

Average Case Analysis of the MST-heuristic for the Power Assignment Problem: Special Cases

Maurits de Graaf
Thales Nederland B.V., University of Twente
P.O. Box 217
7500 AE Enschede, Netherlands
M.deGraaf@utwente.nl

Richard J. Boucherie, Johann L. Hurink,
and Jan-Kees van Ommeren
University of Twente
P.O. Box 217
7500 AE Enschede, Netherlands
{R.J.Boucherie, J.L.Hurink,
J.C.W.vanOmmeren}@utwente.nl

ABSTRACT

We present an average case analysis of the minimum spanning tree heuristic for the power assignment problem. The worst-case approximation ratio of this heuristic is 2. We have the following results: (a) In the one-dimensional case, with uniform $[0, 1]$ -distributed distances, the expected approximation ratio is bounded above by $2 - 2/(p + 2)$, where p denotes the distance power gradient. (b) For the complete graph, with uniform $[0, 1]$ distributed edge weights, the expected approximation ratio is bounded above by $2 - 1/2\zeta(3)$, where ζ denotes the Riemann zeta function.

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General Terms

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1. INTRODUCTION

Ad hoc wireless networks have received significant attention due to their potential applications (see, for example [10]). In such a network, communication takes place either through single-hop transmission, or by relaying through intermediate nodes. The topology (the set of communication links) depends on uncontrollable factors (node mobility) and on controllable parameters (transmit power). We assume an idealized propagation model, with omnidirectional transceivers with adjustable transmit power. For assigning transmit powers, two conflicting effects have to be taken into account: if transmit powers are too low, the resulting topology may be too sparse. On the other extreme, if transmit

powers are too high, the nodes run out of energy quickly.

The goal of the Connected Minimum Power Assignment (CMPA-) problem is to assign transmit powers such that the resulting network is connected and the sum of transmit powers is minimized (see e.g. [10]). This problem is, in general, NP-hard (for some special cases there are polynomial solutions). The intuitive MST-heuristic is known to have a worst-case approximation ratio of 2. This paper analyses the average-case approximation ratio. A directional variant of this problem, the Minimum Energy Broadcast Routing (MEBR) problem is studied in [3]. A related numerical study is carried out in [11]. The results of this work can be used in assessing whether, in a concrete network, a given power assignment can be further optimized.

1.1 Notation, related work and contribution

For a set of points V representing the nodes in a network, a power assignment can be represented as a function $\mathbf{p} : V \rightarrow \mathbb{R}_0^+$. Following the notation of [10], for each ordered pair (u, v) of transceivers, there is a transmit power threshold, denoted by $c(u, v)$, with the following meaning: a signal transmitted by the transceiver u can be received by v only when the transmit power $\mathbf{p}(u)$ is at least $c(u, v)$. We assume that for each pair of points the values $c(u, v)$ are known and symmetric, i.e., $c(u, v) = c(v, u)$ for all pairs $\{u, v\} \in V$. A power assignment \mathbf{p} defines an undirected graph $G_{\mathbf{p}} = (V, E_{\mathbf{p}})$, where $e = \{u, v\} \in E_{\mathbf{p}}$ if and only if $\mathbf{p}(u) \geq c(u, v)$ and $\mathbf{p}(v) \geq c(u, v)$. Note that in the case $\mathbf{p}(v) \geq c(u, v) > \mathbf{p}(u)$ only transmission from v to u is possible. However, in this paper only symmetric links are considered.

This paper deals with the CMPA-problem: given a graph $G = (V, E, c)$, where c denotes the edge weights $c : E \rightarrow \mathbb{R}^+$, one asks for a power assignment $\mathbf{p} : V \rightarrow \mathbb{R}_0^+$ such that $G_{\mathbf{p}}$ is connected and the total power $\sum_{v \in V} \mathbf{p}(v)$ is minimal.

When $V \subset \mathbb{R}^d$, a power attenuation model is assumed, assuming that the signal power decreases with the distance r as r^{-p} , where the *distance-power gradient* $p \in \mathbb{R}^+$ depends on the wireless environment. This implies that $c(u, v) = r^p$ if the distance between u and v is r . Therefore, in this case, the power assignment problem corresponds to assigning a range r_v to node v . This is called the *range assignment* problem.

The range assignment problem is NP-hard in all dimensions $d \geq 2$ for all values of the distance-power gradient p [5]. Based on these complexity results, polynomial time approximation algorithms were studied. The first approximation

algorithm to the CMPA-problem is the Minimum Spanning Tree (MST)-algorithm (see [4], [7]). This work complements [8] and [9], in providing a rigorous proof for the general case with uniform edge weights (Section 4 of this paper), and in analysing the (more complex) MST-heuristic instead of the MST-functional.

MST Power Assignment Algorithm (V, E, c) : (1) Compute a minimum spanning tree T using c as edge costs; (2) For each node $v \in V$ assign

$$\mathbf{p}(v) = \max\{c(e) \mid e \text{ incident to } v \text{ in } T\}.$$

Let T_n denote a minimum spanning tree of a graph on n vertices. In addition, let P_n denote the power assignment corresponding to T_n , i.e., for each $v \in V$: $P_n(v) = \max\{c(e) \mid e \in T_n \text{ and } e \text{ incident to } v\}$. We define $W(T_n)$ to be the total weight of the spanning tree T_n , and $W(P_n)$ the total power of the corresponding power assignment. It is well established (see e.g. [1], [4]) that

$$W(T_n) \leq W(P) \leq W(P_n) \leq 2W(T_n) \quad (1)$$

where $W(P)$ denotes the total power of the optimal power assignment P . In [1] it is shown that the factor 2 is tight.

For the MST algorithm, (1) shows that the worst-case performance ratio is 2. Other approximation algorithms are studied in [1], where a polynomial time approximation scheme with a worst-case performance ratio approaching 5/3 as well as a more practical approximation algorithm with a worst-case approximation factor of 11/6 are given.

While the worst-case performance ratio of 2 might discourage use of the MST algorithm in practice, numerical results indicate that the MST algorithm is often rather close to the optimal solution [12]. This motivates an average case analysis.

Statement of contribution. This paper presents an analysis of the function $W(P_n)/W(T_n)$, in the average case, for $n \rightarrow \infty$, which provides a general upper bound to the average case performance ratio $W(P_n)/W(P)$. Here, we investigate the 1-dimensional situation, as a stepping stone for similar analysis in higher dimensions. Then, we investigate the situation for the complete graph with independent uniform edge weights, where the expected approximation ratio turns out to have a closed form upper bound well below the worst-case upper bound.

The paper is organized as follows. Section 2 provides preliminary results. Section 3 analyzes the 1-dimensional case. Section 4 shows a result on the average case performance for complete graphs with independent uniformly distributed $[0, 1]$ edge weights. Finally, Section 5 presents concluding remarks.

2. PRELIMINARIES ON MINIMUM SPANNING TREES

Let $G = (V, E)$ be a graph with $|V| = n$ and a cost function $c : E \rightarrow \mathbb{R}^+$. Furthermore, for a vertex v , let $G \setminus \{v\}$ denote the graph arising from G by deleting v and all edges incident to v , for an edge e , $G \setminus \{e\}$ denotes the graph arising from G by deleting edge e . Suppose $F = (V, E_F)$ is a forest on G (i.e., a graph with no cycles) with $E_F = \{e_1, \dots, e_m\} \subset E$. We assume $c(e_1) \leq c(e_2) \leq \dots \leq c(e_m)$ and let $S_k(F) = \sum_{j=m-k+1}^m c(e_j)$ denote the sum of the k heaviest edges of F , for $k \in \{1, \dots, m\}$. If $m = n - 1$, then

F is a spanning tree, which we denote by T . For a given tree T , we say that an edge e incident to v covers v , if (a) $e \in T$, i.e., $e = e_i$ for some $i \in \{1, \dots, n - 1\}$, and (b) the index i is maximal among the edges e incident to v . Note that condition (b) ensures that each vertex is covered by exactly one edge and that $c(e_i) \geq c(e_j)$ for all edges $e_j \in T$ incident to v .

Let T_n be a minimum spanning tree and let $f(e)$ denote the number of nodes covered by $e \in T_n$, called the covering number of $e \in E$. Note that $f(e) \in \{0, 1, 2\}$. We immediately see that $\sum_{e \in E} f(e) = n$ as each vertex is covered exactly once. Moreover, $W(P_n) = \sum_{e \in T_n} f(e)c(e)$.

The following observation strengthens (1).

LEMMA 1. *Let the edges e_1, \dots, e_{n-1} of a minimum spanning tree T_n be sorted such that $c(e_1) \leq \dots \leq c(e_{n-1})$. Then*

$$c(e_{n-1}) + S_{n-1}(T_n) \leq W(P_n) \quad (2)$$

and,

$$W(P_n) \leq \begin{cases} c(e_{\lfloor n/2 \rfloor}) + 2S_{\lfloor n/2 \rfloor}(T_n) & \text{if } n \text{ is odd,} \\ 2S_{n/2}(T_n) & \text{if } n \text{ is even.} \end{cases} \quad (3)$$

PROOF. Inequality (2) can be inferred by induction on n , using the fact that each tree contains at least 2 vertices of degree 1. The inequalities of (3) follow from the fact that $f(e) \in \{0, 1, 2\}$ and $\sum_{e \in E} f(e) = n$. Therefore $W(P_n) = \sum_{i=1}^{n-1} f(e_i)c(e_i)$ takes its maximum, when f takes maximum values for the edges with the highest weights. \square

The following example shows that the bounds for the inequalities (2) and (3) are tight.

EXAMPLE 1. *Let $G = (V, E)$ be a path e_1, \dots, e_n such that $c(e_j) = 1$ if j is odd, and $c(e_j) = \epsilon < 1$ if j is even. G has only one spanning tree T which is equal to the graph itself: $T = G$. Sorting the edges according to increasing costs we first obtain $\lceil (n-1)/2 \rceil$ edges of cost ϵ , followed by $\lfloor (n-1)/2 \rfloor$ edges of cost 1. Moreover, $W(T) = \lceil (n-1)/2 \rceil + \lfloor (n-1)/2 \rfloor \epsilon$. Clearly, all edges with an odd index have covering number 2, and, if n is odd, say $n = 2m + 1$, there is only one edge (being e_{2m}) with covering number 1, incident to the last vertex. So $W(P_T) = 2m + \epsilon$, which is a tight bound for the right hand side of (3). If n is even, say $n = 2m$ then $W(P_T) = 2m$, which is a tight bound for the right hand side of (3). An example for tightness of the left hand side of (3) is obtained by considering a graph $G = (V, E)$ where all costs $c(e)$ are 1. In this case $W(T) = n - 1$ and $W(P_T) = n$. \square*

Lemma 1 implies,

$$W(P_n) \leq 2S_{\lceil n/2 \rceil}(T_n). \quad (4)$$

3. ONE DIMENSION: SPANNING TREE IS PATH

We consider the situation where $G = (V, E)$ is a path of length n , $X_1 \leq \dots \leq X_{n+1} \in \mathbb{R}^1$ where the transmit power thresholds $D_i = X_{i+1} - X_i$ to connect neighboring vertices are i.i.d. nonnegative random variables with finite expectation.

THEOREM 1. *Let $G = (V, E)$ be a path as defined above. Then*

$$\frac{W(P_n)}{W(T_n)} \xrightarrow{a.s.} \frac{\mathbb{E}[\max\{D_1, D_2\}]}{\mathbb{E}[D_1]}. \quad (5)$$

PROOF. First we split the sum in odd and even terms:

$$\begin{aligned} W(P_n) &= D_1 + \sum_{i=1}^{n-1} \max\{D_i, D_{i+1}\} + D_n \quad (6) \\ &= D_1 + \sum_{i=1}^{\lfloor n/2 \rfloor} \max\{D_{2i-1}, D_{2i}\} \\ &\quad + \sum_{i=1}^{\lfloor (n-1)/2 \rfloor} \max\{D_{2i}, D_{2i+1}\} + D_n. \end{aligned}$$

Since, by splitting the odd and even terms, in both sums the random variables are i.i.d., it follows by the strong law of large numbers, that

$$\frac{W(P_n)}{n} \xrightarrow{a.s.} \mathbb{E}[\max\{D_1, D_2\}].$$

Being the sum of i.i.d. r.v.'s, $W(T_n)$ also satisfies the strong law of large numbers, and we obtain:

$$\frac{W(T_n)}{n} \xrightarrow{a.s.} \mathbb{E}[D_1],$$

implying almost sure convergence of $\frac{W(P_n)}{W(T_n)}$. \square

Since, $W(P_n)$ and $W(T_n)$ are sequences of r.v.'s, with bounded ratio, the above theorem implies that these ratios also converge in mean and that $\mathbb{E}[W(P_n)/W(T_n)]$ converges. So we have:

COROLLARY 1.

$$\lim_{n \rightarrow \infty} \mathbb{E} \left[\frac{W(P_n)}{W(T_n)} \right] = \frac{\mathbb{E}[\max\{D_1, D_2\}]}{\mathbb{E}[D_1]}$$

Next, we consider the specific situation of n vertices in \mathbb{R}^1 with transmit power threshold $D_i \sim U_i^p$, with $U_i \sim U[0, 1]$, where $U[0, 1]$ denotes the uniform distribution, and p models the distance-power gradient.

COROLLARY 2. *If in the situation of Theorem 1, we have for $i = 1, \dots, n$, $D_i \sim U_i^p$, with $U_i \sim U[0, 1]$ then*

$$\frac{W(P_n)}{W(T_n)} \xrightarrow{a.s.} 2 - \frac{2}{p+2}.$$

PROOF. We have: $\mathbb{E}[D_1] = \frac{1}{p+1}$, and $\mathbb{E}[\max\{D_1, D_2\}] = \frac{2}{p+2}$. \square

4. COMPLETE GRAPH, UNIFORM WEIGHTS

Often there is no simple relation between power and distance, due to obstacles, reflections and interferences. In the extreme case, the power needed for a successful transmission is fully unrelated to the node positions. In this section, we consider a stylized version of this situation, namely the case where we have a complete graph $G = (V, E, \tilde{X})$ with uniformly distributed edge weights. So $V = \{1, 2, \dots, n\}$ and $\tilde{X} = \{X_{ij} : 1 \leq i < j \leq n\}$ is a sample of size $\binom{n}{2}$ with $X_{ij} \sim U[0, 1]$, denoting the edge weights.

As before, we denote with $T_n = T_n(G)$ a minimum spanning tree of G . Let $Y_n = Y_n(G)$ denote the set of $\lfloor n/2 \rfloor$ most expensive, and $U_n = U_n(G)$ the set of $\lfloor n/2 \rfloor$ least expensive edges of T_n , and $W(Y_n)$, $W(U_n)$ the weight of Y_n , U_n . Note that $W(Y_n) = S_{\lfloor n/2 \rfloor}(T_n)$.

In order to analyze $W(Y_n)$ and $W(U_n)$, we follow the exposition of Frieze's result (Theorem 2) of [2]. Let $X_{(1)} \leq$

$X_{(2)} \leq \dots \leq X_{\binom{n}{2}}$ denote the order statistics of the sample $\tilde{X} = (X_{ij})$. With probability 1 we have $X_{(1)} < X_{(2)} < \dots < X_{\binom{n}{2}}$, implying uniqueness of the minimum spanning tree T_n . Now $\tilde{X} = (X_{ij})$ defines a graph process G_t in a natural way, in which edges are added over time, where the edge set of G_t , $0 \leq t \leq \binom{n}{2}$, is given by the t least expensive edges of G :

$$\{\{i, j\} : X_{ij} = X_{(k)} \text{ for some } k \leq t\}.$$

Given G_t , define the r.v. $\psi(\cdot)$ as:

$$\psi(k) = \min\{t : w(G_t) = n - k\}, k = 1, 2, \dots, n-1, \quad (7)$$

where $w(G_t)$ denotes the number of components of G_t . Then $\psi(1) = 1$, $\psi(2) = 2$ and $\psi(n-1)$ is the first time t when the graph G_t is connected. Clearly $\psi(k) \geq k$. By the greedy algorithm of Kruskal, T_n is formed by the edges of G_t appearing at times $\psi(1), \psi(2), \dots, \psi(n-1)$, i.e.,

$$W(T_n) = \sum_{k=1}^{n-1} X_{(\psi(k))}.$$

By a theorem of Frieze [6] we have:

THEOREM 2 ([6]). *Let $G = (V, E, \tilde{X})$ be a complete graph, with uniformly distributed edge weights. Then the weight of the minimum spanning tree T_n satisfies,*

$$W(T_n) \xrightarrow{P} \zeta(3), \quad (8)$$

where $\zeta(\cdot)$ denotes the Riemann zeta function.

With $W(U_n) = \sum_{k=1}^{\lfloor n/2 \rfloor} X_{(\psi(k))}$, we show:

LEMMA 2. *Let $G = (V, E, \tilde{X})$ be a complete graph, with uniformly distributed edge weights. Then :*

$$W(U_n) \xrightarrow{P} 1/4. \quad (9)$$

PROOF. The idea behind the proof of statement (9) is that, for large n and $1 \leq k \leq n/2$, $\psi(k)$ is 'close' to k . In other words: the least expensive edges of large graphs with uniformly distributed edge weights do not contain a circuit. First we show $\lim_{n \rightarrow \infty} \mathbb{E}[W(U_n)] = 1/4$, and then we show convergence in probability. Clearly, as $\mathbb{E}[X_{(i)}] = i / \binom{n}{2} + 1$, we have

$$\mathbb{E}[W(U_n)] = \sum_{k=1}^{\lfloor n/2 \rfloor} \mathbb{E}[\psi(k)] / \left(\binom{n}{2} + 1 \right).$$

Since $\psi(k) \geq k$, one easily finds $\mathbb{E}[W(U_n)] \geq 1/4$. We will show equality.

First we note:

$$\mathbb{P}(k \leq \psi(k) \leq k + n^{8/9}) = 1 - o(n^{-1/4}). \quad (10)$$

We show (10) by combining the inequality $\psi(k) \geq k$ with (6.18) in [2] which states that, with probability $1 - o(n^{-1/4})$,

$$\psi_0(k) - n^{8/9} \leq \psi(k) \leq \psi_0(k) + n^{8/9}, \text{ for } k = 1, \dots, \lfloor n/2 \rfloor, \quad (11)$$

where $\psi_0(k)$ is defined by:

$$u\left(\frac{2\psi_0(k)}{n}\right) = 1 - \frac{k}{n}, \quad (12)$$

where

$$u(x) = \sum_{k=1}^{\infty} \frac{k^{k-2}}{k!} x^{k-1} e^{-kx}.$$

Now, from the fact that u is a one-to-one mapping from \mathbb{R}^+ to $(0, 1]$, and (see [2], page 109) the fact that for $0 \leq x \leq 1$: $u(x) = 1 - x/2$ it follows that: $\psi_0(k) = k$, for $0 \leq k \leq n/2$, which shows (10).

Next, from (10) and (6.14) in [2] which states, $\mathbb{P}(\psi(n-1) \leq 2n \log n) = 1 - O(n^{-3})$, it follows that

$$\begin{aligned} k \leq \mathbb{E}[\psi(k)] &\leq (k + n^{8/9})\mathbb{P}(k \leq \psi(k) \leq k + n^{8/9}) \\ &+ 2n \log n \mathbb{P}(k + n^{8/9} \leq \psi(k) \leq 2n \log n) \\ &+ \binom{n}{2} \mathbb{P}(2n \log n \leq \psi(k) \leq \binom{n}{2}), \end{aligned}$$

which implies $k \leq \mathbb{E}[\psi(k)] \leq k + n^{8/9} + o(n^{3/4} \log n) + O(n^{-1})$, whence,

$$\lim_{n \rightarrow \infty} \mathbb{E}[W(U_n)] = \lim_{n \rightarrow \infty} \frac{1}{\binom{n}{2} + 1} \sum_{k=1}^{\lfloor n/2 \rfloor} \mathbb{E}[\psi(k)] = \frac{1}{4}, \quad (13)$$

completing the first step of the proof. Next, to see convergence in probability, we remark that in [6] equation (15) it is shown that for $1 \leq k \leq \ell \leq n/2$:

$$\mathbb{E}[X_{\psi(k)} X_{\psi(\ell)}] = \frac{\mathbb{E}[\psi(k)\psi(\ell)] + \mathbb{E}[\psi(k)]}{\binom{n}{2} + 1 + \binom{n}{2} + 2},$$

so in order to show that $\text{Var}[W(U_n)] \rightarrow 0$ it suffices to show that

$$\sum_{k=1}^{\lfloor n/2 \rfloor} \sum_{\ell=1}^{\lfloor n/2 \rfloor} \mathbb{E}[\psi(k)\psi(\ell)] \leq (1 + o(1)) \sum_{k=1}^{\lfloor n/2 \rfloor} \sum_{\ell=1}^{\lfloor n/2 \rfloor} \mathbb{E}[\psi(k)]\mathbb{E}[\psi(\ell)]$$

This follows from [6] equation (17) which implies that for some constant c , and $1 \leq k \leq \ell \leq n/2$:

$$\mathbb{E}[\psi(k)\psi(\ell)] \leq \mathbb{E}[\psi(k)]\mathbb{E}[\psi(\ell)] + cn^{11/6}(\log n)^2$$

Now, we obtain (9) from Chebyshev's inequality. \square

This leads to the following theorem for power assignments.

THEOREM 3. *Let $G = (V, E, \tilde{X})$ be a complete graph, with uniformly distributed edge weights. Then,*

$$\limsup_{n \rightarrow \infty} \mathbb{E} \left[\frac{W(P_n)}{W(T_n)} \right] \leq \frac{2(\zeta(3) - 1/4)}{\zeta(3)} \approx 1.58... \quad (14)$$

PROOF. As $W(T_n) \leq W(P_n) \leq 2W(Y_n)$, and $W(T_n) = W(U_n) + W(Y_n)$, it follows from Theorem 2 and Lemma 2 that $\zeta(3) \leq \limsup_{n \rightarrow \infty} \mathbb{E}[W(P_n)] \leq 2(\zeta(3) - 1/4)$. \square

5. CONCLUDING REMARKS

Worst-case, the approximation of the MST heuristic for the power assignment problem can be strengthened to two times the weight of the heavier half of the edges (instead of two times the whole MST). Moreover, in the one-dimensional case, with uniform distributed distances between neighboring vertices, and a distance power gradient of p , the expected approximation ratio is bounded above by $2 - 2/(p+2)$. For the complete graph with uniform $[0, 1]$ -distributed edge weights, the expected approximation ratio is asymptotically bounded above by $2 - 1/2\zeta(3) \approx 1.58...$. Heuristics as presented in [1] are interesting for further analysis.

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