end{smallmatrix}\right]$, $C = \left[\begin{smallmatrix} C \\ 0 \end{smallmatrix}\right]$, $J = \left[\begin{smallmatrix} I & 0 \\ 0 & -\gamma^2 I \end{smallmatrix}\right]$: This is the bounded real lemma. Here $P \geq 0$ and $R \geq 0$ but $J \not\geq 0$.

4. $y = \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$, $D = \left[\begin{smallmatrix} D \\ I \end{smallmatrix}\right]$, $C = \left[\begin{smallmatrix} C \\ 0 \end{smallmatrix}\right]$, $J = \left[\begin{smallmatrix} 0 & I \\ I & 0 \end{smallmatrix}\right]$: This is the positive real lemma. Again $P \geq 0$ and $R \geq 0$ but $J \not\geq 0$.

5. $y = \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$, $u = \left[\begin{smallmatrix} v \\ u \end{smallmatrix}\right]$, $D = \left[\begin{smallmatrix} I & D \\ 0 & I \end{smallmatrix}\right]$, $C = \left[\begin{smallmatrix} C \\ 0 \end{smallmatrix}\right]$, $J = \left[\begin{smallmatrix} I & 0 \\ 0 & -\gamma^2 I \end{smallmatrix}\right]$, and initial time $-\infty$. This is the Nehari problem. Still $P \geq 0$ but $R \not\geq 0$ and $J \not\geq 0$.

6. Same as above, but a smaller value of γ. This is used to compute the n:th singular value of the Hankel operator. In this problem $P \not\geq 0$, $R \not\geq 0$ and $J \not\geq 0$.

7. $y = \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$, $u = \left[\begin{smallmatrix} w \\ u \end{smallmatrix}\right]$, $D = \left[\begin{smallmatrix} D_{11} & D_{12} \\ 0 & I \end{smallmatrix}\right]$, $C = \left[\begin{smallmatrix} C \\ 0 \end{smallmatrix}\right]$, $J = \left[\begin{smallmatrix} I & 0 \\ 0 & -\gamma^2 I \end{smallmatrix}\right]$: This is the H^∞ full information problem. Still $P \geq 0$ but $R \not\geq 0$ and $J \not\geq 0$.

Presently research is going on to extend the finite-dimensional Riccati equation theory to the setting of an infinite-dimensional well-posed linear system. Much has been done, but even more remains to be done. There are several problems which makes the the infinite-dimensional theory significantly more difficult than the finite-dimensional one. On of them is the *lack of compactness* (the closed unit ball is not compact). By taking a closer look at the finite-dimensional system (5.19) we immediately observe another major problem: *how should the Riccati equation be interpreted* (recall that the operators A, B, and C can be unbounded)? For example, to compute the feedback operator K from the equation $K = -R^{-1}(B^*P + D^*JC)$ we need to know for which $x \in \mathcal{D}(C)$ it is true that $Px \in \mathcal{D}(B^*)$. Also the *non-uniqueness of the splitting of the system operator S* into $S = \left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right]$ comes into play: all the operators A, B, C, and D appear in the Riccati equation, and what is the splitting to use in each specific case? (It depends not only on the system itself, but also on the given cost operators.) Is there always a "correct" splitting which makes the Riccati equation valid? Still another difficulty is that in the case where $R \not\geq 0$ the feedback operator K is not always admissible, i.e., the *optimal closed loop system need not be wellposed*.

The difficulties described above have been approached in different ways. Some results on the "*correct splitting*" of S into $S = \left[\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right]$ (i.e., a splitting wich makes the Riccati equation valid) are found in [13] (this reference does not require the optimal closed loop system to be well-posed). If $\widehat{\mathfrak{D}}(\infty)$ exists, then the system is called *regular*, and it is possible to take $D = \widehat{\mathfrak{D}}(\infty)$. In general this splitting is not compatible with the Riccati equation, and we have to add some correction terms to the Riccati equation, similar to those seen in the discrete time theory. Especially, the formula for R becomes more complicated. See [22], [28, 29, 30, 32, 33, 34, 36], and [51]. These results are largely based on *spectral factorization*, and so are [5, 6, 7] and [14, 15, 16]. When the closed loop system is not well-posed we can use the (non-well-posed) *compensators with internal loop* introduced in [11] (see [22]). Another approach is to use the *Caley transform* to map the continuous time system into a discrete time system and use the discrete time Riccati equation theory. Then all the operators become bounded, but some extra correction terms enter the Riccati equations. In many cases *additional smoothing properties* have been used (Pritchard–Salamon and parabolic cases); see, e.g., [8], [9], [18], [41], [23], [27, 26], and [50].

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6

Conservative linear systems from thin air

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Abstract

We present a remarkable class of conservative well-posed linear systems, described by a second order differential equation on a Hilbert space and an algebraic output equation. Any system in this class is generated by two unbounded operators which satisfy very simple assumptions. We devote special attention to questions of stability, controllability and observability.

Keywords

Well-posed linear system, operator semigroup, conservative system, transfer function, exponential stability, strong stability.

6.1 Introduction

By a *well-posed linear system* we mean a linear time-invariant system Σ such that on any finite time interval $[0, \tau]$, the operator Σ_τ from the initial state $x(0)$ and the input function u to the final state $x(\tau)$ and the output function y is bounded. The input, state and output spaces are Hilbert spaces, and the input and output functions are of class $L^2_{\rm loc}$. For any $u \in L^2_{\rm loc}$ and any $\tau \geq 0$, we denote by $\mathbf{P}_\tau u$ its truncation to the interval $[0, \tau]$. Then the well-posed system Σ consists of the family of bounded operators $\Sigma = (\Sigma_\tau)_{\tau \geq 0}$ such that

$$\begin{bmatrix} x(\tau) \\ \mathbf{P}_\tau y \end{bmatrix} = \Sigma_\tau \begin{bmatrix} x(0) \\ \mathbf{P}_\tau u \end{bmatrix}. \tag{6.1}$$

For the detailed definition, background and examples we refer to Salamon [8], [7], Staffans [10], [11], Weiss [13], [14] and Weiss and Rebarber [15]. We follow the notation and terminology of [13, 14, 15]. The well-posed linear system Σ is called *conservative* if for every $\tau \geq 0$, Σ_τ is unitary. Denoting the state space of Σ by X, its input space by U and its output space by Y, the fact that Σ is conservative means that for every $\tau \geq 0$, the following two statements hold:
(i) Σ_τ is an isometry, i.e.,

$$\|x(\tau)\|^2 + \int_0^\tau \|y(t)\|^2 \mathrm{d}t = \|x(0)\|^2 + \int_0^\tau \|u(t)\|^2 \mathrm{d}t, \tag{6.2}$$

(ii) Σ_τ is onto, which means that for every $x(\tau) \in X$ and every $\mathbf{P}_\tau y \in L^2([0,\tau], Y)$, we can find $x(0) \in X$ and $\mathbf{P}_\tau u \in L^2([0,\tau], U)$ such that (6.1) holds.

Our concept of a conservative linear system is equivalent to what Arov and Nudelman [1] call a conservative scattering system and it goes back to the work of Lax and Phillips [3]. A recent survey paper covering also conservative systems (with some new material) is Weiss, Staffans and Tucsnak [16].

To get a better feeling for the concept of a conservative linear system, consider the simple case when Σ is finite-dimensional, i.e., described by

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{cases} \tag{6.3}$$

with A, B, C and D matrices of appropriate dimensions and $t \geq 0$. Then Σ is conservative if and only if these matrices satisfy

$$A + A^* = -C^*C, \quad B = -C^*D, \quad D^*D = I, \quad DD^* = I \tag{6.4}$$

(these imply $A + A^* = -BB^*$ and $C = -DB^*$), see p. 16 in [1]. If the finite-dimensional Σ is conservative, then its transfer function $\mathbf{G}(s) = C(sI - A)^{-1}B + D$ is bounded and analytic on the open right half-plane $\mathbb{C}_0$ and for all $\omega \in \mathbb{R}$,

$$\mathbf{G}^*(i\omega)\mathbf{G}(i\omega) = \mathbf{G}(i\omega)\mathbf{G}(i\omega)^* = I. \tag{6.5}$$

(In a more technical language, $\mathbf{G}$ is inner and co-inner.)

Now let us move to a slightly higher level of generality, namely to well-posed linear systems with bounded B and C. This means that U, X and Y are Hilbert spaces and the system is described by (6.3), where A is the generator of strongly continuous semigroup of operators $\mathbb{T}$ on the Hilbert space X, $B \in \mathcal{L}(U, X)$, $C \in \mathcal{L}(X, Y)$ and $D \in \mathcal{L}(U, Y)$. In this context, it can be shown that the characterization (6.4) of conservativity is still valid. We must have $\mathcal{D}(A^*) = \mathcal{D}(A)$ and the first equation in (6.4) holds on $\mathcal{D}(A)$. The property (6.5) is no longer true at this level of generality (however, it holds if the semigroups $\mathbb{T}$ and $\mathbb{T}^*$ are both strongly stable).

In general, leaving bounded B and C behind, the characterization of conservative well-posed linear systems is a more difficult problem, and interesting recent results in this direction have been obtained by Jarmo Malinen (not yet published) (see also the comments in [16]). For extensions to nonlinear systems see Ball [2], van der Schaft [9] and Maschke and van der Schaft [6].

This talk is about a special class of conservative linear systems, which are described by a second order differential equation (in a Hilbert space) and an output equation. The equations are simple and occur often as models of physical systems. Well-posedness is not assumed a-priori: it can be proved, together with conservativity. The operators B and C are not assumed to be bounded, so that we cannot use the characterization (6.4) of conservativity.

In the next two sections we outline our construction and state the main results. No proofs will be given here, but this talk is essentially a summary of the recent papers [12] and [17] by the same authors.

Let H be a Hilbert space, and let $A_0 : \mathcal{D}(A_0) \to H$ be a self-adjoint, positive and boundedly invertible operator. We introduce the scale of Hilbert spaces H_α, $\alpha \in \mathbb{R}$, as follows: for every $\alpha \geq 0$, $H_\alpha = \mathcal{D}(A_0^\alpha)$, with the norm $\|z\|_\alpha = \|A_0^\alpha z\|_H$. The space $H_{-\alpha}$ is defined by duality with respect to the pivot space H as follows: $H_{-\alpha} = H_\alpha^*$ for $\alpha > 0$. Equivalently, $H_{-\alpha}$ is the completion of H with respect to the norm $\|z\|_{-\alpha} = \left\|A_0^{-\alpha} z\right\|_H$. The operator A_0 can be extended (or restricted) to each H_α, such that it becomes a bounded operator

$$A_0 : H_\alpha \to H_{\alpha-1} \qquad \forall\, \alpha \in \mathbb{R}. \tag{6.6}$$

The second ingredient needed for our construction is a bounded linear operator $C_0 : H_{\frac{1}{2}} \to U$, where U is another Hilbert space. We identify U with its dual, so that $U = U^*$. We denote $B_0 = C_0^*$, so that $B_0 : U \to H_{-\frac{1}{2}}$.

We study of the system described by

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2} z(t) + A_0 z(t) + \frac{1}{2} B_0 \frac{\mathrm{d}}{\mathrm{d}t} C_0 z(t) = B_0 u(t), \tag{6.7}$$

$$y(t) = -\frac{\mathrm{d}}{\mathrm{d}t} C_0 z(t) + u(t), \tag{6.8}$$

where $t \in [0, \infty)$ is the time. The equation (6.7) is understood as an equation in $H_{-\frac{1}{2}}$, i.e., all the terms are in $H_{-\frac{1}{2}}$. Most of the linear equations modelling the damped vibrations of elastic structures can be written in the form (6.7), where z stands for the displacement field and the term $B_0 \frac{\mathrm{d}}{\mathrm{d}t} C_0 z(t)$, informally written as $B_0 C_0 \dot{z}(t)$, represents a viscous feedback damping. The signal $u(t)$ is an external input with values in U (often a displacement, a force or a moment acting on the boundary) and the signal $y(t)$ is the output (measurement) with values in U as well. The state $x(t)$ of this system and its state space X are defined by

$$x(t) = \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}, \qquad X = H_{\frac{1}{2}} \times H.$$

This means that in order to solve (6.7), initial values for $z(t)$ and $\dot{z}(t)$ at $t = 0$ have to be specified, and we take $z(0) \in H_{\frac{1}{2}}$ and $\dot{z}(0) \in H$. As we shall see, if $u \in L^2([0, \infty), U)$ then also $y \in L^2([0, \infty), U)$.

6.2 Well-posedness and energy balance

We need some notation: for any Hilbert space W, the Sobolev spaces $H^p(0, \infty; W)$ of W-valued functions (with $p \in \mathbb{N}$) are defined in the usual way, see [4]. The larger spaces $H^p_{loc}(0, \infty; W)$

(with $p \in \mathbb{N}$) are defined recursively: $H^0_{loc}(0,\infty;W) = L^2_{loc}([0,\infty),W)$ and for $p \in \mathbb{N}$, $f \in H^p_{loc}(0,\infty;W)$ if f is continuous and

$$f(\tau) - f(0) = \int_0^\tau v(t)\mathrm{d}t \qquad \forall\, \tau \geq 0\,,$$

for some $v \in H^{p-1}_{loc}(0,\infty;W)$. The notation $C^n(0,\infty;W)$ (with $n \in \{0,1,2,\ldots\}$) for n times continuously differentiable W-valued functions on $[0,\infty)$ is also quite standard (at $t=0$ we consider the derivative from the right, of course). We denote by $BC^n(0,\infty;W)$ the space of those $f \in C^n(0,\infty;W)$ for which $f, f', \ldots f^{(n)}$ are all bounded on $[0,\infty)$. We write C instead of C^0. Our main result is the following:

Theorem 6.2.1. *With U, H, A_0, H_α, C_0, B_0 and X as above, the equations* (6.7) *and* (6.8) *determine a conservative linear system Σ, in the following sense:*

There exists a conservative linear system Σ whose input and output spaces are both U, whose state space is X and which has the following properties: If $u \in L^2([0,\infty),U)$ is the input function, $x_0 = \begin{bmatrix} z_0 \\ w_0 \end{bmatrix} \in X$ is the initial state, $x = \begin{bmatrix} z \\ w \end{bmatrix}$ is the corresponding state trajectory and y is the corresponding output function, then

(1) $z \in BC(0,\infty;H_{\frac{1}{2}}) \cap BC^1(0,\infty;H) \cap H^2_{loc}(0,\infty;H_{-\frac{1}{2}})$.

(2) *The two components of x are related by $w = \dot{z}$.*

(3) *$C_0 z \in H^1(0,\infty;U)$ and the equations* (6.7) *(in $H_{-\frac{1}{2}}$) and* (6.8) *(in U) hold for almost every $t \geq 0$ (hence, $y \in L^2([0,\infty),U)$).*

Note that the property $C_0 z \in H^1(0,\infty;U)$ (contained in point (3) of the theorem) implies that $\lim_{t\to\infty} C_0 z(t) = 0$ (for any initial state and any L^2 input function). This is remarkable, because the system Σ is not strongly stable in general.

If we make additional smoothness assumptions on the input signal u and the initial conditions z_0 and w_0, as well as a compatibility assumption, then we get smoother state trajectories and output functions. Then, the equations (6.7) and (6.8) can be rewritten in a somewhat simpler form, as the following theorem shows.

Theorem 6.2.2. *With the assumptions and the notation of Theorem* 6.2.1*, introduce the Hilbert the space $Z_0 = H_1 + A_0^{-1}B_0U \subset H_{\frac{1}{2}}$, with the norm*

$$\|z\|^2_{Z_0} = \inf\left\{ \|z_1\|_1^2 + \|v\|^2 \;\middle|\; z = z_1 - A_0^{-1}B_0 v \quad z_1 \in H_1\,, \quad v \in U \right\}.$$

Suppose that $u \in H^1(0,\infty;U)$, $z_0, w_0 \in H_{\frac{1}{2}}$ and

$$A_0 z_0 + \frac{1}{2}B_0 C_0 w_0 - B_0 u(0) \in H \tag{6.1}$$

(this implies $z_0 \in Z_0$). If we denote by z the solution of (6.7) *with $z(0) = z_0$ and $\dot{z}(0) = w_0$, and if we denote by y the output defind by* (6.8)*, then*

$$z \in BC(0,\infty;Z_0) \cap BC^1(0,\infty;H_{\frac{1}{2}}) \cap BC^2(0,\infty;H), \tag{6.2}$$

$y \in H^1(0,\infty;U)$ and we have for every $t \geq 0$

$$\ddot{z}(t) + A_0 z(t) + \frac{1}{2}B_0 C_0 \dot{z}(t) = B_0 u(t)\,, \quad y(t) = -C_0\dot{z}(t) + u(t)\,. \tag{6.3}$$

It is easy to verify that the equations (6.3) are equivalent to the following system of first order equations:

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t)\,, \\ y(t) &= \overline{C}x(t) + u(t)\,, \end{cases} \tag{6.4}$$

where

$$A = \begin{bmatrix} 0 & I \\ -A_0 & -\frac{1}{2}B_0C_0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ B_0 \end{bmatrix},$$

$$\mathcal{D}(A) = \left\{ \begin{bmatrix} z \\ w \end{bmatrix} \in H_{\frac{1}{2}} \times H_{\frac{1}{2}} \;\middle|\; A_0 z + \frac{1}{2}B_0C_0 w \in H \right\},$$

$$\overline{C} : Z_0 \times H_{\frac{1}{2}} \to U\,, \qquad \overline{C} = [0 \;\; -C_0]\,.$$

We denote by C the restriction of $\overline{C}$ to $\mathcal{D}(A)$. For the concepts of semigroup generator, control operator, observation operator and transfer function of a well-posed linear system, we refer again to [13], [14]. We denote by $\mathbb{C}_0$ the open right half-plane in $\mathbb{C}$ (where $\mathrm{Re}\, s > 0$), and $V(s) = s^2 I + A_0 + \frac{s}{2}B_0C_0$.

Theorem 6.2.3. *With the notation from Theorem* 6.2.1 *together with the above notation, the semigroup generator of* Σ *is* A*, its control operator is* B *and its observation operator is* C*. The transfer function of* Σ *is given for all* $s \in \mathbb{C}_0$ *by*

$$\mathbf{G}(s) = \overline{C}(sI - A)^{-1}B + I = I - C_0 sV(s)^{-1}B_0 \tag{6.5}$$

and we have $\|\mathbf{G}(s)\| \leq 1$ *for all* $s \in \mathbb{C}_0$*. The function* $\mathbf{G}$ *has an analytic continuation to a neighborhood of* 0 *and, denoting* $T = C_0 A_0^{-1} B_0$*, we have*

$$\mathbf{G}(0) = I\,, \quad \mathbf{G}'(0) = -T\,, \quad \mathbf{G}''(0) = T^2\,. \tag{6.6}$$

Note that the operator $T \in \mathcal{L}(U)$ introduced above is self-adjoint and $T \geq 0$. The formulas (6.6) show that the first three terms in the Taylor expansion of $\mathbf{G}$ around zero agree with the first three terms in the expansion of $\exp(-Ts)$.

The last part of Theorem 6.2.3 shows that not every conservative system with equal input and output spaces is isomorphic to a system of the type discussed in Theorem 6.2.1. Indeed, for conservative systems in general, $\mathbf{G}$ may be any analytic function on $\mathbb{C}_0$ whose values are contractions, see [1, pp. 32–33], and such transfer functions do not have to satisfy $\mathbf{G}(0) = I$ or $\mathbf{G}''(0) = [\mathbf{G}'(0)]^2$.

The following theorem refers to a special subclass of the systems treated in the first three theorems. If the system Σ originates from a partial differential equation with boundary control, then it usually belongs to this subclass.

Theorem 6.2.4. *With the assumptions and the notation of Theorem* 6.2.1*, and with the Hilbert space* Z_0 *defined as in Theorem* 6.2.2*, suppose that there exists an operator* $G_0 \in \mathcal{L}(Z_0, U)$ *such that*

$$G_0 H_1 = \{0\}\,, \qquad G_0 A_0^{-1} B_0 = I\,.$$

For every $z \in Z_0$*, we define* $L_0 z = A_0 z - B_0 G_0 z$ *(here we have used the extension of* A_0 *to* $H_{\frac{1}{2}}$*, as in* (6.6)*). Then* $\mathrm{Ker}\, G_0 = H_1$*,* $L_0 \in \mathcal{L}(Z_0, H)$ *and the system* Σ *can be described by the equations*

$$\begin{cases} \ddot{z}(t) + L_0 z(t) &= 0\,, \\ G_0 z(t) + \frac{1}{2} C_0 \dot{z}(t) &= u(t)\,, \\ G_0 z(t) - \frac{1}{2} C_0 \dot{z}(t) &= y(t)\,, \end{cases} \tag{6.7}$$

in the following sense:
(1) *If* $u \in H^1(0,\infty;U)$, $z_0 \in Z_0$ *and* $w_0 \in H_{\frac{1}{2}}$, *then the condition*

$$G_0 z_0 + \frac{1}{2} C_0 w_0 = u(0) \tag{6.8}$$

is equivalent to (6.1)*. Hence, if the above condition holds, if we denote by z the solution of* (6.7) *with* $z(0) = z_0$ *and* $\dot{z}(0) = w_0$*, and if we denote by y the output defined by* (6.8)*, then (by Theorem* 6.2.2*)* (6.2) *holds and* $y \in H^1(0,\infty;U)$.
(2) *The equations* (6.7) *are equivalent to* (6.3) *(and hence also to* (6.4)*). This means that a function* $z \in C(0,\infty;Z_0) \cap C^1(0,\infty;H_{\frac{1}{2}})$ *together with* $u, y \in C(0,\infty;U)$ *satisfy* (6.7) *if and only if they satisfy* (6.3)*.*

6.3 Controllability, observability and stability

The following theorems use various controllability, observability and stability concepts, which we shall assume to be known by the reader.

Theorem 6.3.1. *With the above notation, the following assertions are equivalent:*
(1) *The pair* (A, B) *is exactly controllable (in some finite time).*
(2) *The pair* (A, C) *is exactly observable (in some finite time).*
(3) *The semigroup* $\mathbb{T}$ *is exponentially stable.*
(4) *The pair* (A, B) *is optimizable.*
(5) *The pair* (A, C) *is estimatable.*
(6) *We have* $\sup_{s\in\mathbb{C}_0} \|A_0^{\frac{1}{2}} V(s)\|_{L(H)} < \infty$ *and* $\sup_{s\in\mathbb{C}_0} \|sV(s)\|_{L(H)} < \infty$.
(7) *We have* $i\mathbb{R} \subset \rho(A)$ *and*

$$\sup_{\omega\in\mathbb{R}} \|A_0^{\frac{1}{2}} V(i\omega)\|_{L(H)} < \infty\,, \qquad \sup_{\omega\in\mathbb{R}} \|\omega V(i\omega)\|_{L(H)} < \infty\,.$$

We mention that (1)–(5) are equivalent for *every* conservative system.

The version of this theorem corresponding to bounded B and C, i.e., with $C_0 \in L(H, U)$, is in Liu [5, Sections 2-3] (without conditions (4)–(6)).

A result similar to Theorem 6.3.1 holds also for strong stability, with an additional assumption on the spectrum $\sigma(A_0)$:

Theorem 6.3.2. *With the above notation, assume that* $\sigma(A_0)$ *is countable (this happens, e.g., if* A_0^{-1} *is compact). Then the following assertions are equivalent:*
(1) $\mathbb{T}$ *is strongly stable.*
(2) *The pair* (A, C) *is exactly observable in infinite time.*
(3) *The pair* (A, C) *is approximately observable in infinite time.*
(4) $\mathbb{T}$ *is weakly stable (equivalently,* $\mathbb{T}^*$ *is weakly stable).*

(5) $\mathbb{T}^*$ *is strongly stable.*
(6) *The pair* (A, B) *is exactly controllable in infinite time.*
(7) *The pair* (A, B) *is approximately controllable in infinite time.*
(8) *For any* $z \in H_1$, *if* z *is an eigenvector of* A_0, *then* $Cz_0 \neq 0$.

Note that "A_0^{-1} is compact" does *not* imply that $(sI - A)^{-1}$ is compact.

If the system described by (6.7)–(6.8) is not exponentially stable to begin with, then this cannot be helped by connecting a controller (another linear system) from its output to its input, as the following proposition shows:

Proposition 6.3.3. *With the notation of Theorem* 6.2.1, *if* Σ *is not exponentially stable, then it is also not dynamically stabilizable.*

For the precise meaning of dynamic stabilization and background material on this concept we refer to Weiss and Rebarber [15]. Proposition 6.3.3 is a direct consequence of Proposition 6.2.3 (the boundedness of **G**) and of [15, Remark 4 in Section 6].

We also examine the well-posedness of the undamped system corresponding to (6.7)–(6.8), by which we mean the system described by

$$\ddot{z}(t) + A_0 z(t) = B_0 u(t), \tag{6.1}$$

with the same assumptions on A_0 and B_0, and with the output signal given again by (6.8). It is interesting that in this special context, only the well-posedness of the transfer function has to be checked, the admissibility of the control and observation operators follows. For any $\omega \in \mathbb{R}$, we denote $\mathbb{C}_\omega = \{s \in \mathbb{C} \mid \operatorname{Re} s > \omega\}$.

Theorem 6.3.4. *The following statements are equivalent:*
(1) *The function* $\mathbf{G}^o : \mathbb{C}_0 \to \mathcal{L}(U)$ *defined by*

$$\mathbf{G}^o(s) = I - C_0 s \left(s^2 I + A_0\right)^{-1} B_0$$

is bounded on a vertical line contained in $\mathbb{C}_0$.
(2) *The function* $\mathbf{G}^o$ *defined above is bounded on* $\mathbb{C}_\omega$ *for every* $\omega > 0$.
(3) *The equations* (6.1) *and* (6.8) *determine a well-posed linear system* Σ^0 *with the state space* $X = H_{\frac{1}{2}} \times H$, *and with input and output space* U.

If the above statements are true, then the transfer function of the system from point (3) *is* $\mathbf{G}^o$ *from point* (1). *Moreover, for every* $T > 0$, *the system* Σ^0 *is exactly (or approximately) controllable (or observable) in time* T *if and only if the system* Σ *from Theorem* 6.2.1 *has the same property.*

Note that the above theorem, together with Theorem 6.3.1 implies that Σ^0 is exactly controllable if and only if it is exactly observable, and this is further equivalent to the exponential stability of Σ.

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7

Control of waves: heterogenous media and numerical simulation

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It is by now well known that the wave equation is controllable in the energy space provided a Geometric Control Condition is satisfied. Roughly speaking, this condition requires every ray of Geometric Optics to intersect the control region. This result is valid for equations with sufficiently smooth coefficients (say C^2). In one space dimension the same result turns out to be true for BV coefficients.

In the first part of this lecture we will describe some pathological highly heterogeneous media associated to Hölder continuous coefficients in which the controllability property fails. This is part of a joint work with Carlos Castro.

In the second part of the lecture we shall discuss the problem of the numerical discretizations of the wave equation. Knowing that the continuous wave equation is controllable it seems natural to ask whether its discrete versions are alo controllable and if the control for the continuous model may be obtained as limit of the controls for the discret systems when the discretization parameter tends to zero. The interest of this question is twofold. First of all, obvioulsy, it is relevant from a computational point of view. Second, it is also of interest from the modelling point of view since it may be considered as a test of the accuracy of finite dimensional mechanical models for the modelling and control of continuous structures.

As we shall see, high frequency pathologies may produce the divergence of the controls as the mesh size of the discretization tends to zero.

We shall discuss the analogies between these two phenomena: lack of controllability for Hölder continuous coefficients and lack of convergence of controls under discretizations.

We shall also present a class of viscous mechanical and numerical damping terms that may avoid these pathological behavior and some preliminary results in this direction.

Contributed Talks

8

Lossless transmission systems with strongly regular J-inner transfer functions

Z.D. Arova

8.1 Abstract

A linear stationary system is called conservative transmission if for it the conservativeness condition

$$(Ju(t), u(t)) - (Jy(t), y(t)) = n(x(t)), \quad t \geq 0$$

take place, where J be a signature fixed operator in the input-output space $Y (= U)$ and

$$n(x(t)) = \begin{bmatrix} \|x(t+1)\|^2 - \|x(t)\|^2 & \text{for discrete-time systems,} \\ \frac{d}{dt}\|x(t)\|^2 & \text{for continuous time systems} \end{bmatrix} \tag{8.1}$$

and if the such kind property have adjoint system, too. We consider such systems with $Y = \mathbb{C}^m$. The transfer matrix function Θ_Σ of such a system belongs to the Potapov class $P(J, \Omega_+)$ of meromorphic J - contractive in the domain Ω_+ $m \times m$ matrix functions (m.f.'s), where Ω_+ is upper half plane $\mathbb{C}_+$ or the unit disk $\mathbb{K}$ for continuous- or discrete- time systems, resp..

A conservative transmission system Σ is called lossless if Θ_Σ belongs to the class $U(J, \Omega_+)$ of J -inner in Ω_+ m.f.'s, i.e. if Θ_Σ has J -unitary boundary values a.e. on $\partial\Omega_+$. We consider only such systems Σ of this class for which Θ_Σ belongs to the nice class $U_{sR}(J, \Omega_+)$ that was introduced and investigated by D.Z. Arov and H. Dym in [1].

Our previous results on such systems are in the works [2]–[4]. They are formulated in the terms of the operator nodes, Livsic- Brodskii and J - unitary, that corresponds to the conservative transmission systems with continuous- and discrete-time, resp. Here we stop on the case, when $\Theta_\Sigma (\in U(J, \Omega_+))$ isn't from the Smirnov classes $N_+^{m\times m}$ and $N_-^{m\times m}$ in the domain Ω_+ and $\Omega_- = \mathbb{C} - \overline{\Omega_+}$.

A semigroup $T(t)$ is called bi-stable, if

$$s - \lim_{t\to+\infty} T(t) = 0, \qquad s - \lim_{t\to+\infty} T^*(t) = 0.$$

A group $T(t)$ is called antistable, if

$$s - \lim_{t\to-\infty} T(t) = 0, \qquad s - \lim_{t\to-\infty} T^*(t) = 0.$$

A semigroup $T(t)$ in the space X has a two-side dichotomy property, if X has a representation

$$X = X_+ \dot{+} X_-, \qquad X_\pm \neq \{0\}.$$

such that $T(t)X_+ \subset X_+$, $T(t)X_- = X_-$ and for $T_\pm(t) = T(t) \mid X_\pm$ we have that $T_+(t)$ is a bi- stable semigroup and $T_-(t)$ is a bi-antistable group.

Theorem 8.1.1. *Let Σ is a minimal lossless transmission system with $\Theta_\Sigma \in U_{sR}(J, \Omega_+)$. Then:*

1. *The evolution semigroup has two-side dichotomy property iff $\Theta_\Sigma \notin N_+^{m\times m}$ and $\Theta_\Sigma \notin N_-^{m\times m}$.*
2. *The evolution semigroup is bi-stable, iff $\Theta_\Sigma \in N_+^{m\times m}$ and bi-antistable iff $\Theta_\Sigma \in N_-^{m\times m}$.*

There are considered the necessary and sufficient condition on $\Theta_\Sigma(\in U(J, \Omega_+))$ under which $\Theta_\Sigma \in U_{sR}(J, \Omega_+)$ in the terms of solution of the Stein (Lyapunov) equation that corresponds to the considering discrete (continuous, resp.) - time system Σ.

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9

Riesz bases of exponential divided differences in control and inverse problems for distributed parameter systems

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Abstract

We present new results on Riesz bases of exponentials and their applications to control problems for distributed parameter systems

Keywords

Riesz bases, Exponentials, Divided differences, Controllability

The method of moments is a powerful tool in control theory of partial differential equations. It is based on properties of exponential families (usually in space $L^2(0,T)$) such as minimality and the Riesz basis property. In recent years interest in the method of moments in control theory has increased, and this is connected with investigations into new classes of distributed parameter systems such as hybrid systems, structurally damped systems and systems with singularities in control or equation. The other challenging subject concerns simultaneous controllability of several systems. Control problems for these systems have raised a number of new difficult problems in the theory of exponential families.

The principal questions which we consider in this talk are connected with the basis property of linear combinations from exponentials $e^{i\lambda_n t}$ in the case when the distance between some points λ_n tends to zero and therefore the family of exponentials $\left\{e^{i\lambda_n t}\right\}$ does not form a Riesz basis in $L^2(0,T)$. Using a new approach we have generalized the classical Ingham inequality for the case when the set $\{\lambda_n\}$ is the union of a finite number of separated sets [1]. Moreover, we obtained a full description of Riesz bases of special kinds linear combinations of exponentials — generalized divided differences [2]

We applied our results to problems of simultaneous and partial controllability of several elastic strings and beams [1]–[4].

The talk is based on joint papers with S. Ivanov, W. Moran, and M. Tucsnak.

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10

A reduced order computational methodology for eddy current based nondestructive evaluation techniques

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In the field of nondestructive evaluation, new and improved techniques are constantly being sought to facilitate the detection of hidden corrosion and flaws in structures such as air foils and pipelines. For the past several years, our research group has collaborated with NASA Langley Research Center scientists to develop and test theoretically sound and experimentally implementable real-time computational methods for the use of electromagnetic probes in damage location and characterization. We use eddy current based nondestructive evaluation techniques and reduced order modeling to explore the feasibility of detecting subsurface damages. To explicitly identify the geometry of a damage, an optimization algorithm is employed which requires solving the forward problem numerous times. To implement these methods in a practical setting, the forward algorithm must be solved with extremely fast and accurate solution methods. Therefore, our computational methods are based on reduced order Proper Orthogonal Decomposition (POD) techniques. For proof-of-concept, we implement the methodology first on a 2-D simulated problem and then on an experimental test problem using a giant magnetoresistive (GMR) sensor and in both instances find the methods to be efficient and robust. Furthermore, the methods were fast; our findings suggest a significant reduction in computational time.In some examples, the algorithms were 4000 times faster,while the overall inverse problem could be solved in less than 10 seconds. In this lecture we will summarize our findings to date (some of which can be found in the references listed below-some of which are new and as yet unpublished). Theoretical, computational and experimental results will be discussed. The results presented are from joint collaborations between the speaker, M.L.Joyner (NCSU), B.Wincheski (NASA LaRC), and W.P.Winfree (NASA LaRC).

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11

Stabilization, nuclearity and realization of various fractional differential systems

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Abstract

We will present in this talk several properties of a large class of fractional differential systems which are linked to their robust BIBO stabilization.

Keywords

fractional systems, neutral systems, stability conditions, nuclearity, diffusive realizations.

We are considering systems with transfer functions which involve fractional powers of s and 'standard exponentials' possibly in combination with exponentials of fractional powers, that is of the form :

$$P(s) \quad = \quad \frac{q_0(s) + \sum_{i=1}^{n_2} q_i(s) e^{-\beta_i s} + \sum_{i=1}^{\tilde{n}_2} \tilde{q}_i(s) e^{-v_i(s)}}{p_0(s) + \sum_{i=1}^{n_1} p_i(s) e^{-\gamma_i s} + \sum_{i=1}^{\tilde{n}_1} \tilde{p}_i(s) e^{-u_i(s)}}$$

where $0 < \gamma_1 \cdots < \gamma_{n_1}$, $0 < \beta_1 \cdots < \beta_{n_2}$, the $p_i,\ q_i,\ \tilde{p}_i,\ \tilde{q}_i$ being polynomials of the form $\sum_{k=0}^{l_i} a_k s^{\alpha_k}$ with $\alpha_k \in \mathbb{R}^+$ and $u_i,\ v_i$ being polynomials of the form $\sum_{k=1}^{m_i} b_k s^{\delta_k}$ with $0 < \delta_k \leq 1$ and $b_k \geq 0.$ We suppose of course that u_i and v_i are not of the form αs that is, are not standard polynomials of degree one.

As is done in the standard delay case, we can, depending on the link between the degrees of the $p_i's$ and the $q_i's$, divide this class into the class of retarded and neutral systems.

Building on the work of [1] and [2] where the cases of standard retarded delay systems and fractional differential systems without delays were treated, we have determined in [3] the set of all stabilizing contollers of retarded fractional differential systems.

We are now focusing on the case of neutral systems for which we only have at present a sufficient condition ensuring BIBO stability, moreover this condition is expressed in terms of the location of the poles of the system in a half-plane delimited by an axis which is strictly to the left of the imaginary axis. We will present two examples which prove that, on the one hand this sufficient condition is not necessary in general, on the other hand the usual condition 'no poles in the right-half-plane' cannot be sufficient in this case.

Considering now the implementation of controllers for fractional differential systems with delays we will give some conditions ensuring the nuclearity of such systems as it is well known that nuclear systems are well approximated in the H_∞-norm.

In the same spirit, we will consider the approximation of our systems by finite dimensional diffusive systems (see [4]).

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12

Transfer functions and input-output maps of boundary control systems in factor form

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Abstract

Some facts from the theory of boundary control systems in factor form are reported. The transfer function and input–output map are discussed. Boundary control systems in factor form lead naturally to regular Salamon–Weiss abstract linear systems with a well–defined state–space description.

Keywords

Infinite–dimensional control systems, semigroups, input–output relations, abstract linear systems.

12.1 Main concepts

In a Hilbert space H with a scalar product $\langle\cdot,\cdot\rangle_{\mathrm{H}}$ consider the SISO model of boundary control in factor form [1],

$$\left\{\begin{array}{rcl}\dot{x}(t) & = & A[x(t)+du(t)]\\ y(t) & = & c^{\#}x(t)\end{array}\right\} . \tag{12.1}$$

We assume that $A : (D(A) \subset \mathrm{H}) \longrightarrow \mathrm{H}$ generates a linear *exponentially stable* (**EXS**), C_0–semigroup $\{S(t)\}_{t\geq 0}$ on H, $d \in \mathrm{H}$ is a factor control vector, $u \in \mathrm{L}^2(0,\infty)$ is a scalar control function, y is a scalar output defined by an A–bounded linear observation functional $c^\#$ with $D(A) \subset D(c^\#)$. Restriction $c^\#\big|_{D(A)}$ can be represented as

$$c^\# x = \langle Ax, h\rangle_{\mathrm{H}} = h^* Ax, \qquad x \in D(A) \tag{12.2}$$

for $h \in \mathrm{H}$, $h^* = c^\# A^{-1}$.

Definition 12.1.1. *The linear A–bounded observation functional $c^\#$ is called* admissible *if there exists $\gamma > 0$ such that*

$$\int_0^\infty \left|c^\# S(t)x_0\right|^2 dt \leq \gamma \, \|x_0\|_H^2 \qquad \forall x_0 \in D(A)$$

i.e. the observability map $P : D(A) \ni x_0 \longmapsto c^\# S(\cdot)x_0 \in L^2(0,\infty)$ *has a bounded extension to H, denoted by $\overline{P}$.*

Theorem 12.1.2. [1, Theorem 4.1]. Let $c^\#$ be admissible. Then for $x_0 \in \mathrm{H}$, $(\overline{P}x_0)(t) = \dfrac{d}{dt}\left[h^* S(t)x_0\right]$ with Laplace transform $\left(\widehat{\overline{P}x_0}\right)(s) = c^\#(sI - A)^{-1}x_0$.

Definition 12.1.3. *The factor control vector $d \in H$ is called* admissible *if*

$$\left[\int_0^\infty S(t)du(t)dt\right] \in D(A) \qquad \forall u \in L^2(0,\infty) \ . \tag{12.3}$$

The operator W given by $Wu := \displaystyle\int_0^\infty S(t)du(t)dt$ belongs to $\mathbf{L}(\mathrm{L}^2(0,\infty),\mathrm{H})$. Since A is closed and (12.3) is equivalent to the inclusion $R(W) \subset D(A)$ then applying the closed graph theorem we get $Q = AW \in \mathbf{L}(\mathrm{L}^2(0,\infty),\mathrm{H})$. The operator Q is called the *reachability map*.

Lemma 12.1.4. [1, Theorem 4.2]. Let $d \in \mathrm{H}$ be an admissible factor control vector and let $u \in \mathrm{L}^2(0,\infty)$. Then the function

$$\begin{aligned} x(t) &:= QR_t u = A\int_0^t S(t-\tau)du(\tau)d\tau, \qquad (12.4)\\ (R_t u)(\tau) &:= \left\{\begin{array}{cc} u(t-\tau), & \tau \in [0,t] \\ 0, & \tau \geq t \end{array}\right\} \end{aligned}$$

is a *weak* solution of $\dot{x}(t) = A[x(t) + du(t)]$ for $x(0) = 0$, i.e. it is a continuous H–valued function of t such that for all $y \in D(A^*)$, $t \longmapsto \langle x(t), y\rangle$ is absolutely continuous and for almost all t and all $y \in D(A^*)$ we have $\dfrac{d}{dt}\langle x(t), y\rangle_{\mathrm{H}} = \langle x(t) + du(t), A^* y\rangle_{\mathrm{H}}$. Actually, $x \in \mathrm{BUC}_0[0,\infty;\mathrm{H})$, where $\mathrm{BUC}_0[0,\infty;\mathrm{H})$ stands for a Banach space of bounded strongly uniformly continuous functions defined on $[0,\infty)$, taking values in H, and vanishing at infinity.

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13

On-line fault detection and diagnosis in distributed parameter systems

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Abstract

In this work, fault detection techniques based on finite dimensional results are extended and applied to a class of infinite dimensional dynamical systems. This special class of systems assumes linear plant dynamics having either an abrupt or incipient additive perturbation as the component fault. This component fault is assumed to be linear in the (unknown) constant (and possibly functional) parameters. An adaptive detection observer is proposed which serves to monitor the system's dynamics for unanticipated failures, and its well posedness is summarized. Using a Lyapunov synthesis approach extended and applied to infinite dimensional systems, a stable adaptive fault diagnosis (fault parameter learning) scheme is developed. The resulting parameter adaptation scheme is able to "sense" the instance of the fault occurrence. In addition, it identifies the component fault parameters using the additional assumption of persistence of excitation. Simulations studies are presented to illustrate the applicability of the theoretical results.

Keywords

Fault detection, fault diagnosis, distributed parameter systems, adaptive detection observer

13.1 Introduction

In this work, a procedure for designing an on-line fault detection scheme for a class of infinite dimensional systems is proposed. Specifically, we will be concerned with the following class of dynamical systems

$$\dot{x}(t) + Ax(t) + \beta(t-t^*)D(\theta)x(t) = Bu(t), \qquad x(0) = x_0 \in H, \tag{13.1}$$

where H is an infinite dimensional state space, x denotes the state, and A, D, B denote the system operator, the failure operator and the input operator, respectively. At first, the component failure is assumed to be *abrupt* [1], and specifically the function $\beta(t-t^*)$ that represents the *time profile* of the failure is assumed to be a step function that is given by

$$\beta(t-t^*) = \begin{cases} 1 & \text{for } t \geq t^* \\ 0 & \text{for } t < t^*. \end{cases} \tag{13.2}$$

Equations (13.1), (13.2) above describe dynamical systems which prior to any unanticipated failures are given by $\dot{x}(t) + Ax(t) = Bu(t)$, $t < t^*$, and after the failure occurrence are expressed by $\dot{x}(t) + [A + D(\theta)]x(t) = Bu(t)$, $t \geq t^*$. The *nominal* system dynamics given by $\dot{x}(t) + Ax(t) = Bu(t)$ are assumed to be known. The $\theta-$*parameterized* operator $D(\theta)$ models the unanticipated component failure and it is assumed that the structure of the failure is known, *i.e.* for a given parameter θ the operator $D(\cdot)$ is known, but the parameter θ is unknown. We will provide the mathematical preliminaries required for the analysis and well-posedness of the plant and the derivation of the on-line estimated model of (13.1). This estimated model, or detection observer, will use as its inputs the plant output $x(t)$ and the adjustable (on-line) estimates $\hat{\theta}(t)$ of the (unknown) parameter θ.

The objective of the proposed scheme is the detection of the unknown failure time t^* by the detection observer and the subsequent diagnosis/isolation of the component fault via the on-line estimation of the parameter θ. If, for $\varphi \in V$, where V is a reflexive Banach space densely and continuously embedded in H, we define the linear operator $G(\varphi) : V \to Q$ by

$$\langle G(\varphi)\psi, \theta \rangle_Q = \langle D(\theta)\varphi, \psi \rangle, \quad \psi \in V, \quad \theta \in Q, \tag{13.3}$$

where Q is the parameter space, then the detection observer along with the proposed diagnostic scheme take the form of an initial value problem and are given by

$$\dot{\widehat{x}}(t) + L\widehat{x}(t) + G^*(x(t))\widehat{\theta}(t) = Bu(t) - Ax(t) + Lx(t), \quad \widehat{x}(0) = x(0), \tag{13.4}$$

$$\dot{\widehat{\theta}}(t) - G(x(t))\widehat{x}(t) = -G(x(t))x(t), \quad a.e. \quad t > 0, \ \widehat{\theta}(0) = 0, \tag{13.5}$$

where the filter operator L is designed to be a coercive and bounded operator. The observation error $e(t) = x(t) - \widehat{x}(t)$ will be zero prior to any faults (healthy) and will assume a nonzero value at the occurrence of the fault. This will serve as a means for monitoring the system for possible faults. A fault will be declared by the monitoring scheme once the observation attains a nonzero value.

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14

What can we learn from PDE models of reactors

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Abstract

The present communication aims at giving an overview of different results that have been obtained in the field of analysis of distributed parameter models of chemical and biochemical reactors, including the transfer of properties from the infinite dimensional systems from the reduced finite dimensional models.

Keywords

Chemical processes, biochemical processes, tubular reactors, observability, reachability, stability

14.1 Introduction

The dynamics of tubular reactors are typically described by nonlinear partial differential equations derived from mass and energy balance principles.

The study of the dynamical properties of nonisothermal reactors with a view to process control has been the object of active research over the last decades. If the control-oriented contributions were mainly dedicated to lumped parameter nonisothermal reactors (i.e. Continuous Stirred Tank Reactors (CSTR's)) (see e.g. [6] and the references therein), a large research activity has been dedicated to the analysis of the properties of Partial Differential Equations (PDE's) tubular reactor models (see e.g. [7] for a survey), and more recently to the control design based on distributed parameter models and to system theoretical properties of such models (see e.g. [2], [3], [9]).

Here we intend to give an overview of different results in the analysis of tubular reactor models after the past five years.

1. Analysis of linear tubular reactors. The analysis is based on a model with one reactant and one product in an isothermal reactor with first-order kinetics, both for hyperbolic (plug flow) and parabolic (axial diffusion) systems. Approximate observability and reachability notions have been considered. Both systems are shown to be observable when both reactant and product are measured at the reactor output. The axial dispersion (plug flow) reactor is also (H^+-)observable when the product is measured at the reactor output and (H^+S)-reachable if the control input is the inlet reactant concentration [9]. These results are comparable to those obtained with reduced finite dimensional models obtained by finite differences.

2. Extension of the results. The above results can be extended to sequential reactions where reactants appears in only one reaction. However they cannot be extended to models of tubular reactors when one component is fixed (no convection, no diffusion) in the tank (case of bioreactors where the biomass may be fixed) or when one reactant appears as a reactant in several reactions [1].

3. Analysis of the existence and the stability of (nonlinear) non-isothermal models of chemical reactors [4].

4. Transfer of the properties from the infinite dimensional models to the reduced finite dimensional models

14.2 Acknowledgments

In order to avoid a too long list of authors, the names of the different persons who have had a significant contribution in the presented results have not been put the author list. Let me duly acknowledge their essential contribution to the present work : Elarbi Achhab, Jean-Pierre Babary, Sylvie Bourrel, Mohammed Laabissi, Laurent Lefèvre, Philippe Ligarius, Alphonse Magnus, Walter Waldraff, Joseph Winkin.

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15

On composite semigroups with applications

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Abstract

In the paper we present the concept of a *composite semigroup.* It is a strong-operator continuous semigroup defined on a Banach space of linear bounded operators obtained as a composition of two 'classical' strongly continuous semigroups defined on a Hilbert space. We think that it is a very attractive notion with a potentially wide range of applications in the infinite-dimensional control and systems theory. We start with some basic properties of the composite semigroup. Then we apply this concept to study the operator differential Sylvester equation which arises in various control problems on finite time horizon. We investigate basic properties of the solution to this equation in the case when the operators occurring in in the equation are unbounded. On the way we derive a necessary and sufficient condition for an unbounded control operator to be admissible in the sense of Weiss. Although this result, expressing the admissibility in terms of the infinite-dimensional algebraic Lyapunov equation, can be found in the literature our proof is completely different.

Keywords

Composite semigroup, Sylvester differential equation, control operator admissibility

15.1 Short presentation of the composite semigroup

Notation and assumptions

- $\mathbf{A}_1$ and $\mathbf{A}_2$ are linear unbounded operators on a Hilbert space H, with domains $D(\mathbf{A}_1) \subset H$ and $D(\mathbf{A}_2) \subset H$. $\mathbf{A}_1$ and $\mathbf{A}_2$ generate strongly continuous semigroups $\mathbf{T}_1(t) \in L(H)$ and $\mathbf{T}_2(t) \in L(H)$, $t \in [0, \infty)$.

- $H_1(\mathbf{A}_1) = D(\mathbf{A}_1)$ are Hilbert spaces with suitably defined scalar products, and $H_{-1}(\mathbf{A}_1) = D(\mathbf{A}_1^*)^*$. Analogously, $H_1(\mathbf{A}_2)$ and $H_{-1}(\mathbf{A}_2)$.

Using the semigroups $\mathbf{T}_1(t), \mathbf{T}_2(t) \in L(H)$ generated by $\mathbf{A}_1$ and $\mathbf{A}_2$ we define the *composite semigroup* $\mathbb{T}(t) \in L(L(H))$ (ie. $\mathbb{T}(t) : L(H) \to L(H)$), as follows

$$\mathbb{T}(t)\mathbf{X} = \mathbf{T}_1(t)\mathbf{X}\mathbf{T}_2(t), \quad \mathbf{X} \in L(H), \quad t \in [0, \infty) . \tag{15.1}$$

- The family $\mathbb{T}(t) \in L(L(H))$, $t \in [0, \infty)$, satisfies

$$\begin{aligned} \mathbb{T}(0)\mathbf{X} &= \mathbf{X}, \quad \mathbf{X} \in L(H), \\ \mathbb{T}(t+s)\mathbf{X} &= \mathbb{T}(t)(\mathbb{T}(s)\mathbf{X}) = \mathbb{T}(s)(\mathbb{T}(t)\mathbf{X}), \quad \mathbf{X} \in L(H), \end{aligned}$$

  for $t, s \in [0, \infty)$.

- $\mathbb{T}(\cdot) \in L(L(H))$ is strong-operator continuous at every $t \in [0, \infty)$.
- The *infinitesimal generator* $\mathbb{A}$ of $\mathbb{T}(t)$ is defined as the limit

$$(\mathbb{A}\mathbf{X})h = \lim_{t\to 0^+} \frac{(\mathbb{T}(t)\mathbf{X})h - \mathbf{X}h}{t}\}, \quad \mathbf{X} \in D(\mathbb{A}), \tag{15.2}$$

  for $h \in H$, where $D(\mathbb{A}) \subset L(H)$ is its *domain* defined as follows

$$D(\mathbb{A}) = \{\mathbf{X} \in L(H) : \quad \lim_{t\to 0^+} \frac{(\mathbb{T}(t)\mathbf{X})h - \mathbf{X}h}{t}\} . \tag{15.3}$$

- For $\mathbf{X} \in D(\mathbb{A})$ we have

$$\mathbb{T}(t)\mathbf{X} \in D(\mathbb{A}), \quad \frac{d}{dt}(\mathbb{T}(t)\mathbf{X}) = \mathbb{A}(\mathbb{T}(t)\mathbf{X}) = \mathbb{T}(t)(\mathbb{A}\mathbf{X}) .$$

- For $\mathbf{X} \in D(\mathbb{A})$ and $h \in D(\mathbf{A}^*)$

$$(\mathbb{A}\mathbf{X})h = \mathbf{A}_1\mathbf{X}h + \mathbf{X}\mathbf{A}_2 h. \tag{15.4}$$

If we suitably define space $H_{-1}(\mathbb{A})$ then every element of $H_{-1}(\mathbb{A})$ can be identified with an element of $L(D(\mathbf{A}_2), D(\mathbf{A}_1^*)^*)$ and thus we get

$$L(H) \subset H_{-1}(\mathbb{A}) \subset L(D(\mathbf{A}_2), D(\mathbf{A}_1^*)^*) . \tag{15.5}$$

In particular, every element $\mathbf{Y} \in H_{-1}(\mathbb{A})$ can be uniquely expressed in the form $\mathbf{Y} = (\lambda\mathbb{I} - \mathbb{A})\mathbf{X}$, where $\mathbf{X} \in L(H)$. This fact leads to the following corollary.

Corollary 15.1.1. $\mathbf{Y} \in L(D(\mathbf{A}_2), D(\mathbf{A}_1^*)^*)$ *is an element of* $H_{-1}(\mathbb{A})$ *if and only if the following Sylvester algebraic equation has a (unique) solution* $\mathbf{X} \in L(H)$

$$\lambda\langle \mathbf{X}x, y\rangle_H - \langle \mathbf{X}x, \mathbf{A}_1^* y\rangle_H - \langle \mathbf{X}\mathbf{A}_2 x, y\rangle_H = \langle \mathbf{Y}x, y\rangle_{D(\mathbf{A}_1^*)^*, D(\mathbf{A}_1^*)} ,$$

where $x \in D(\mathbf{A}_2)$, $\quad y \in D(\mathbf{A}_1^*)$ *and* $\langle \cdot, \cdot \rangle_{D(\mathbf{A}_1^*)^*, D(\mathbf{A}_1^*)}$ *denotes the duality pairing between* $D(\mathbf{A}_1^*)$ *and* $D(\mathbf{A}_1^*)^*$.

16

The circle criterion for boundary control systems in factor form: Lyapunov approach

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Abstract

A criterion of absolute strong asymptotic stability of the null equilibrium point of a Lur'e feedback control system is derived using Lyapunov functional. The construction of such quadratic form functional is reduced to solving a Lur'e system of equations. The solvability of the latter system is investigated. The results are illustrated in detail by electrical transmission line.

Keywords

Infinite–dimensional control systems, semigroups, Lyapunov functionals, circle criterion

16.1 Asymptotic stability of the Lur'e feedback system

Our aim is to prove a criterion of *strong asymptotic stability* (**AS**) for the Lur'e feedback system depicted in Figure 16.1,
which consists of a linear part described by

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & A[x(t)+u(t)d] \\ y & = & c^{\#}x \end{array} \right\} ,$$

and a scalar static controller nonlinearity $f : \mathbb{R} \to \mathbb{R}$. For this purpose we assume the following linear subsystem assumptions:

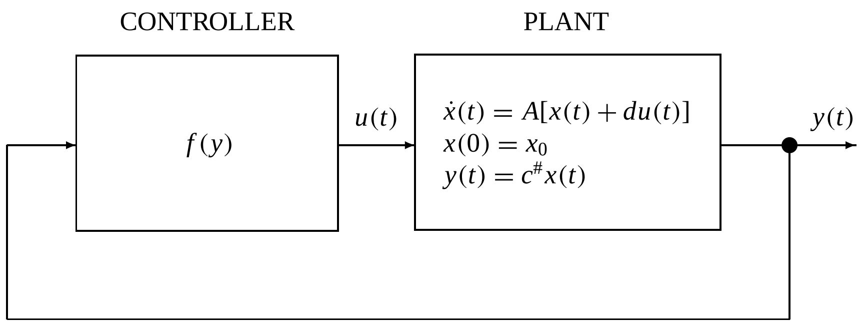

Figure 16.1: The Lur'e control system

(A1) The operator A generates on H an **AS** C_0–semigroup with $A^{-1} \in \mathbf{L}(\mathrm{H})$,

(A2) The *compatibility condition* holds,

$$d \in D(c^{\#}) \tag{16.1}$$

(A3) There exist constants k_1 and $k_2 > k_1$ such that with $q := k_1 k_2$, $e := \dfrac{k_1+k_2}{2} + k_1 k_2 c^{\#} d$, $\delta := (1+k_1 c^{\#} d)(1+k_2 c^{\#} d)$ the *Lur'e system*

$$\left\{ \begin{array}{rcl} H A^{-1} + \left(A^{-1}\right)^{*} H - q h h^{*} &=& -g g^{*} \\ -H d + e h &=& -\sqrt{\delta} g \end{array} \right\} \tag{16.2}$$

has a solution (H, g), $g \in \mathrm{H}$, $H \in \mathbf{L}(\mathrm{H})$, $H = H^{*} \geq 0$.

Next for the controller two sets describe restrictions to be imposed on the static nonlinearity $f : \mathbb{R} \longrightarrow \mathbb{R}$, namely

- For sufficiently small $\varepsilon > 0$ we consider the sector

$$\begin{aligned} S_{\varepsilon} \quad := \quad & \Big\{ f \in \mathrm{C}(\mathbb{R}) : \; -\infty < k_1 < \frac{1}{2}\left[k_1 + k_2 - \sqrt{(k_2-k_1)^2 - 4\varepsilon}\right] \\ & \leq \frac{f(y)}{y} \leq \frac{1}{2}\left[k_1 + k_2 + \sqrt{(k_2-k_1)^2 - 4\varepsilon}\right] < k_2 < \infty \\ & \forall y \in \mathbb{R} \setminus \{0\}, \;\; f(0) = 0 \Big\}. \end{aligned}$$

- We denote by M the class of those functions $f \in \mathrm{C}(\mathbb{R})$ which are sufficiently smooth to ensure that the solutions of the *closed–loop system* equations

$$\left\{ \begin{array}{rcl} A^{-1}\dot{x} &=& x + d f(y) \\ y &=& c^{\#} x = c^{\#}(A^{-1}\dot{x} - d f) = c^{\#} A^{-1}\dot{x} - c^{\#} d f = h^{*}\dot{x} - c^{\#} d f \end{array} \right\}$$

generate a *local dynamical system* on the state space H.

Definition 16.1.1. *Let A generate a C_0–semigroup $\{S(t)\}_{t\geq 0}$ which is* **AS**. *The factor control vector $d \in H$ is said to be* admissible *if $W \in \mathbf{L}(L^2(0,\infty), H)$ and* $\text{Range}(W) \subset D(A)$, $Wu := \int_0^\infty S(t)du(t)dt$. *The* reachability operator *Q then satisfies $Q := AW \in \mathbf{L}(L^2(0,\infty), H)$.*

(**A4**) The factor control vector $d \in \mathrm{H}$ is admissible according to Definition 16.1.1.

Theorem 16.1.2. Let assumptions (**A1**)–(**A4**) hold. Let f belong to $S_\varepsilon \cap M$. Then the origin of the space H is *globally strongly asymptotically stable* (**GAS**).

The result above improves that of [1, Theorem 3.3].

16.2 Solvability of the Lur'e system of equations (16.2)

Our criterion of solvability of the Lur'e system of equations (16.2) is partially based on the results of Oostveen and Curtain [4, Theorem 19 and Corollary 20].

Theorem 16.2.1. Let the following assumptions hold:

(**OS1**) The operator A generates an **EXS** linear C_0–semigroup on H and the semigroup generated by A^{-1} is uniformly bounded;

(**OS2**) The observation functional $c^\#$ is admissible, $c^\#|_{D(A)} = h^*A$ and satifies the compatibility condition (16.1);

(**OS3**) The open–loop transfer function $\hat{g}(s) := sc^\#(sI - A)^{-1}d - c^\# d = sh^*A(sI - A)^{-1}d - c^\# d$ is in $\mathrm{H}^\infty(\Pi^+)$;

(**FDI**) There exist k_1, k_2, $k_1 < k_2$ such that

$$\pi(\omega) := 1 - (k_1 + k_2)\,\mathrm{Re}[\hat{g}(j\omega)] + k_1 k_2 \left|\hat{g}(j\omega)\right|^2 \geq \varepsilon > 0 \quad \forall \omega \in \mathbb{R} \ ; \tag{16.3}$$

(**CS**) For $\mu_0 = (k_1 + k_2)/2$ we have: $s \longmapsto \dfrac{1}{1 - \mu_0 \hat{g}(s)}$ is in $\mathrm{H}^\infty(\Pi^+)$, and the semigroup generated by $A_0 := A^{-1} - \dfrac{\mu_0}{1 + \mu_0 c^\# d} dh^* \in \mathbf{L}(\mathrm{H})$ is **AS**.

Then the system (16.2) has a solution (H, g), $H \in \mathbf{L}(\mathrm{H})$, $H = H^* \geq 0$, $g \in \mathrm{H}$ and g^*A is admissible with respect to the semigroup generated by A.

16.3 Example

The results will be illustrated and in detail by electrical transmission lines, where contrary to [3] the nonlinearity argument is an unbounded linear observation functional. For similar results using the input–output approach see [2].

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17

Riesz basis property of a second order hyperbolic system with scalar input/output and application to a connected Euler-Bernoulli beam equation

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Abstract

The closed-loop form of a second order hyperbolic system with scalar collocated sensor/actuator is considered. The Riesz basis property of the system is verified for the diagonal semigroup based on an abstract result of rank one perturbation of discrete type operators in Hilbert spaces. A simplified proof for the abstract result is presented. Finally, the result is applied to a connected Euler-Bernoulli beam equation with joint control.

Keywords

Riesz basis, perturbation of linear operators, beam equation, collocated input/output, second order system

17.1 Introduction

The Riesz basis, meaning that the generalized eigenvectors of the system form an unconditional basis for the state Hilbert space, is one of the fundamental properties of a linear vibrating system. The establishment of the basis property will naturally lead to solutions to such problems as the spectrum-determined growth condition and the exponential stability for infinite dimensional systems.

In this paper we study the following closed-loop form of second order hyperbolic system with scalar collocated input/output in a Hilbert space X:

$$y_{tt} + Ay + kbb^* y_t = 0 \tag{17.1}$$

where k is a constant. $A : D(A)(\subset X) \to X$ is an unbounded positive selfadjoint operator in X. $b \in [D(A^{1/2})]'$. $b^* \in L([D(A^{1/2})], \mathbb{C})$ is defined by

$$b^* x = < x, b >_{[D(A^{1/2})]\times[D(A^{1/2})]'}, \forall x \in D(A^{1/2}). \tag{17.2}$$

We assume that A is diagonal, that is there is an orthonormal basis $\{\phi_n\}_1^\infty$ for X such that

$$A\phi_n = \omega_n^2 \phi_n, \omega_n > 0, \ n \geq 1. \tag{17.3}$$

Hence

$$b = \sum_{n=1}^{\infty} 2b_n\phi_n, \ \ \sum_{n=1}^{\infty} \frac{|b_n|^2}{\omega_n^2} < \infty, \ \ b^*\phi_n = 2b_n, n \geq 1. \tag{17.4}$$

17.2 Main result

Our main result can be stated as following theorem 17.2.1.

Theorem 17.2.1. *Suppose that ω_n in (17.3) satisfy*

$$\omega_{n+1} - \omega_n \geq M\omega_{n+1}^\delta, \ n \geq 1. \tag{17.5}$$

for some $M, \delta > 0$ and

$$\sum_{n=1}^{\infty} \frac{1}{\omega_n^{2\delta}} < \infty. \tag{17.6}$$

If b is admissible, then system (17.1) is a Riesz spectral system: that is, there is a set of eigenvectors but finite number of generalized eigenvectors of the system, which forms a Riesz basis in the state space $H = [D(A^{1/2})] \times X$. Moreover, there exists a constant $C > 0$ such that the eigenpairs $\{(\mu_n, \psi_n)\} \cup \{$ their conjugate$\}$ of system (17.1) satisfy

$$\begin{cases} |\mu_n - i\omega_n^2| \leq C, \\ \psi_n = \phi_n + O(\max\{\omega_n^{-1/2}, \omega_{n-1}^{-2\delta}\}). \end{cases} \tag{17.7}$$

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18

Boundary control of exponentially stable infinite-dimensional systems in Hilbert space

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Abstract

Finite-dimensional robust control of exponentially stable infinite-dimensional systems in Hilbert space will be considered. Both distributed and boundary control are present. Distributed, boundary and measurement disturbances are allowed. It is shown that signals that are linear combinations of sinusoids with polynomial coefficients can be tracked and rejected.

Keywords

Boundary control, Low gain tracking

18.1 Introduction

Consider the control system in Fig. 18.1 where the plant P is described by the equations

$$\begin{aligned} \dot{z}(t) &= A_0 z(t) + B_d u_d(t) + E_d w_d(t), \\ S_b z(t) &= B_b u_b(t) + E_b w_b(t), \\ y(t) &= C z(t) + D_d u_d(t) + D_b u_b(t) + E_m w_m(t), \end{aligned}$$

with closed operator $A_0 : X \supset D(A_0) \to X$, boundary operator $S_b : X \supset D(S_b) \to H$, where X and H are Hilbert spaces, finite-dimensional input and output spaces, and bounded B, E, C and D. The system operator $A : X \supset D(A) \to X$, with $D(A) = D(A_0) \cap \ker S_b$ and $Ax = A_0 x$ for $x \in D(A)$, is assumed to be the infinitesimal generator of an exponentially stable C_0-semigroup.

We show that if the reference signal r and the disturbance signals w_d, w_b and w_m are of the form

$$\sum_{k=-n}^{n} a_k(t) e^{i\omega_k t},$$

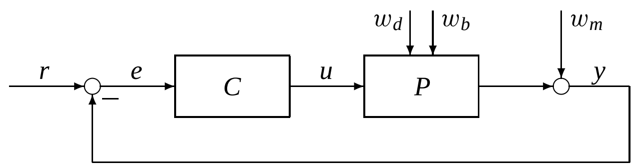

Figure 18.1: The closed-loop system with the stable plant P, the reference signal r, the disturbance signals w_d, w_b and w_m, and the controller C.

where the coefficients $a_k(t)$ are polynomials, then the finite-dimensional controller described by the authors in [2] stabilizes the closed-loop system and achieves asymptotic tracking and disturbance rejection.

The results are extentions of those in [1], [3], [4] to more general reference and disturbance signals. The advantage of the time domain formulation in this paper over the frequency domain formulation in [2] is that the reference and disturbance signals can directly enter the state. Rebarber and Weiss [5] have extended [2] to well-posed systems in the case of sinusoidal reference and disturbance signals. However, they achieve tracking only in the sense that the exponentially weighted error signal e is in $L^2[0,\infty)$. The tracking results given in this paper achieve asymptotic tracking and are hence stronger, but apply to a smaller class of systems than in [5].

The stabilization proof is based on showing that the resolvent of the extended system operator is in H^∞, rather than on perturbation theory as in [3]. This simplifies the proofs, but they are valid only in a Hilbert space.

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19

Equalizing vectors as a "tool" in H_∞-control

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Abstract

In this paper we present the equalizing vectors as an important "tool" in H_∞-control.

Keywords

Infinite-dimensional systems, Wiener algebra, H_∞-control, equalizing vectors, J-spectral factorization.

19.1 Introduction and main results

We consider the following class of transfer functions, known as the *Wiener algebra*

$$\hat{W} = \left\{ \hat{f} \mid \hat{f} = \hat{f}_1 + \hat{f}_2, \text{ with } \hat{f}_1, \hat{f}_2^{\sim} \in \hat{A} \right\},$$

where $\hat{A}$ (the proper-stable class of transfer functions) consists of all the Laplace transform of functions of the form

$$f(t) = \begin{cases} f_a(t) + f_0\delta(t), & t \geq 0, \\ 0, & t < 0, \end{cases}$$

with f_0 a complex number, $\int_0^\infty |f_a(t)|dt < \infty$, δ represents the delta distribution at zero, and $f^{\sim}(s) = \overline{f(-\overline{s})}$ for any complex number s. The elements of $\hat{W}$ are bounded and continuous on the imaginary axis, and their limit at infinity is well-defined. Note that proper rational functions which have no poles on the imaginary axis belong to the class $\hat{W}$. We denote by $\hat{A}^{n\times m}$, $\hat{W}^{n\times m}$, the classes of $n \times m$ matrices with entries in $\hat{A}$, $\hat{W}$, respectively, and by H_2 ($H_2^{\perp}$) the set of all vector-valued functions f analytic in the right (left) half-plane, such that $\sup_{\sigma>0}\int_{-\infty}^{\infty}\|f(\sigma+j\omega)\|^2\,d\omega<\infty$ ($\sup_{\sigma<0}\int_{-\infty}^{\infty}\|f(\sigma+j\omega)\|^2\,d\omega<\infty$).

We say that a matrix-valued function $Z\in\hat{W}^{n\times n}$, defined on the imaginary axis, admits a J-spectral factorization if there exists a matrix-valued function V, invertible over $\hat{A}^{n\times n}$, such that

$$Z(s)=V^{\sim}(s)\begin{bmatrix} I_p & 0\\ 0 & -I_q\end{bmatrix}V(s)=V^{\sim}(s)JV(s)$$

on the imaginary axis. Here $p+q=n$ and $V^{\sim}(s)=\overline{V(-\overline{s})}^T$.

By definition, a vector u is an equalizing vector of $Z\in\hat{W}^{n\times m}$ if u is a nonzero element of H_2 such that Zu is in $H_2^{\perp}$. We show that $Z=Z^{\sim}\in\hat{W}^{n\times n}$ (with nonzero determinant on the imaginary axis) admits a J-spectral factorization if and only if Z has no equalizing vectors (see [1]). This extends the result from [5] to non-rational functions.

Using equalizing vectors, a simple frequency domain solution is obtained for the suboptimal Nehari extension problem for our class of infinite-dimensional systems. Moreover, the equalizing vector is fixing the solution of the Nehari extension problem in the direction of the eigenvector corresponding to the largest singular value of the Hankel operator. For the scalar case, an equalizing vector can be used to prove the uniqueness of the solution for the Nehari extension problem (see [3]). Similar technics can be used to solve the suboptimal Hankel-norm approximation problem (see [4]).

In order to provide necessary and sufficient conditions for the solvability of the standard H_∞-suboptimal control problem for systems with the transfer function in a subalgebra of the quotient field of the Wiener algebra, equalizing vectors play also a very important role (see [2]).

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20

Stabilizability of systems defined on the full time axis

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Abstract

In this talk we study time-invariant, discrete-time systems over the signal space $\ell_2(\mathbb{Z})$. Since there are serious difficulties using the standard definition of stabilizability in this context, we suggest an alternative definition of stabilizability in order to overcome these difficulties.

Keywords

Linear systems, discrete-time systems, time-invariant systems, full time axis, stabilizability, causality

20.1 Introduction

The goal of this talk is the study of discrete-time, time-invariant systems with input space $\ell_2(\mathbb{Z})^m$ and output space $\ell_2(\mathbb{Z})^p$ from an input-output point of view. In this context an operator

$$P : D(P) \subset \ell_2(\mathbb{Z})^m \to \ell_2(\mathbb{Z})^p$$

is called a *linear time-invariant system* with input space $\ell_2(\mathbb{Z})^m$ and output space $\ell_2(\mathbb{Z})^p$ (or *LTI*$^{p\times m}$*-system* for short) if P is *linear* and *shift-invariant*. Moreover, we say P is *closed*, if the graph of P, denoted by $G(P)$, is a closed subspace of $\ell_2(\mathbb{Z})^{m+p}$, and we say P is *causal*, if $u \in \ell_2(\mathbb{N}_0)^m \cap D(P)$ implies $Pu \in \ell_2(\mathbb{N}_0)^p$.

It was noted by Georgiou and Smith [1], see also Mäkilä [2], that there appear to be some serious difficulties in treating unstable systems defined on $\mathbb{Z}$. Here a LTI$^{p\times m}$-system P is called *stable*, if it is closed and $D(P) = \ell_2(\mathbb{Z})^m$. One problem is that the simple and well-studied system

$$(P_1 u)(t) := \sum_{n=-\infty}^{t} 2^{t-n} u(n), \quad u \in D(P_1),$$

where $D(P_1)$ consists of all $u \in \ell_2(\mathbb{Z})^m$ such that $P_1 u$ is an element of $\ell_2(\mathbb{Z})^p$, is not closed. However, using the standard definition of closed-loop stabilizability, closedness is a necessary condition for stabilizability. This implies that the system P_1 would not be stabilizable, contradicting the common belief that P_1 is stabilized by proportional negative feedback of gain greater than one. Another problem is that a stable closed-loop system is not automatically causal, as it is the case for systems defined on $\mathbb{N}$. Again, there are simple examples showing this effect.

Thus we seek for an alternative definition of closed-loop stabilizability for LTI$^{p\times m}$-systems which agrees with the common belief that P_1 is stabilized, and which guarantees that the resulting closed-loop system is causal. Our alternative definition of stabilizability, called *causal stabilizability*, is as follows. We say a LTI$^{p\times m}$-system P is *closable*, if the operator P is closable. If P is closable, then we denote the closure of P by $\bar{P}$.

Definition 20.1.1. We say a LTI$^{p\times m}$-system P is *causally stabilizable* if P is closable and if there exists a LTI$^{m\times p}$-system C such that

$$F_{[\bar{P},C]} := \begin{pmatrix} I & C \\ \bar{P} & I \end{pmatrix} : D(\bar{P}) \times D(C) \to \ell_2(\mathbb{Z})^{p+m}$$

is bounded invertible, and $F^{-1}_{[\bar{P},C]}$ is causal.

It is easy to see that a LTI$^{p\times m}$-system P can only be causally stabilizable if P is closable and $D(P)$ is dense in $\ell_2(\mathbb{Z})^m$. Moreover, in this situation $\bar{P}$ can uniquely described by the transfer function $\mathrm{P} \in L_\infty(\mathbb{T})^{p\times m}$. We obtain the following equivalent conditions for causal stabilizability.

Theorem 20.1.2. *Let P be closable LTI$^{p\times m}$-system with dense domain. Then the following statements are equivalent.*

1. *P is causally stabilizable.*
2. *$\mathrm{P} \in H_\infty(\mathbb{D})^{p\times m}$ and P possesses a strong right coprime factorization as well as strong left coprime factorization over $H_\infty(\mathbb{D})$.*
3. *$\mathrm{P} \in H_\infty(\mathbb{D})^{p\times m}$ and P possesses a strong right coprime factorization over $H_\infty(\mathbb{D})$.*

Moreover, if P is causally stabilizable, then we are able to parametrize all causally stabilizing controllers C.

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21

Control of nanostructures

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Abstract

Numerical approximations for control via oscillating potential barriers at the quantum- mechanical level will be discussed.

Keywords

quantum, numerical

The development of new solid-state devices at nanometer scale is proceeding rapidly in scientific and engineering communities. Examples include quantum dots, quantum cellular automata, carbon nanotubes and miniature circuits made via lithography or self-assembly. At this level quantum-mechanical effects come into play, and are described by the Schrodinger equation

$$i\frac{\partial \Psi}{\partial t} = H\Psi + V\Psi.$$

Here, Ψ represents the wave function, H the Hamiltonian, and V the potential.

Some issues of control of these structures are arising [1-3] and others remain to be formulated. In general the control will be through the potential. The simplest type of potential is that of a square barrier or well. Oscillating barriers have been studied in their own right and as controls [4 and references therein]. In [3], an extra quantum dot placed between two quantum dots in a quantum cellular automata half-cell acts as a tunable barrier to control single-electron tunneling between the two end dots.

We will discuss numerical approaches to control via an oscillating barrier, asking in what ways initial states may be transferred to terminal states via chosen oscillations. In [4], the Crank-Nicolson method is used for the Schrodinger equation in one space dimension, discretizing with respect to space and time. One obtains the difference equations

$$\begin{aligned} &\frac{-i}{2h^2}\Psi_{j+1}^{n+1} + \left(\frac{1}{k} + \frac{i}{h^2} + \frac{iV_j^{n+1}}{2}\right)\Psi_j^{n+1} - \frac{i}{2h^2}\Psi_{j-1}^{n+1} \\ = \ &\frac{i}{2h^2}\Psi_{j+1}^{n} + \left(\frac{1}{k} - \frac{i}{h^2} - \frac{iV_j^{n}}{2}\right)\Psi_j^{n} + \frac{i}{2h^2}\Psi_{j-1}^{n}, \quad j = 1\dots J-1\,. \end{aligned} \tag{21.1}$$

Here, h is the space step, k is the time step and Ψ_j^n approximates $\Psi(x_j, t_n)$, $j = 1..J-1, n > 0$. We take $\Psi_0^n = \Psi_J^n = 0$ for all n and $\Psi_j^0 = \Psi^0(x_j)$, $j = 1..J-1$, where $\Psi^0(x)$ is initial data. Using Maple, one may obtain discretized controls, V_j^n, in certain situations. Several questions will be examined: what occurs with further time steps, how to smooth in time and space?

Discretizing with respect to space only, i.e. semi-discretizing, gives a system of finitely many equations

$$i\frac{d\Psi_j}{dt} = -\frac{1}{h^2}(\Psi_{j+1} - 2\Psi_j + \Psi_{j-1}) + V_j(t)\Psi_j(t).$$

Now, controlling via the $V_j(t)$ amounts to the problem of control of a finite-dimensional bilinear system. This offers the advantage that the controls are no longer discretized in time. Such systems have been extensively studied via techniques of Lie algebras and differential geometry. Also, such methods have been used to treat finite-dimensional quantum systems (not as a result of a numerical approximation) off and on for at least 20 years, see e.g., [5]. We will consider what geometric methods tell us about control of the semi-discretized system.

Additional issues concern the relation of the numerical methods to physical implementation. A long-standing problem in quantum physics is that of determination of the tunneling time of a quantum particle through a potential barrier. This was one motivation of the study of oscillating barriers. Thus it would be of interest to determine if well-chosen oscillations could increase or decrease tunneling times. Another problem relates to the length scale at which wavepackets become important. In [3], while working at the ”huge” micron level, one may consider an electron as a localized particle once it is ”latched” onto an end quantum dot. How small can one go before the propagation of quantum waves becomes important?

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22

Absolute stability results in infinite dimensions with applications to low-gain integral control

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Abstract

We derive absolute stability results for well-posed infinite-dimensional systems which, in a sense, extend the well-known Popov criterion to the case that the underlying linear system is the series interconnection of a strongly stable well-posed infinite-dimensional system and an integrator and the nonlinearity φ satisfies a sector condition of the form $\langle \varphi(v), v\rangle \geq \|\varphi(v)\|^2/a$ for some constant $0 < a \leq \infty$. These results are used to prove convergence and stability properties of low-gain integral feedback control applied to strongly stable, linear, well-posed systems subject to actuator nonlinearities. The class of actuator nonlinearities under consideration contains standard nonlinearities which are important in control engineering such as saturation and deadzone.

Keywords

Absolute stability; actuator nonlinearities; integral control; Popov stability criterion; positive real; robust tracking; sector-bounded nonlinearities; strong stability; well-posed infinite-dimensional systems.

23

A Hamiltonian formulation of boundary control systems

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Abstract

A port controlled Hamiltonian formulation of the dynamics of distributed parameter systems is presented, which incorporates the energy flow through the boundary of the domain of the system, and which allows to represent the system as a boundary control Hamiltonian system. This port controlled Hamiltonian system is defined with respect to a Dirac structure associated with the exterior derivative and based on Stokes' theorem. The definition is illustrated on the examples of the telegrapher's equations, Maxwell's equations, the vibrating string and Euler's equations for fluid dynamics.

Keywords

Distributed parameter systems, Hamiltonian systems, Dirac structures, boundary control

Recently, for *finite-dimensional* nonlinear systems we have proposed a generalized Hamiltonian formulation of physical systems' dynamics with external (input and output) variables. This has led to the notions of *port-controlled Hamiltonian (PCH) systems* [7] [6], and *port-controlled Hamiltonian systems with dissipation* (PCHD systems) [5] defined with respect to a geometric structure, called Dirac strucure, which expresses the dynamic invariants and contraints arizing from the balance and continuity equations of physical systems' models. This theory is aimed at applications in the modelling and simulation of complex *interconnected* physical systems, and in the design and *control* of such systems, exploiting the Hamiltonian and passivity structure in a crucial way [5], [3].

In the present paper we present an extension of finite-dimensional PCH and PCHD systems to the distributed parameter (or, infinite-dimensional) case. It extends also the Hamiltonian formulations theory as for instance exposed in [4] to distributed parameter systems *with external variables* (inputs and outputs)by including boundary conditions inducing *energy exchange through the boundary*.

Therefore, in the present paper we the definition *Dirac structure* on certain spaces of differential forms on the spatial domain and its boundary proposed in previous publications [1] [2]. This construction of the Dirac structure is based on the use of Stokes' theorem. Then we employ the definition of a port-controlled Hamiltonian system with respect to a Dirac structure, as already given in previous papers (see e.g. [7]) for the finite-dimensional case, to describe *implicit* PCH systems, in order to formalize distributed parameter systems with boundary external variables as infinite-dimensional PCH systems. This framework is then applied to the port-controlled Hamiltonian formulation of Maxwell's equations on a bounded domain, the telegrapher's equations for an ideal transmission line, and the vibrating string. Furthermore, by modifying the Stokes-Dirac structure with an additional term corresponding to three-dimensional convection we provide an extension of port-controlled Hamiltonian systems suitable for the formulation of the ideal adiabatic fluid (e.g. Euler's equations).

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24

Damping models for mechanical systems using diffusive representation of pseudo-differential operators: theory and examples

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Abstract

A class of damping models is introduced for linear mechanical systems, which preserve the modal structure of the undamped system $\partial_{tt}^2 X + A\,X = 0$, with A a Riesz-spectral operator. The damping models involve pseudo-differential time-operators of diffusive type that are dissipative, applied either to the time-derivative $\partial_t X$ or to $A\,\partial_t X$. When A is a positive self-adjoint operator, an energy analysis of the damped system can be made on an augmented state-space. When A is a Riesz spectral-operator, a spectral analysis can be performed.

Keywords

Diffusive representation of pseudo-differential operators, Lyapunov functionals, Riesz spectral-operators.

24.1 Introduction

Pseudo-differential operators (PDO) D of *diffusive type* have first been introduced for fractional derivatives and integrals, and their theory has been elaborated in [6]; the use of the diffusive symbol μ_D in a diffusive *realisation* – in the sense of systems theory – helps transforming a non-local in time pseudo-differential equation into a first order differential equation on an infinite-dimensional state-space, endowed with a Hilbert structure, which allows for stability analysis (see [4]) and straightforward finite-dimensional approximations. This approach reveals useful for both theoretical and numerical treatment of pseudo-differential equations (not only fractional differential equations), even time-varying and non-linear ones (see [7]).

The aim of this presentation is to show how the simple fractional oscillators examined in [4] and the non-standard oscillators studied in [2] can also be analysed in the case of PDEs, thus generalizing the examples of damping introduced in [5] for the wave equation, or in [3] for the beam equation. The damped systems under consideration are:

$$\partial^2_{tt} X + \varepsilon_1\, \partial_t X + D_1(\partial_t X) + \varepsilon_2 A\, \partial_t X + D_2\,(A\, \partial_t X) + A\, X = 0 \tag{24.1}$$

where $D_i = D_{\mu_i} + \frac{d}{dt} D_{\nu_i}$ are *positive* PDOs (i.e. μ_i and ν_i are positive diffusive symbols, that characterize the PDO of symbol $\widehat{D}_i(s) = \int_0^{+\infty} \frac{\mu_i(\xi)+s\,\nu_i(\xi)}{s+\xi}\, d\xi$, for $\Re e(s) > 0$) and $\varepsilon_i \geq 0$.

24.2 Energy analysis

As suggested in [6, sec. 6], the case where A is a positive self-adjoint operator with continuous inverse, the following energy functional is introduced on an augmented state-space:

$$\begin{aligned} E(t) &= \frac{1}{2}\|\partial_t X\|^2_{L^2(\Omega)} + \frac{1}{2}\|A^{\frac{1}{2}} X\|^2_{L^2(\Omega)} + \frac{1}{2}\int_0^{+\infty} \|\varphi(.,\xi,t)\|^2_{L^2(\Omega)}\, d\xi + \\ &\quad \frac{1}{2}\int_0^{+\infty} \xi\, \|\psi(.,\xi,t)\|^2_{L^2(\Omega)}\, d\xi \end{aligned} \tag{24.2}$$

It helps prove the asymptotic internal stability of the original system, thanks to the dissipativity of the extended system, making use the balanced diffusive realisations of the PDOs D_i with state variables φ for the μ_i-parts and ψ for the ν_i-parts.

24.3 Spectral analysis

In the case where A is a Riesz spectral operator, as defined in [1, chap. 2, sec. 3], the solution X can be decomposed onto the Riesz basis $\{\varphi_n\}_{n\geq 0}$; a scalar product with the biorthogonal Riesz basis $\{\psi_n\}_{n\geq 0}$ leads to the following equation for the poles $s_n \in \mathbb{C}$:

$$s_n{}^2 + \left[\varepsilon_1 + \widehat{D_1}(s_n)\right] s_n + \lambda_n \left[\varepsilon_2 + \widehat{D_2}(s_n)\right] s_n + \lambda_n = 0 \tag{24.3}$$

where λ_n are the eigenvalues of A. In the case of § 24.2, the eigenvalues λ_n are real and strictly positive, allowing for a straightforward analysis of the location of the damped poles.

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25

H^∞-output feedback of infinite-dimensional systems via approximation

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Abstract

infinite-dimensional H^∞ disturbance-attenuation problem may be calculated by solving two Riccati equations. We approximate the original infinite-dimensional system by a sequence of finite-dimensional systems. The solutions to the corresponding finite-dimensional Riccati equations are shown to converge to the solution of the infinite-dimensional Riccati equations. Furthermore, the corresponding finite-dimensional controllers yield performance arbitrarily close to that obtained with the infinite-dimensional controller.

Keywords

infinite-dimensional systems, gap topology, H^∞, approximation, stability

25.1 Introduction

In this paper we discuss H^∞ control problems for the linear system in a separable Hilbert space X

$$\frac{d}{dt}x(t) = Ax(t) + B_1v(t) + B_2u(t), \quad x(0) = x_o \in X \tag{25.1}$$

$$y(t) = C_1x(t) + D_{12}u(t) \tag{25.2}$$

$$z(t) = C_2x(t) + D_{21}v(t)\,. \tag{25.3}$$

The linear closed operator A generates the C_0-semigroup $S(t)$ on X. Let W, U, Y and Z be separable Hilbert spaces. We assume that $B_1 \in \mathcal{L}(W, X)$, $B_2 \in \mathcal{L}(U, X)$, $C_1 \in \mathcal{L}(X, Y)$ and $C_2 \in \mathcal{L}(X, Z)$. Let G_{ij} be the transfer function with state-space realization (A, B_j, C_i, D_{ij}), and let H be the transfer function of a controller so that the closed loop is well-posed. The closed loop transfer function from uncontrolled input v to cost y is

$$\Delta(G, H) = G_{11} + G_{12}H\,(I - G_{22}H)^{-1}G_{21}.$$

This paper is concerned with the problem of constructing a stabilizing feedback controller H so that

$$\|\Delta(G, H)\|_\infty < \gamma. \tag{25.4}$$

Such problems arise in a variety of contexts; robust stabilization is one of the most important.

If the H^∞ disturbance-attenuation problem is solvable, then it can be solved by calculating the solutions to two algebraic Riccati equations. The infinite-dimensional Riccati equations can rarely be solved exactly. In the special case of full-information control only one Riccati equation needs to be solved. It was shown in [2] that in this case the sequence of solutions to the finite-dimensional Riccati equation converge to the solution of the infinite-dimensional Riccati equation. Furthermore, performance arbitrarily close to that obtained with infinite-dimensional state-feedback can be obtained using finite-dimensional state feedback.

These results are extended here to include output feedback. The assumptions on the approximation scheme are similar to those arising in the approximation theory for linear quadratic problems *e.g.* [5]. The gap topology used in [4, 5] is also used here. The resulting proofs are short. It is also proven that if the original system is stabilizable with attenuation γ, then so are the approximating systems.

The optimal attenuation problem for the infinite-dimensional system is to find $\hat{\gamma} = \inf \gamma$ where the infinum is calculated over all γ such that the problem is stabilizable with attenuation γ. Letting $\hat{\gamma}^N$ indicate the optimal attenuation for the corresponding approximating system, we have that $\lim_{N\longrightarrow\infty} \hat{\gamma}^N = \hat{\gamma}$.

25.2 Approximation framework

Let X^N be a finite-dimensional subspace of X and p^N be the orthogonal projection of X onto X^N. The space X^N is equipped with the induced norm from X. Consider a sequence of operators $A^N \in L(X^N, X^N)$, $B_i^N = p^N B_i$, $C_i^N =$ the restriction of C_i onto X^N, for $i = 1, 2$. The operator A^N can be extended to all of X by $A^N p^N x$.

(A1) For each $x \in X$, we have $e^{A^N t} p^N x \to S(t)x$, and $(e^{A^N t})^* p^N x \to S^*(t)x$, uniformly in t on bounded intervals.

(A2) (i) The family of pairs (A^N, B_2^N) is uniformly exponentially stabilizable, *i.e.*, there exists a uniformly bounded sequence of operators $K^N \in L(X^N, U)$ such that $\left|e^{(A^N - B_2^N K^N)t} p^N x\right|_X \le M_1 e^{-\omega_1 t} |x|_X$ for constants $M_1 \ge 1$ and $\omega_1 > 0$.

(ii) The family of pairs (A^N, C_1^N) is uniformly exponentially detectable, *i.e.*, there exists a uniformly bounded sequence of operators $F^N \in L(Y, X^N)$ such that $\left|e^{(A^N - F^N C_1^N)t} p^N x\right|_X \le M_2 e^{-\omega_2 t}$, for constants $M_2 \ge 1$ and $\omega_2 > 0$.

(iii) The family of pairs (A^N, B_1^N) is uniformly exponentially stabilizable (as in (A2i)).

(iv) The family of pairs (A^N, C_2^N) is uniformly exponentially detectable (as in (A2ii)).

(A3) (i) The disturbance operator B_1 is compact.

(ii) The input operator B_2 is compact.

(iii) The observation operator C_2 is compact.

25.3 Main results

We will assume throughout this paper that (A, B_1) and (A, B_2) are stabilizable and that (A, C_1) and (A, C_2) are detectable. These assumptions ensure that an internally stabilizing controller exists; and that internal and external stability are equivalent for the closed loop if the controller realization is stabilizable and detectable.

Theorem 25.3.1. *[1, 3] The system is stabilizable with attenuation $\gamma > 0$ if and only if the following two conditions are satisfied:*

1. *There exists a nonnegative self-adjoint operator Σ on X satisfying the Riccati equation*

$$A^*\Sigma + \Sigma A + \Sigma\left(\frac{1}{\gamma^2}B_1B_1^* - B_2B_2^*\right)\Sigma + C_1^*C_1 = 0 \tag{25.5}$$

such that $A + (\frac{1}{\gamma^2}B_1B_1^* - B_2B_2^*)\Sigma$ generates an exponentially stable semigroup on X.

2. *Define $\tilde{A} = A + \frac{1}{\gamma^2}B_1B_1^*\Sigma$ and $\hat{K} = B_2^*\Sigma$. There exists a nonnegative self-adjoint operator $\tilde{\Pi}$ on X satisfying the Riccati equation*

$$\tilde{A}\tilde{\Pi} + \tilde{\Pi}\tilde{A}^* + \tilde{\Pi}\left(\frac{1}{\gamma^2}\hat{K}^*\hat{K} - C_2^*C_2\right)\tilde{\Pi} + B_1B_1^* = 0 \tag{25.6}$$

such that $\tilde{A} + \tilde{\Pi}\left(\frac{1}{\gamma^2}\hat{K}^*\hat{K} - C_2^*C_2\right)$ generates an exponentially stable semigroup on X.

Moreover, if both conditions are satisfied, define $F = \tilde{\Pi}C_2^*$ and $A_c = A + \frac{1}{\gamma^2}B_1B_1^*\Sigma - B_2\hat{K} - \hat{F}C_2$. The controller with state-space description

$$\begin{aligned} \dot{x}_c(t) &= A_cx_c(t) + \hat{F}z(t) \\ u(t) &= -\hat{K}x_c(t) \end{aligned} \tag{25.7}$$

solves the H^∞ disturbance-attenuation problem.

Condition (2) above is more often written as the following two equivalent conditions: (a) There exists a nonnegative, self-adjoint operator Π on X satisfying the Riccati equation

$$A\Pi + \Pi A^* + \Pi\left(\frac{1}{\gamma^2}C_1^*C_1 - C_2^*C_2\right)\Pi + B_1B_1^* = 0 \tag{25.8}$$

such that $A + \Pi(\frac{1}{\gamma^2}C_1^*C_1 - C_2^*C_2)$ generates an exponentially stable semigroup on X, and (b) $r(\Pi\Sigma) < \gamma^2$.

Theorem 25.3.2. *Assume that (A1) to (A3) hold. If the original system is stabilizable with attenuation γ then the approximating systems are stabilizable with attenuation γ for all N sufficiently large.*

Suppose that the algebraic Riccati equations (25.5,25.6) with A replaced by A^N etc. have solutions $\Sigma^N$and $\tilde{\Pi}^N$ respectively. Define $\hat{K}^N = (B_2^N)^*\Sigma^N$, $\hat{F}^N = \tilde{\Pi}^N(C_2^N)^*$ and $A_c^N = A^N + \frac{1}{\gamma^2}B_1^N(B_1^N)^*\Sigma^N - B_2^N\hat{K}^N - \hat{F}^N C_2^N$. The corresponding finite-dimensional state-space based controller is

$$\begin{aligned} \dot{x}_c(t) &= A_c^N x_c(t) + \hat{F}^N z(t) \\ u(t) &= -\hat{K}^N x_c(t)\,. \end{aligned} \tag{25.9}$$

In [2] it was shown that, under the given assumptions, $\hat{K}^N$ converges uniformly to $\hat{K}$ and also that $A^N - B_2^N\hat{K}^N$ generate uniformly exponentially stable semigroups. At this point convergence of the solution Π^N to the filtering Riccati equation (25.8) will follow using a straightforward duality argument if C_1 is compact and assumptions (A1), (A2iii), (A2iv) and (A3iii) hold. However, since we only have strong convergence of $\Sigma^N \longrightarrow \Sigma$ and of $\Pi^N \longrightarrow \Pi$, convergence of $\tilde{\Pi}^N = (I - \frac{1}{\gamma^2}\Pi^N\Sigma^N)^{-1}\Pi^N$ is not implied and so we do not have controller convergence. Convergence of the solution $\tilde{\Pi}^N$ to the estimation Riccati equation (25.6) is shown, and this leads to controller convergence.

Theorem 25.3.3. *Let γ be such that the infinite-dimensional problem is solvable. Assume that assumptions (A1)-(A3) hold. Then the finite-dimensional controllers (25.9) converge in the gap topology to the infinite-dimensional controller (25.7). For sufficiently large N, the finite-dimensional controllers (25.9) stabilize the infinite-dimensional system and provide γ attenuation.*

Theorem 25.3.4. *Assume that (A1)-(A3) hold. Then $\lim_{N\longrightarrow\infty}\hat{\gamma}^N = \hat{\gamma}$.*

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26

Input–output gains for linear and nonlinear systems

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Abstract

Various notions of finite gain for linear and nonlinear systems are compared. Fuller details appear in [4].

Keywords

Input–output gains, bounded power, finite memory system

26.1 Main results

Let $1 \le p < \infty$. Define the norm

$$\|u\|_{Sp} = \left(\sup_{n \ge 0} \frac{1}{n+1} \sum_{t=0}^{n} |u(t)|^p \right)^{1/p}, \tag{26.1}$$

and the seminorm

$$\|u\|_{Bp} = \left(\limsup_{n \to \infty} \frac{1}{n+1} \sum_{t=0}^{n} |u(t)|^p \right)^{1/p}, \tag{26.2}$$

when they exist for the real (or complex) sequence $u = \{u(t)\}_{t\geq 0}$. Let S_p and B_p denote the linear vector spaces of all sequences u such that $\|u\|_{Sp} < \infty$ and $\|u\|_{Bp} < \infty$, respectively. These two sequence spaces are actually the same.

We consider first causal, linear, time-invariant convolution operators G:

$$y(t) = (Gu)(t) = \sum_{k\geq 0} g(k)u(t-k), \qquad t \geq 0, \tag{26.3}$$

where $\{g(k)\}_{k\geq 0}$ denotes the unit impulse response of G. The induced S_p, B_p and ℓ_p norms of G are denoted as $\|G\|_{i,Sp}$, $\|G\|_{i,Bp}$ and $\|G\|_{i,p}$, respectively. The following theorem provides a generalization of a result in [3].

Theorem 26.1.1. *Let $1 \leq p < \infty$. A causal, linear, time-invariant operator G satisfies $\|Gu\|_{Sp} \leq K\|u\|_{Sp}$ for any $u \in S_p$, for some constant $K > 0$ (independent of u), if and only if G is a bounded operator on ℓ_p. Moreover, the induced operator norm $\|G\|$ is the same when we consider G acting on S_p or ℓ_p.*

The following theorem generalizes material in [5]; even in the case $p = 2$ the result is stronger than those previously obtained (see [5, Remark 3.1]).

Theorem 26.1.2. *Let $1 \leq p < \infty$. A causal, linear, time-invariant operator G satisfies $\|Gu\|_{Bp} \leq K\|u\|_{Bp}$ for any $u \in B_p$, for some constant $K > 0$ (independent of u), if and only if G is a bounded operator on ℓ_p. Moreover, the induced operator norm $\|G\|$ is the same when we consider G acting on B_p or ℓ_p.*

Moving on to the nonlinear case, consider the induced norms $\|G\|_{i,p}$, for $1 \leq p \leq \infty$, defined by

$$\|G\|_{i,p} = \sup_{u\in\ell_p, u\neq 0} \frac{\|Gu\|_p}{\|u\|_p}. \tag{26.4}$$

It is well known that for linear time-invariant systems G we always have the inequality $\|G\|_{i,p} \leq \|G\|_{i,\infty}$. This holds also for a large class of nonlinear systems, the Hammerstein–Wiener systems (see [1]). These include systems incorporating saturators of the form $(\phi(u))(t) = \operatorname{sign} u(t) \min\{1, |u(t)|\}$. (see, for example, [2]). For finite memory systems the idea of finite gain does not depend on the choice of signal space.

Theorem 26.1.3. *Let G be a finite memory nonlinear system. Then for each p with $1 \leq p < \infty$ we have the inequality*

$$n^{-1/p}\|G\|_{i,\infty} \leq \|G\|_{i,p} \leq n^{1/p}\|G\|_{i,\infty},$$

where n is the tap length of the system. Thus finite ℓ_p gain and finite ℓ_∞ gain are equivalent in this case.

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27

A fractional representation approach of synthesis problems: an algebraic analysis point of view

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Abstract

Using an algebraic analysis point of view on the fractional representation approach of synthesis problems, we give the complete answers of two open questions on internal stabilization problems, namely a necessary and sufficient condition of internal stabilization and the characterization of the classes of the systems over which any plant is internally stabilizable. Finally, we give a sufficient condition so that all the stabilizing controllers of an internal stabilizable plant can be parametrized by means of the Youla parametrization.

Keywords

Fractional representation of synthesis problems, internal stabilization, parametrization of all the stabilizing controllers, doubly coprime factorizations, coherent rings and modules, $H_\infty(\mathbb{C}_+)$, Prüfer domains.

27.1 Main results

The *fractional representation approach of synthesis problems* [1, 4] is a general framework which was developed to formulate and solve different questions on feedback stabilization (e.g. internal/strong/simultaneous stabilizations, Youla parametrization of all the stabilizing controllers of a plant, left/right/doubly coprime factorizations, graph topology...). Despite the success of this approach (robust control of finite dimensional systems), it seems that the following questions are still open [4]:

1. Does it exist a necessary and sufficient condition of internal stabilization for a plant defined by a transfer matrix with entries in the quotient field $K = Q(A)$ of a domain A of SISO stable systems?

2. Is it possible to characterize the domains A of SISO stable systems satisfying that any plant − defined by a transfer matrix with entries in the field of fractions $K = Q(A)$ of A − is internally stabilizable?

3. Is it possible to characterize all the domains A of SISO stable systems satisfying that all the stabilizing controllers of a stabilizing plant can be parametrized by means of the Youla parametrization?

The purpose of this talk is to give following answers to these questions:

1. [2] A plant defined by the transfer matrix $T = P^{-1}\,Q \in K^{q\times(p-q)}$, with $P \in A^{p\times p}$ and $Q \in A^{q\times(p-q)}$, is internally stabilizable if and only if the A-module $M = A^p/A^q\,(P\;-\;Q)$ is such that $M/t(M)$ is a *projective* A-module, where $t(M) = \{m \in M \mid \exists\, 0 \neq a \in A : a\,m = 0\}$ is the torsion submodule of M [3].

   A torsion element $m \in t(M)$ corresponds to an unstable pole/zero simplification in the $P^{-1}\,Q$.

   This result gives an explicit form of a stabilizing controller C of T which can be effectively calculated if we can solve scalar Bézout identities, namely $\sum_{i=1}^{n} a_i\,x_i = 1$ ($a_i \in A$ given, $x_i \in A$), in the domain A of SISO stable systems.

2. [2] We shall prove the following equivalences:

   - any MIMO plant defined by the transfer matrix $T = P^{-1}\,Q \in K^{q\times(p-q)}$ − with $P \in A^{p\times p}$ and $Q \in A^{q\times(p-q)}$ − is internally stabilizable,
   - any SISO plant defined by $T = q/p \in K$, with $p \neq 0,\ q \in A$, is internally stabilizable, i.e. it satisfies

     $$(p,\,q)\,(p,\,q)^{-1} = (p\,:\,q) + (q\,:\,p) = A,$$

     where $(p,\,q)^{-1} = \{r \in K \mid r\,(p,\,q) \subset A\}$ is the *fractional ideal* associated with the ideal $(p,\,q)$ and $(p : q) = \{c \in A \mid c\,q \in (p)\}$ is an ideal of A [3],
   - A is a *Prüfer domain*, i.e. a domain which satisfies that any finitely generated ideal $I = (a_1, \ldots, a_n)$ is a projective A-module [3].

   Many examples of Prüfer domains exist in the literature of algebraic geometry (any non-singular surface defines a Prüfer affine domain), number theory (any integral closure of $\mathbb{Z}$ into a finite extension of $\mathbb{Q}$ is a Prüfer domain) and functions theory (the domain of entire functions/locally bounded rational functions, the meromorphic bounded Nash functions...).

   The previous result is in the same spirit as the one proved by M. Vidyasagar [4]: any MIMO plant defined by $T = P^{-1}\,Q \in K^{q\times(p-q)}$, − $P \in A^{p\times p}$ and $Q \in A^{q\times(p-q)}$ − has doubly coprime factorizations if and only if A is a *Bézout domain*, namely a domain which satisfies that any finitely generated ideal of A is defined by a unique element [3]. Therefore, if A is a Prüfer domain which is not a Bézout domain (any Bézout domain is a Prüfer domain), then there exist stabilizable plants which have no doubly coprime factorizations, i.e. it is not possible to parametrize their stabilizing controllers by means of the Youla parametrization. Hence, using the fact that generically a Prüfer domain A is not a Bézout one (*class group* $C(A)$), most of internally stabilizable plants generally fails to have doubly coprime factorizations, i.e. it is not possible to parametrize their stabilizing controllers by means of the Youla parametrization.

3. [2] If A is a *Hermite domain* − namely a domain which satisfies that any finitely generated projective A-module is free [3] (e.g. $RH_\infty, H_\infty(\mathbb{C}_+), E$ ...) − then it is possible to parametrize all the stabilizing controllers of a stabilizable plant by means of the Youla parametrization.

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Low gain tracking and disturbance rejection for stable well-posed systems

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Keywords

Tracking; Well-posed linear system; Internal model principle; Stability; Dynamic stabilization; Positive transfer function.

28.1 Abstract

We solve a tracking and disturbance rejection problem for stable well-posed linear systems, using a low gain controller suggested by the internal model principle. Our first result is a partial extension of Hämäläinen and Pohjolainen [1]. In [1], the plant is required to have an exponentially stable transfer function in the Callier-Desoer algebra, while in this paper we only require the transfer function of the plant to be exponentially stable and well-posed. The conditions for a transfer function to be well-posed are sufficiently unrestrictive to be verifiable for many partial differential equations in more than one space variable. In our second result, we assume that the transfer function for part of the plant is *positive*, and obtain results which do not require the gain to be small.

We assume that the plant Σ_p is a well-posed linear system (see, for instance Weiss [2]) and that it is exponentially stable. The plant has two inputs, w and u. The input w consists of the external signals (typically both references and disturbances) and u is the control input. These signals take values in the Hilbert spaces W and U, respectively. The output signal of Σ_p, denoted by z, which represents the tracking error, takes values in the Hilbert space Y. The transfer function of the plant is

$$\mathbf{P} = [\mathbf{P}_1 \;\; \mathbf{P}_2],$$

where $\mathbf{P}_1(s) \in \mathcal{L}(W, Y)$ and $\mathbf{P}_2(s) \in \mathcal{L}(U, Y)$. Let $\mathbf{C}$ be the transfer function of the well-posed controller Σ_c which is to be determined, where $\mathbf{C}(s) \in \mathcal{L}(Y, U)$. The connection between these systems and signals is shown in the figure.

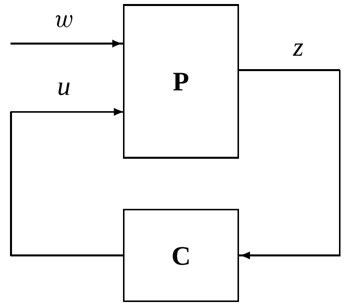

Figure 28.1: The closed loop system

Let J be a finite index set of integers. We assume that w is of the form

$$w(t) = \sum_{j\in J} \mathrm{w}_j e^{i\omega_j t}, \quad \mathrm{w}_j \in W, \quad \omega_j \in \mathbb{R}. \tag{28.1}$$

Thus, w is a superposition of constant and sinusoidal signals. The frequencies ω_j are assumed to be known (for design purposes), but the vectors w_j (which determine the amplitudes and the phases) are not known in advance.

For any Banach space Z, we denote by $H_a^\infty(Z)$ the space of bounded analytic Z-valued functions on $\mathbb{C}_a := \{\mathrm{Re}(z) > a\}$. The transfer function of a well-posed plant is in $H_a^\infty(\mathcal{L}(U, Y))$ for some $a \in \mathbb{R}$. For any $\alpha \in \mathbb{R}$, set

$$L_\alpha^2[0,\infty) = \left\{ f \in L_{loc}^2[0,\infty) \;\middle|\; \int_0^\infty e^{-2\alpha t}\|f(t)\|^2 \mathrm{d}t < \infty \right\}.$$

The corresponding space of Y-valued functions is denoted by $L_\alpha^2([0,\infty), Y)$. However, we write $L_\alpha^2[0,\infty)$ when the range space Y is clear from the context.

Our objective is to find a controller Σ_c so that the closed-loop system in Figure 28.1 is exponentially stable, and the output z (the tracking error) decays exponentially to zero, by which we mean that $z \in L_\alpha^2[0,\infty)$ for some $\alpha < 0$.

Theorem 28.1.1. *Let Σ_p satisfy the conditions described above. Let Σ_c be an optimizable and estimatable (see [3]) realization of a transfer function $\mathbf{C}$ of the form*

$$\mathbf{C}(s) = -\varepsilon\left(\mathbf{C}_0(s) + \sum_{j\in J}\frac{K_j}{s - i\omega_j}\right), \tag{28.2}$$

where $\mathbf{C}_0 \in H_\alpha^\infty(\mathcal{L}(Y, U))$ with $\alpha < 0$, $K_j \in \mathcal{L}(Y, U)$ and $\sigma(\mathbf{P}_2(i\omega_j)K_j) \subset \mathbb{C}_0$.

Then the feedback system in Figure 28.1 is exponentially stable for all sufficiently small $\varepsilon > 0$. If w is if the form (28.1), then $z \in L_a^2[0,\infty)$ for some $a < 0$.

We apply this result to a model for structure/acoustics interaction, showing that external noise at fixed positions in the cavity can be rejected.

Let $\mathbf{P}$ be an $\mathcal{L}(U)$-valued transfer function defined on (a set containing) the half-plane $\mathbb{C}_0$. We say that $\mathbf{P}$ is a *positive* transfer function if

$$\operatorname{Re}\mathbf{P}(s) := \frac{1}{2}\left[\mathbf{P}(s) + \mathbf{P}(s)^*\right] \geq 0 \quad \forall s \in \mathbb{C}_0.$$

When the second component of the plant transfer function (from control input to error) is positive, the following theorem states that certain simple controllers will stabilize the system in the sense of Theorem 28.1.1 and also achieve tracking. Moreover, in this situation, there is no need to adjust an unknown small gain.

Theorem 28.1.2. *Suppose that Σ_p is an exponentially stable well-posed linear system with transfer function $\mathbf{P} = [\mathbf{P}_1\ \mathbf{P}_2]$, where $\mathbf{P}_1(s) \in \mathcal{L}(W, U)$, $\mathbf{P}_2(s) \in \mathcal{L}(U)$, $\mathbf{P}_2$ is a positive transfer function, and $\operatorname{Re}\mathbf{P}_2(i\omega_j)$ is invertible for all $j \in J$. Let Σ_c be an optimizable and estimatable realization of a transfer function $\mathbf{C}$ of the form*

$$\mathbf{C}(s) = -\left(\mathbf{C}_0(s) + \sum_{j\in J} \frac{K_j}{s - i\omega_j}\right), \tag{28.3}$$

where $K_j \in \mathcal{L}(U)$, $K_j \geq 0$, $\mathbf{C}_0 \in H^\infty_\alpha(\mathcal{L}(U))$ with $\alpha < 0$ and

$$\operatorname{Re}\mathbf{C}_0(s) \geq \frac{1}{2}I \quad \forall s \in \mathbb{C}_0.$$

Then the feedback system in Figure 28.1 is exponentially stable. If w is if the form (28.1), then $z \in L^2_a[0,\infty)$ for some $a < 0$.

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29

Conditions for time-controllability of behaviours

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Abstract

We study systems whose dynamics are described by systems of linear constant coefficient partial differential equations in a behavioural framework. Questions about the notion of controllability are addressed with special importance given to the time-evolution. Algebraic conditions concerning time-controllability are obtained.

Keywords

Behaviours, time-controllability, partial differential equations.

29.1 Introduction

We study autonomy and controllability of dynamical systems described by linear constant coefficient partial differential equations in the behavioural theory of Willems (see Willems [1]). Traditionally, behaviours arising from systems of partial differential equations are studied in a general setting in which the time-axis does not play a distinguished role in the formulation of the definitions pertinent to control theory (see for example, Pillai and Shankar [2].) However, it is reasonable to suggest that in the study of systems with "dynamics" arising from (engineering) applications, it is useful to give special importance to the time variable in defining system theoretic concepts. This also highlights the similarities with the definitions in the case of $1-\mathrm{D}$ dynamical systems. (For an excellent elementary introduction to the behavioural theory in the $1-\mathrm{D}$ case, we refer to Polderman and Willems [3].) Here we study time-controllability of the so-called "separated-variable dynamical systems". A *separated-variable dynamical system* Σ is defined as a triple $(\mathbb{R}, C^\infty(\mathbb{R}^{\mathtt{m}}, \mathbb{C}^{\mathtt{w}}), \mathfrak{B})$, where $C^\infty(\mathbb{R}^{\mathtt{m}}, \mathbb{C}^{\mathtt{w}})$ is called the *signal space*, and $\mathfrak{B}$ is a subset of $C^\infty(\mathbb{R}, C^\infty(\mathbb{R}^{\mathtt{m}}, \mathbb{C}^{\mathtt{w}}))$, where $C^\infty(\mathbb{R}, C^\infty(\mathbb{R}^{\mathtt{m}}, \mathbb{C}^{\mathtt{w}}))$ denotes the set of all functions $w : \mathbb{R} \to C^\infty(\mathbb{R}^{\mathtt{m}}, \mathbb{C}^{\mathtt{w}})$ such that the associated map $\Omega_w(t, x) := (w(t))(x),\ t \in \mathbb{R},\ x \in \mathbb{R}^{\mathtt{m}}$, is an element in $C^\infty\left(\mathbb{R}^{\mathtt{m}+1}, \mathbb{C}^{\mathtt{w}}\right)$. The set $\mathfrak{B}$ is called the *behaviour of* Σ. A separated-variable dynamical system Σ is said to be described by a *kernel representation* given by $R \in \mathbb{C}^{\mathtt{g}\times\mathtt{w}}[\xi, \eta_1, \ldots, \eta_{\mathtt{m}}]$ if

$$\mathfrak{B} = \left\{ w \in C^{\infty}\left(\mathbb{R}, C^{\infty}\left(\mathbb{R}^{\mathtt{m}}, \mathbb{C}^{\mathtt{w}}\right)\right) \mid R\left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x_1}, \ldots, \frac{\partial}{\partial x_{\mathtt{m}}}\right)\Omega_w = 0 \right\}.$$

In this case we say that $\mathfrak{B}$ is the behaviour corresponding to R. The behaviour $\mathfrak{B}$ of a separated-variable dynamical system is said to be *time-controllable* if for any w_1 and w_2 in $\mathfrak{B}$, there exists a $w \in \mathfrak{B}$ and a $\tau > 0$ such that

$$w(t) = \begin{cases} w_1(t) & t \leq 0 \\ w_2(t-\tau) & t \geq \tau \end{cases}.$$

Let us denote the ring $\mathbb{C}[\xi, \eta_1, \ldots, \eta_{\mathtt{m}}]$ by A.

Given a polynomial matrix $R \in \mathbb{C}^{\mathtt{g}\times\mathtt{w}}[\xi, \eta_1, \ldots, \eta_{\mathtt{m}}]$, with each entry in A, we can consider each row of R as an element of the free module $A^{\mathtt{w}}$. We denote the submodule of $A^{\mathtt{w}}$ generated by the rows of R by $\langle R \rangle$. The following are our main results.

Theorem 29.1.1. (Sufficient condition.) *Let* $R \in \mathbb{C}^{\mathtt{g}\times\mathtt{w}}[\xi, \eta_1, \ldots, \eta_{\mathtt{m}}]$. *If the* A*-module* $A^{\mathtt{w}}/\langle R \rangle$ *is torsion free, then the behaviour* $\mathfrak{B}$ *of the separated-variable dynamical system corresponding to* R *is time-controllable.*

Example 29.1.2. This condition is not necessary for time-controllability: for example, consider the behaviour corresponding to $\eta \in \mathbb{C}[\xi, \eta]$ (thus $\mathtt{m} = \mathtt{g} = \mathtt{w} = 1$). It can be shown that this behaviour is time-controllable, but the $\mathbb{C}[\xi, \eta]-$module $\mathbb{C}[\xi, \eta]/\langle \eta \rangle$ is not torsion free.

Theorem 29.1.3. (Necessary condition.) *Let* $R \in \mathbb{C}^{\mathtt{g}\times\mathtt{w}}[\xi, \eta_1, \ldots, \eta_{\mathtt{m}}]$. *The behaviour* $\mathfrak{B}$ *of the separated-variable dynamical system corresponding to* R *is time-controllable only if the* $\mathbb{C}[\xi]$*-module* $A^{\mathtt{w}}/\langle R \rangle$ *is torsion free.* [1]

Example 29.1.4. This condition is not sufficient for time-controllability: for example, consider the behaviour corresponding to $\xi + \eta \in \mathbb{C}[\xi, \eta]$ (thus $\mathtt{m} = \mathtt{g} = \mathtt{w} = 1$). It can be shown that this behaviour is not time-controllable, but the $\mathbb{C}[\xi]-$module $\mathbb{C}[\xi, \eta]/\langle \xi + \eta \rangle$ is torsion free.

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[1] $A^{\mathtt{w}}/\langle R \rangle$ is a $A-$module. Since $\mathbb{C}[\xi]$ is a subring of A, we can consider $A^{\mathtt{w}}/\langle R \rangle$ as a $\mathbb{C}[\xi]-$module: we simply restrict scalar multiplication to elements belonging to $\mathbb{C}[\xi]$ instead of the whole ring A.

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On determination of strongly stabilizing controls

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Abstract

Necessary and sufficient conditions for the strong stability of a control system with a skew-adjoint operator in a Hilbert space are presented.

Keywords

Strong stabilizing control, Skew-adjoint operator

30.1 Main results

The strong stabilizability of contractive systems is an extensively studied subject in systems theory (see, e.g., [1, 2] and references therein). We consider a system in a Hilbert space H:

$$\dot{x}(t) = Ax(t) + bu, \qquad b \in H, \qquad t \geq 0, \tag{30.1}$$

under the assuptions:

i) A is a skew-adjoint unbounded densely defined operator with a compact inverse (we denote its eigenvalues $\{\lambda_k\}_{k=1}^{\infty}$ and eigenfunctions $\{\phi_k\}_{k=1}^{\infty}$);

ii) there exists a constant $C_\sigma \equiv \frac{1}{2} \min\limits_{i \neq j} |\lambda_i - \lambda_j| > 0$;

iii) the vector $b \in H$ satisfies $\langle b, \phi_n \rangle \neq 0$, $n \in \mathbf{N}$.

Our goal is to examine for feedback controls $u = \langle x, q \rangle \equiv q^* x$, $q \in H$ the property to be stabilizing. The first step is to analyse the necessary condition for the stability of (30.1) which is presented in

Theorem 30.1.1. *Let the system $\dot{x} = (A + bq^*)x$ be stable and vectors b and q be such that $||b|| \cdot ||q|| < C_\sigma/2$ or $||b|| + ||q|| \le \sqrt{C_\sigma}$. Then there exist a Hilbert norm $\langle F\cdot, \cdot\rangle$ and a compact operator $W_1 \ge 0$ such that for any solution $x(t)$ of the system one has*

$$\frac{d}{dt}\langle Fx(t), x(t)\rangle = -\langle W_1 x(t), x(t)\rangle,$$

here operator $F > 0$ is bounded and invertible. Moreover, W_1 is a nonnegative kernal operator.

Now we give sufficient conditions for the stability of (30.1).

Consider a feedback control $u(x) = -b^*x + p^*x$, i.e. $u(x) = \langle x, -b + p\rangle$. Then (30.1) takes the following form

$$\dot{x}(t) = (A - bb^* + bp^*)x \equiv (\widetilde{A} + bp^*)x, \qquad b \in H, \qquad t \ge 0, \tag{30.2}$$

where $\widetilde{A} \equiv A - bb^*$.

Consider arbitrary finite or infinite orthonormal system $\{\omega_i\}_{i=1}^N \subset H$ and $\{\mu_i\}_{i=1}^N \subset \ell_1;\ \mu_i \ge 0,\ N \le \infty$. Define a compact operator $W_0 = \sum_{i=1}^N \mu_i \omega_i \omega_i^* \ge 0$. For any positive δ and stable operator $\widetilde{A} \equiv A - bb^*$ we can consider Lyapunov equation with the right hand side $-W_0$:

$$[D + (1 - \delta/2)I]\widetilde{A} + \widetilde{A}^*[D + (1 - \delta/2)I] = -W_0. \tag{30.3}$$

and its unique operator solution $D + (1 - \delta/2)I \ge 0$.

We denote by $\lambda_\pm$ and $x_\pm$ the eigenvalues and eigenvectors of the two-dimensional selfadjoint operator $R_2 \equiv (I + D)bp^* + pb^*(I + D)$. They are given by $\lambda_\pm = \langle (I + D)b,\ p\rangle \pm \|(I + D)b\| \cdot \|p\|, \qquad \lambda_+ \ge 0,\ \lambda_- \le 0,$

$$x_\pm = (I + D)b\|p\| \pm p\|(I + D)b\|, \qquad \langle x_+, x_-\rangle = 0. \tag{30.4}$$

Theorem 30.1.2. *Let the vectors b and p be such that $||b|| \cdot ||q|| < C_\sigma/2$ or $||b|| + ||q|| \le \sqrt{C_\sigma}$. Then for any vector p for which there exist finite or infinite orthonormal system $\{\omega_i\}_{i=1}^{N(p)} \subset H$ and $\{\mu_i\}_{i=1}^{N(p)} \subset \ell_1;\ \mu_i \ge 0$ such that*

$$x_+ \in Span\{\omega_i\}_{i=1}^{N(p)} \qquad \text{and} \qquad \lambda_+ |\langle x_+, \omega_i\rangle| < \mu_i ||x_+||, \quad i = 1, \ldots, N(p),$$

the system $\dot{x} = Ax + bu$ is asymptotically stabilizable with the aid of the control $u(x) = \langle x, -b + p\rangle$. Here the vector x_+ is defined (4).

Remark 30.1.3. Let us note that under the conditions of Theorem 30.1.2 on vectors p and b we in fact show (c.f. Theorem 30.1.1), that for any solution of the system

$$\frac{d}{dt}\langle Fx(t), x(t)\rangle = -\langle W_1 x(t), x(t)\rangle,$$

where W_1 is a nonnegative kernal operator.

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Reduced order modeling and control of thin film growth in an HPCVD reactor

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Keywords

Chemical vapor deposition, reduced order model, nonlinear feedback tracking control, nonlinear compensator, state-dependent Riccati equations.

31.1 Introduction

Chemical vapor deposition (CVD) is a technique used to grow very thin films with certain desired properties, involving the deposition of source vapors onto a heated substrate surface where they then react chemically to form the desired material. This process is used in the manufacture of many computer hardware products, including high-speed (GaAs) integrated circuits, transistors, and DRAM chips, as well as UV detectors and green and blue light emitting diodes. Precise control of the film layer thickness and composition is extremely important, and the increasing demands on the precision of the desired properties make real-time feedback control of the CVD process very desirable.

Low-pressure chemical vapor deposition processes are the preferred choice for manufacturing many of the devices mentioned above. Previous work within our N.C. State University research group has successfully implemented feedback control of film thickness and composition in $GaP/Ga_{1-x}In_xP$ films, during experiments in a low-pressure pulsed chemical beam epitaxy (PCBE) reactor using real-time optical monitoring by *p*-polarized reflectance spectroscopy (PRS) measurements.

However, there are some materials (such as InN or $Ga_{1-x}In_xN$ films) which have potential industrial uses, but cannot be effectively produced at desirable temperatures under low-pressure conditions. Extending the CVD procedure to higher pressures increases our ability to control the thermal decomposition of certain source gases, and expands the range of compositions which can be produced at optimal process temperatures. This has applications to flat panel displays covering the entire visible wavelength range, and optoelectronics in the visible to UV wavelength range, as well as radiation-resistant high power electronics. In addition, higher pressures give the advantage of a fuller ability to intentionally introduce controlled defects into the film or dope the film with impurities (for example, to give the film a positive charge, in the case of the speaker application). Control of defect chemistry/residual absorption and laser damage of nonlinear optical materials (such as $ZnGeP_2$) is also important for wave-guided nonlinear optical sensors and advanced optical parametric oscillators. Higher pressures can also result in faster film deposition and throughput, an advantage in time-intensive applications in the semiconductor industry. The challenge in high-pressure chemical vapor deposition (HPCVD) is that it is significantly more difficult to control than the low-pressure process, as the higher pressure introduces source vapor gas flow dynamics in the place of low-pressure ballistic source vapor pulses.

As part of a research team at N.C. State working on the design and construction of an HPCVD reactor with real-time sensors to use in feedback control of the film growth process, we have worked to create an effective mathematical model of the more complicated high-pressure deposition process. We also have developed closed-loop control methods to use on the nonlinear model, including estimation of the system state from the sensor measurements and tracking of desired properties such as film thickness and composition.

One part of the HPCVD process is the gas flow dynamics in the high-pressure reactor, as the source vapors travel from the reactor inlet to the substrate surface in a carrier gas at pressures of up to 100 atmospheres. A dilute approximation is used in our work, leading to a quasi-steady model with steady-state nonlinear continuity, momentum and energy equations being decoupled from the transient linear species equations. We use the reduced order method known as proper orthogonal decomposition (POD) to obtain a reduced order system from the species equations, so that real-time model-based feedback control is possible.

The second part of the CVD model is the description of the surface kinetics, including the decomposition of source vapors deposited on the surface and their reactions forming the compound which is integrated into the growing film. These reactions are represented by a reduced order surface kinetics (ROSK) model which assumes that among the many reactions involved there are a small number of significant limiting steps. The ROSK model is then coupled to the gas phase model through the flux of species to the surface.

The combined model is nonlinear due to the reactions on the surface, so a nonlinear feedback control method must be used. To control the growing film thickness we use a nonlinear tracking control methodology, which is based on the state-dependent Riccati equation (SDRE). In addition, only nonlinear partial measurements of the growth process are available. Therefore a state estimator/compensator is developed to reconstruct from these measurements an estimated value of the full state on which to base the feedback control. In this talk, we describe these methods in the context of the HPCVD tracking control problem. In addition, simulation results are given and analyzed, to study the effectiveness of the reduced order POD and ROSK models, the reduced order model-based nonlinear tracking control, and the state estimation process using nonlinear partial observations of the actual state. Finally, some preliminary theoretical results will be presented on the state-dependent Riccati equation based nonlinear feedback tracking control.

32

Optimal location of the actuator in some pointwise stabilization problems

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Abstract

We study the large time behavior of the solutions of a homogenous string equation with a homogenous Dirichlet boundary condition at the left end and a homogenous Neuman boundary condition at the right end. A pointwise interior actuator gives a linear viscous damping term. We give a complete characterization of the positions of the actuator for which the system becomes exponentially stable in the energy space. Moreover we show that the fastest decay rate is obtained if the actuator is located at the middle point of the string. Possible extensions to beam systems are then discussed.

Keywords

stabilization,beams

32.1 Introduction

Consider the following initial and boundary value problem :

$$\frac{\partial^2 w}{\partial t^2}(x,t) - \frac{\partial^2 w}{\partial x^2}(x,t) + \frac{\partial w}{\partial t}(\xi,t)\,\delta_\xi = 0,\ 0 < x < \pi,\ t > 0, \tag{32.1}$$

$$w(0,t) = \frac{\partial w}{\partial x}(\pi,t) = 0,\ t > 0, \tag{32.2}$$

$$w(x,0) = w^0(x),\ \frac{\partial w}{\partial t}(x,0) = w^1(x),\ 0 < x < \pi. \tag{32.3}$$

Here above we suppose that the string is of length π and we denote by δ_ξ the Dirac mass concentrated in the point $\xi \in (0, \pi)$.

If w is a solution of (32.1)-(32.3) we define the energy of w at instant t by

$$E(w(t)) = \frac{1}{2}\int_0^\pi \left(\left|\frac{\partial w}{\partial t}(x,t)\right|^2 + \left|\frac{\partial w}{\partial x}(x,t)\right|^2 \right) dx. \tag{32.4}$$

Simple formal calculations show that a sufficiently smooth solution of (32.1)-(32.3) satisfies the energy estimate

$$E(w(0)) - E(w(t)) = \int_0^t \left[\frac{\partial w}{\partial t}(\xi, s)\right]^2 ds, \quad \forall t \geq 0. \tag{32.5}$$

In the next section we describe precisely the behavior of the solutions of (32.1)-(32.3) when $t \to \infty$ and we study the dependence of this behavior on the parameter ξ (the location of the actuator).

32.2 Main results

In order to state the result on the optimal location of the actuator, we define the decay rate, depending on ξ, as

$$\begin{aligned} \omega(\xi) \quad = \quad & \inf\{\omega \mid \text{ there exists } C = C(\omega) > 0 \text{ such that} \\ & E(w(t)) \leq C(\omega)e^{2\omega t}E(w(0)), \text{ for every solution of (32.1)-(32.3)} \\ & \text{with initial data in } V \times L^2(0,\pi)\}, \end{aligned} \tag{32.6}$$

where $E(w(t))$ is defined in (32.4). It can be easily checked that $\omega(\xi) \leq 0$ for all $\xi \in (0, \pi)$. Our first main result, on the optimal location of the actuator, is

Theorem 32.2.1.

1. *The inequality $\omega(\xi) < 0$ holds true if and only if $\xi \in (0, \pi)$ admits a coprime factorization*

$$\frac{\xi}{\pi} = \frac{p}{q} \text{ with } p \text{ odd}. \tag{32.7}$$

   In other words all finite energy solutions of (32.1)-(32.3) *are exponentially stable if and only if ξ satisfies* 32.7.

2. $\omega(\frac{\pi}{2}) = -\frac{1}{2\pi}\ln(3)$ *and $\omega(\xi) > \omega(\frac{\pi}{2})$ for any $\xi \in (0,\pi) \setminus \frac{\pi}{2}$. In other words the fastest decay rate of the solutions of* 32.1-32.3 *is obtained if the actuator is located at the middle of the string.*

The problem of finding the optimal decay rate for strings with distributed interior damping is difficult and has not a complete answer in the case of a variable (in space) damping coefficient. We refer to [1], [2] and to references therein.

Finally we consider a similar problem for the Bernoulli-Euler beam in the presence of one or two pointwise feedbacks.

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On optimal measurement locations for parameter estimation in distributed systems

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Abstract

The problem of locating pointwise sensor measurements so as to optimally estimate unknown parameters in a class of distributed systems is studied. Based on a scalar measure of performance defined on the corresponding Fisher information matrix, three approaches are delineated and compared for this problem: introduction of continuous designs, clusterization-free designs, and application of standard non-linear programming techniques.

Keywords

Sensor Location, Parameter Estimation, Optimum Experimental Design

33.1 Introduction

In this note we wish to focus on locating discrete pointwise sensors so as to estimate unknown parameters in the underlying DPS models as accurately as possible. This is an appealing problem since in most applications sensor locations are not pre-specified and therefore provide design parameters. The importance of sensor planning has already been recognized in many application domains, e.g. in optimization of air quality monitoring networks. The problem was attacked from various angles (cf. [5, 6, 2] for reviews), but few works have appeared about the results regarding two- or three-dimensional spatial domains, spatially-varying parameters, as well as the dependence of the solutions on the values assumed for the unknown parameters to be identified. The main purpose here is to show how some well-known methods which have been successful in akin fields of optimum experimental design can be extended to the setting of the sensor location problem so as to overcome the above-mentioned limitations.

33.2 Main results

We assume that measurements at the sensors are available continuously in time, and the design criterion is the minimization of a scalar measure of the Fisher Information Matrix related to the identified parameters. In the same spirit as in the classical optimum experimental design theory for lumped systems [1, 3, 7] and the approach delineated in [4], we introduce first the notion of continuous designs. Their advantage lies in the fact that the problem dimensionality is dramatically reduced. Moreover, with some minor changes, sequential numerical design algorithms, which have been continually refined since the early 1960s, can be employed here. Unfortunately, this approach does not prevent sensors from clustering which is a rather undesirable phenomenon in potential applications. Clusterization is a consequence of the assumption that the measurement noise is spatially uncorrelated. This means that in an optimal solution different sensors often tend to take measurements at the same point, which is most often unacceptable from the technical point of view.

Alternatively, we may seek to find an optimal design, not within the class of all designs, but rather in a restricted subset of competing clusterization-free designs. To implement this idea, some recent advances in spatial statistics are employed, and in particular Fedorov's idea of directly constrained design measures [1] is adapted to our framework. As a consequence, this leads to a very efficient and particularly simple exchange-type algorithm. Bear in mind, however, that this approach should in principle be used if the number of sensors is relatively high. If this is not the case, we can resort to standard optimization routines which ensure that the constraints on the design measure and region are satisfied (in particular, adaptive random search coupled with a sequential constrained quadratic programming method can be used).

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Spectral factorization by symmetric extraction for semigroup state-space systems

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Abstract

The spectral factorization problem of a scalar coercive spectral density is considered in the framework of the Callier-Desoer algebra of distributed parameter system transfer functions. Criteria for the convergence of the symmetric extraction method solving this problem are described and commented for semigroup Hilbert state-space systems with a Riesz-spectral generator.

Keywords

Distributed parameter systems, spectral factorization, coercivity, symmetric extraction, convergence analysis, Riesz-spectral systems.

The spectral factorization problem plays a central role in feedback control system design, see e.g. [11], [8] and the references therein; in particular, it constitutes an essential step in the solution of the Linear-Quadratic optimal control problem for infinite-dimensional state-space systems, see e.g. [2], [4], [7], [9], [12]. This contribution is devoted to the description and the convergence analysis of the symmetric extraction method for the spectral factorization of a coprime fraction (coercive) spectral density. The analysis is performed in the framework of C_0-semigroup Hilbert state-space systems, whose infinitesimal generator is a Riesz-spectral operator, with eigenvalues satisfying some asymptotic conditions (see [10], [5]), and with transfer function in the Callier-Desoer algebra (see e.g. [3],[6]). Criteria for the elementary (rational) factor infinite product representation of a coercive spectral density and for the convergence of such spectral factorization procedure are reported. These criteria are based on the knowledge of the comparative asymptotic behavior of the spectral density poles and zeros, i.e. basically on the pole-zero absolute and relative errors. The proofs of the main results are based on standard results from the theory of entire functions like the Weierstrass factorization theorem (see e.g. [1], [13] and the references therein). Our present investigations are oriented towards the analysis and the numerical simulation of the method on some typical examples like the damped vibrating string model (see e.g. [6]). It turns out that the convergence conditions hold for such model, as they do for the heat diffusion equation, implying therefore the convergence of the symmetric extraction method for these examples.

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Eigenvalues and eigenvectors of infinite-dimensional closed-loop systems

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Abstract

We study eigenvalues and eigenvectors of the infinite-dimensional closed-loop system obtained after an admissible output feedback for a well-posed linear system. Using controllability we propose some simple sufficient conditions to guarantee that the closed-loop system is spectral when the open-loop is. Many feedback-stabilized vibration systems can be dealt with by our proposed method.

Keywords

Infinite-dimensional system, C_0 semigroup, Riesz bases, eigenvectors.

35.1 Introduction and main results

We consider static output feedback control of infinite-dimensional well-posed linear systems. The aim of our work is to investigate the relationship between the eigenvalues of an open-loop system and the controlled system, and to give a useful characterization of the eigenvalues of the controlled system in terms of the available data on the open-loop system. Especially, we are interested to know when the eigenvectors of the closed-loop system form a Riesz basis and we obtain some results in this direction.

We illustrate the usefulness of our results through several examples in the stabilization of systems described by partial differential equations. For these systems we show that some sequence of generalized eigenvectors of the closed-loop system form a Riesz basis in the state space. Our approach leads to a unified treatment for different cases which have been studied separately in the literature. Our approach enables us to simplify or avoid long computations which were necessary otherwise. The exact controllability (or observability) needed to be verified in our approach can be checked using the multiplier method or the Carleson measure criterion.

For example we consider the following coupled Euler-Bernoulli beams with the stabilizing feedback $-\kappa\, w_t(d,t)$ acting at the junction point $d \in (0,1)$:

$$\begin{cases} w_{tt}(x,t) + w_{xxxx}(x,t) = 0, \quad x \in (0,d) \cup (d,1), \\ w(0,t) = w_{xx}(0,t) = 0, \;\; w_x(1,t) = w_{xxx}(1,t) = 0, \\ w(d^+,t) = w(d^-,t), \;\; w_x(d^+,t) = w_x(d^-,t), \\ w_{xx}(d^+,t) = w_{xx}(d^-,t), \\ w_{xxx}(d^+,t) - w_{xxx}(d^-,t) = -\kappa\, w_t(d,t) + v(t). \end{cases}$$

Using our approach we prove that if d is a rational number such that p is odd in the coprime factorization $d = p/q$, then, for any $\kappa > 0$ with $v = 0$, some sequence of generalized eigenvectors of the stabilized system form a Riesz basis in its state space. Hence the spectral bound determines the exponential decay rate of the trajectories for the stabilized system.

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36

Control using delay elements

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Abstract

It is well known that the presence of delays in a control system is often detrimental: They can cause instability, lack of robustness, and they increase the complexity of control problems. The idea of this talk is that time delays, if reasonably used, may be good for control. We give an overview of the uses of delay elements in controllers.

Keywords

Time delay control, repetitive control, stability, input shaping.

36.1 Introduction

Delay systems form the simplest, but perhaps the most widely applied, class of distributed parameter systems. It is well known that the presence of delays makes the system analysis and control design much more complex. However, in recent years, some researchers have explored the use of delay elements in control system design. They have shown that, if reasonably used, time delays can be useful for control. In this talk we present some of these approaches.

Some of the properties that controllers including delays may have are:

* infinitely many poles and/or zeros
* finite impulse response
* behavior similar to time variant systems

36.2 Applications of delay in control

Input shaping techniques

Input shaping is a feed-forward control technique for reducing vibrations in systems. The method works by creating a command signal that cancels its own vibration. From the frequency domain point of view, this uses some of the infinitely many zeros in a filter with time delay to cancel the damped oscillating poles of the process. If they are exactly cancelled, the vibration is eliminated entirely.

This technique has been applied to coordinate measuring machines, cranes etc. Some experiments have been done in the space shuttle.

Repetitive control

Repetitive control uses a periodic signal generator implemented with a delay element to track periodic commands or to reject periodic disturbances. The periodic signal generator has infinitely many poles $\pm j\frac{2\pi}{L}l$ $(l = 0, 1, 2, \cdots)$ on the $j\omega$-axis (L is the delay used) . Hence, it can be used as the internal model to track or reject any periodic signals with period L since these can be decomposed into a Fourier series with frequencies $\frac{l}{L}$ $(l = 0, 1, 2, \cdots)$.

This approach has been originally applied to a proton synchrotron magnet power supply and later to industrial robots, CD players, PWM inverters etc.

Time delay observer

K. Youcef-Toumi presented an approach to observe the uncertainties and/or disturbances of a plant by using a delay. This approach assumes that, in a sufficiently small time interval, a continuous signal remains almost unchanged. This approach has good performance when there exist uncertainties and disturbances of a certain structure.

It has been applied to servo systems, robots, cranes etc. It is very interesting that this approach results in a similar structure as repetitive control.

Some other applications

Nobuyama, Watanabe and others used delays in controllers to achieve deadbeat tracking for continuous time systems. One or more delay elements are used in these controllers.

K.Pyragas and others presented a delayed feedback control law to stabilize chaotic systems. This method has been successfully applied to non-autonomous as well as autonomous electronic chaos oscillators and to a laser system.

N. Olgac and others presented a delayed resonator to suppress the undesirable vibration in bridges, machines, aircraft, etc. He uses delayed feedback to destabilize the system just enough so that it becomes a perfect resonator in the steady state. Hence, the vibration is suppressed perfectly. (Of course, all the vibrations will not disappear in reality.)

In general, it seems that delay elements should be used in a feed-forward path or in a positive feedback loop, but not in a negative feedback loop.

Theme of the workshop

Distributed parameter systems (dps) is an established area of research in control which can trace its roots back to the sixties. While the general aims are the same as for lumped parameter systems, to adequately describe the distributed nature of the system one needs to use partial differential equation models. The modelling issue is in itself nontrivial, especially when there is boundary control action and sensing on the boundary. Controllability and observability concepts are subtle and investigating these for a single p.d.e. example leads to a sophisticated mathematical problem. The action of controlling the system introduces feedback into the p.d.e. model which results in a more complicated mathematical model; the resulting closed-loop system may not be well-posed and this issue has only recently become well understood. At this stage, the mathematical machinery for formulating the basic control problems is available (although not so well known), and this has led to a wealth of new system theoretic results for dps.

If this theory is to be applied, it needs to be tested by numerical simulations of feedback connections of p.d.e. systems, which requires another area of mathematical expertise. Over the past decades considerable experience has been acquired in numerical modelling, simulation and control of dps for various applications. In particular, much work has been done on the numerical implementation of LQG (and minimax) algorithms to various classes of p.d.e. systems. This involves an analytical study of approximation of solutions of operator Riccati equations, which is reasonably well understood. These approximations lead to a finite-dimensional controller which is designed to stabilize a finite-dimensional approximation of the p.d.e. model. If, however, the controller is to stabilize the original system and not just a simulation of the p.d.e. model, it needs to be robust. Various theories for robust controllers have been proposed, but many open questions remain. More recently, another practical issue, sampled data-control has been addressed. New technology has introduced new control paradigms. In particular, the advent of smart materials for sensors and actuators and micro electro-mechanical actuators and sensors has introduced challenging new modelling and control problems for distributed parameter systems.

Due to the mathematical sophistication of even simply formulated control problems for distributed parameter systems there has been an increasing tendency to specialize on one particular aspect of control, for example

- exact controllability of linear p.d.e's
- abstract systems theory of linear infinite-dimensional linear systems
- p.d.e. modelling of smart actuators and sensors
- H_∞ problems and relations to operator theory
- stability of nonlinear p.d.e.'s

- model reduction
- compensator design for dps
- spatially distributed systems
- experimental results on computer simulations of dps
- proper orthogonal decomposition (POD) in reduced order controller design
- experimentation of controlling physical dps in real time
- control of specific dps, such as fluid flow

to mention just a few.

Unfortunately this increasing specialization leads to ignorance of existing expertise in other specializations which could be very appropriate for the problem at hand. The aim of this workshop is perhaps unusual: it is to bring together scientists who are all studying distributed parameter systems, but from different points of view and possessing different types of expertise. In this way, we hope to make scientists aware of new developments in this fast expanding field of research and to promote cross-fertilization of ideas across artificial boundaries. We hope this will open up new directions for future research.