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Teaching and Learning Problem Solving in Science

Part I: A general strategy

The Problem

Anyone teaching science, both at secondary and tertiary level, is familiar with the problem that the authors have been working on these last years. This problem is that students have to learn to *work with* the concepts, laws and formulas that have been taught in chemistry, physics, and technology. These students need to learn to use these concepts, laws, and formulas in solving problems.

Students usually work a number of problems in assigned exercises, but as a rule, only a few students do this well. Many students do not know how to start. They turn over the pages of the textbook to find a suitable formula, or just wait for the teacher to give a hint or the complete solution. Many often use a trial and error method and hope that this will build up sufficient routine to enable them to pass the exam by sheer recognition of a familiar set problems.

The authors find this approach to solving problems undesirable, because it does not help students learn to understand the subject and to apply the subject matter in new contexts. After the course the students are not able to approach new problems in a systematic way. Therefore, we have specifically focused our attention on developing a systematic approach to solving problems and on designing instruction where students learn this approach.

Overview

In this paper, Part I of the series, we will specify the systematic approach to problem solving that we have developed. Part II (to follow in next month's issue) will discuss how we teach students this systematic approach. Our plan of instruction is based on a learning theory which applies Gal'perin's theory of stage by stage formation of mental actions in combination with the contributions of Talyzina and Landa. The theoretical background of this approach has been described previously (*1*).

Our research and development work originated in a course in thermodynamics at Twente University of Technology. Some evaluation data of experimental courses will be given.

The systematic approach to problem solving has been formulated in very general terms and is—with minor adaptations—applicable to problem solving in many fields of science and technology. The plan of instruction developed for thermodynamics appears to be a good model for instruction in other fields as well. At Twente University of Technology, courses which apply the same approach in electricity and magnetism have been field tested with good results (*2*). Projects for adaptation of the model to mechanics and materials science, to a chemistry course in the vocational training of laboratory technicians, and to the training of laboratory students have also been initiated (*3, 4*).

Many authors have made valuable contributions describing problem solving in chemistry and chemical engineering (*5, 6, 7, 8*). Most of these publications, however, mention only one aspect of problem solving and lack a theoretical basis to their experiments. We have tried to make use of the findings of others and to avoid these shortcomings.

A System of Heuristics for Problem Solving in Thermodynamics

A Program of Actions and Methods

Teachers regularly give students directions regarding the way they should do the problems: e.g., read the problem carefully, check the answer. (Hereafter, we will call this type of directions *heuristics*.) Heuristics increase the chances of finding the solution to the problem but give no guarantee of reaching that solution. These directions usually are concerned with only a small part of the problem-solving process. We believe that offering a few random heuristics here and there is not sufficient; the heuristics offered should form a *system of heuristics*.

Polya developed a well-known set of heuristics for problem solving in mathematics (*12*). In science, however, we found that Polya's set of heuristics is unsatisfactory. We also consider the reasoning by analogy of related problems too risky; this method offers too little chance of success. Hence, we developed a *Program of Actions and Methods* (PAM) for solving those problems in science that require the specification of the situation (e.g., the calculation of a specific temperature, work, force, equilibrium-constant, concentration). In a systematic way, this program lists the actions and methods that should be executed in solving those so-called specification problems. As mentioned in the introduction, PAM is formulated in very general terms and, therefore, can be applied with minor adaptations in other fields of science and technology.

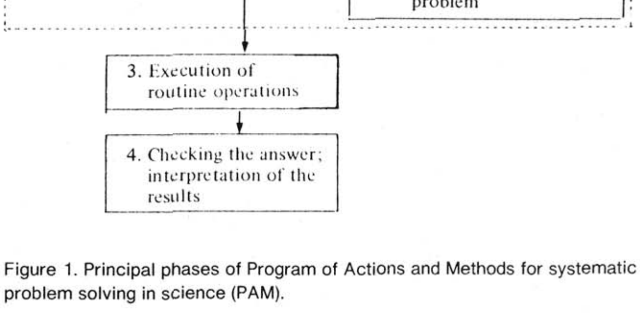

Figure 1. Principal phases of Program of Actions and Methods for systematic problem solving in science (PAM).

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The following gives a description of the final version of PAM. The way PAM was developed, tested, and improved we described elsewhere (*1, 10*).

Description of PAM

Four principal phases can be distinguished when problems in thermodynamics and related fields are being systematically solved:

Phase 1: Reading the problem thoroughly; careful analysis of the data and the unknown by making a scheme.
Phase 2: Establishing whether it is a standard problem, i.e., a problem that can be solved by mere routine operations; if not: Looking for relations between the data and the unknown that can be of use in the transformation of the problem to a standard problem; conversion of the problem to a standard problem.
Phase 3: Execution of routine operations.
Phase 4: Checking the answer, interpretation of the results.

Figure 1 shows these principal phases combined in a block diagram. These principal phases will now be presented in more detail. For every phase we first mention its purpose, and then list a number of desired actions. We only list the actions that can be expressed in general terms. For different fields, different specifications of the actions are needed.

Phase 1: Analysis of the Problem

Purpose: Getting an overall picture of the data and the unknown. The problem solver should first understand the problem well before he starts to solve it.

Desired actions:

1.1. Reading the problem carefully, e.g. by putting a slant line after every datum.
1.2. Transformation of the text of the problem into a scheme, using paper and pencil to develop an image of the problem situation and to get a schematic survey of the data and unknown.
All data should be mentioned in the scheme, in correct symbols and units (e.g. boundaries and characteristics of the system, variables of state, angles, characteristics of processes).
In some cases, plotting or sketching a graph may help to get a better image of the problem situation.
1.3. Writing down the unknown in symbols, if possible.
1.4. Estimation of the answer: probable sign, magnitude, dimension, special cases. An estimation facilitates checking the answer later on.

Phase 2: Transformation of the Problem

Purpose: Conversion of the problem to a standard problem by linking the unknown and the data with given relations between quantities.

Desired actions:

As you have seen in the block scheme of PAM (Fig. 1), this phase is divided into 3 parts. These will be discussed here.

2a. Establishing whether the problem is a standard problem
If so, the problem solver can go on to phase 3.
If not, the next step is 2b.
2b. Writing down possibly useful relationships
2b.1. Splitting up the problem (if necessary) into sub problems; choice of the first sub problem to solve (e.g. the easiest or one where results are expected that can be used later).
2b.2. Writing down possibly useful relationships from the following sources (taking the unknown and/or data as the starting point):
  a. Charts with key relationships for this subject. Key relationships contain the very core of the subject matter in a formulation which makes them a good starting point for solving problems. (Part II will discuss this in more detail and given an example.
  b. Charts with relationships for other fields (e.g. mathematics, prerequisite subjects).
  c. Relationships which follow from data, directly and indirectly.
  d. Relationships which the problem solver at this stage only can indicate in general terms (e.g. in a rubber band the length (L) is a function of the force (K) and the temperature (T): $F(K,L,T) = 0$).
2b.3. Checking the relationships found for their validity in this problem situation.
2c. Conversion of the problem to a standard problem
2c.1. Trying to interrelate unknown and data by applying the relationships to the problem situation and by linking them. This can be done in many ways, but experience shows that using the unknown as the starting point gives a better chance for a successful solution to the problem. When this is done, chances of transformations that are irrelevant or come to a dead end are less than when the data are used as a starting point. In a block diagram (Fig. 2) the strategy for transformation to a standard problem, using the unknown as the starting point, is summarized.
If the strategy is not used (e.g., because the problem solver sees a route, starting from the data), the actions d and f are necessary.
2c.2. If it is not possible to arrive at a standard problem by the actions in 2c.1., the following actions might be tried:
  a. Trying to simplify the problem, e.g., by solving it for an infinitesimally small change, after which integration might be justified.
  b. Trying to restate the problem or to consider it from a different point of view (e.g. larger or smaller scale; setting up the analysis of the problem in a different way).
  c. Trying to solve an analogous problem in a different field; this might generate ideas about how to solve this problem.
  d. Letting the problem rest for some time; difficult problems generally are not solved in one go.

Phase 3: The Execution of Routine Operations

Purpose: To work out the solution that has been found in the preceding phase.

Desired actions:

3.1. Writing down the routine operations and the answer in a well-organized way. Many unnecessary mistakes in this phase are caused by sloppiness with signs, powers, and units.
3.2. Checking very frequently whether all signs, powers, and units are taken along, and whether the results still make sense.

The execution of these routine operations is much simpler than the transformation of the problem.
The execution, however, requires patience and precision. It is usually advisable to put off calculations until the end, that is to leave all results in the form of formulas and to fill in the values only at the end.

Phase 4: Checking the Answer and Interpretation of the Results

Purpose: To check if the problem has been solved correctly and completely. By looking at the answer and retracing the way

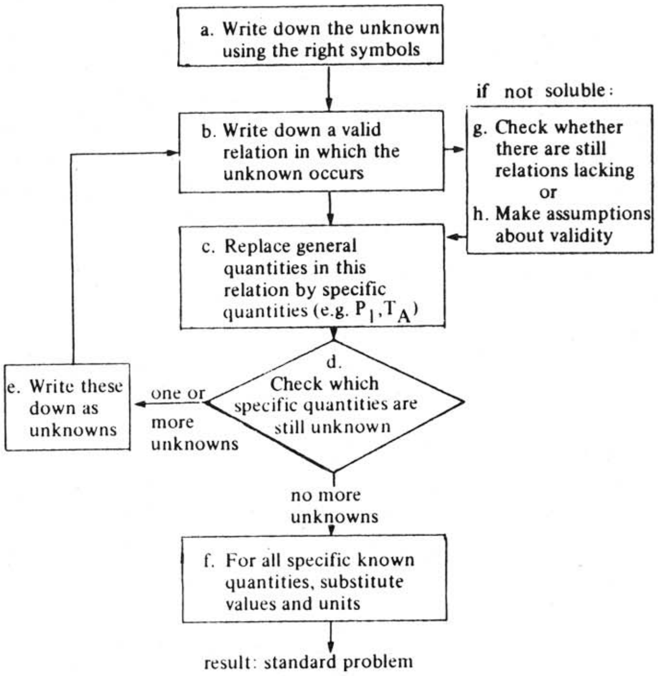


Figure 2. Diagram of strategy: transformation, using the unknown as point of departure.

Starting from a concentrated HCl solution/, 5.000 liter/of a dilute/0.100 M/solution has to be prepared. The concentrated acid has a density of 1.13 g/ml;/it contains 25.5 % (by weight) HCl/. How many milliliters/of the concentrated HCl solution/are needed?

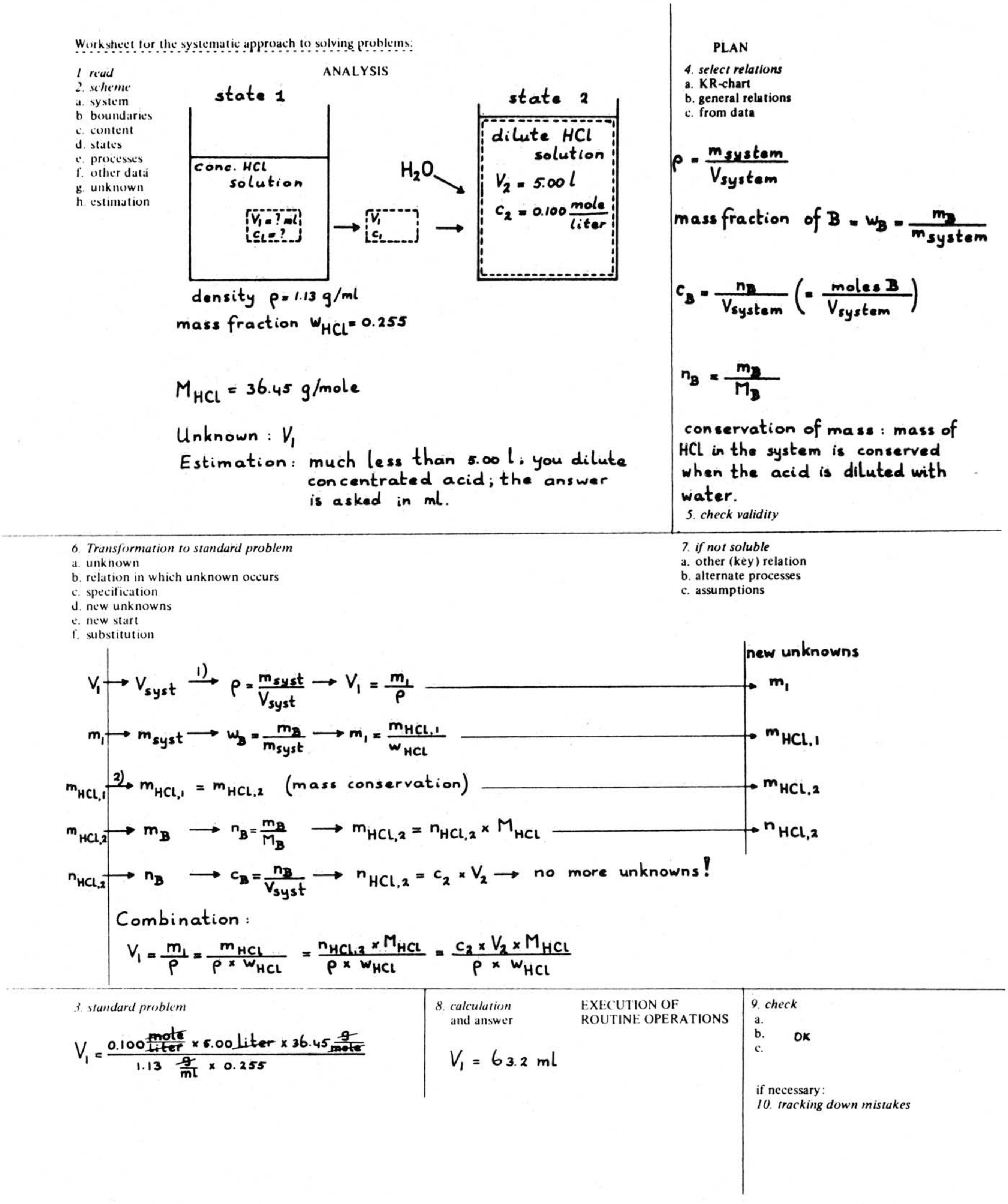
Figure 3. Text of the problem elaborated on the worksheet in Figure 4.

the problem has been solved, possible mistakes can be tracked down and corrected.

A second purpose is to develop and improve problem-solving skills.

Desired actions:

4.1. Checking the answer by comparing it with the estimation that has been made in the analysis (probable sign, magnitude, dimension, special cases, extrapolations). If a discrepancy shows up, checking whether mistakes have been made in the estimation,



Worksheet for the systematic approach to solving problems:

ANALYSIS

1. read
2. scheme
a. system
b. boundaries
c. content
d. states
e. processes
f. other data
g. unknown
h. estimation

state 1

conc. HCl solution

$V_1 = ?\ ml$
$c_1 = ?$

V_1
c_1

H_2O

state 2

dilute HCl solution

$V_2 = 5.00\ l$

$c_2 = 0.100 \frac{mole}{liter}$

density $\rho = 1.13\ g/ml$

mass fraction $w_{HCl} = 0.255$

$M_{HCl} = 36.45\ g/mole$

Unknown : V_1

Estimation: much less than 5.00 l; you dilute concentrated acid; the answer is asked in ml.

PLAN

4. select relations
a. KR-chart
b. general relations
c. from data

$$\rho = \frac{m_{system}}{V_{system}}$$

$$\text{mass fraction of } B = w_B = \frac{m_B}{m_{system}}$$

$$c_B = \frac{n_B}{V_{system}} \left(= \frac{\text{moles } B}{V_{system}} \right)$$

$$n_B = \frac{m_B}{M_B}$$

conservation of mass : mass of HCl in the system is conserved when the acid is diluted with water.

5. check validity

6. Transformation to standard problem
a. unknown
b. relation in which unknown occurs
c. specification
d. new unknowns
e. new start
f. substitution

7. if not soluble
a. other (key) relation
b. alternate processes
c. assumptions

new unknowns

$$V_1 \rightarrow V_{syst} \xrightarrow{1)} \rho = \frac{m_{syst}}{V_{syst}} \rightarrow V_1 = \frac{m_1}{\rho} \longrightarrow m_1$$

$$m_1 \rightarrow m_{syst} \rightarrow w_B = \frac{m_B}{m_{syst}} \rightarrow m_1 = \frac{m_{HCl,1}}{w_{HCl}} \longrightarrow m_{HCl,1}$$

$$m_{HCl,1} \xrightarrow{2)} m_{HCl,1} = m_{HCl,2} \text{ (mass conservation)} \longrightarrow m_{HCl,2}$$

$$m_{HCl,2} \rightarrow m_B \rightarrow n_B = \frac{m_B}{M_B} \rightarrow m_{HCl,2} = n_{HCl,2} \times M_{HCl} \longrightarrow n_{HCl,2}$$

$$n_{HCl,2} \rightarrow n_B \rightarrow c_B = \frac{n_B}{V_{syst}} \rightarrow n_{HCl,2} = c_2 \times V_2 \rightarrow \text{no more unknowns!}$$

Combination :

$$V_1 = \frac{m_1}{\rho} = \frac{m_{HCl}}{\rho \times w_{HCl}} = \frac{n_{HCl,2} \times M_{HCl}}{\rho \times w_{HCl}} = \frac{c_2 \times V_2 \times M_{HCl}}{\rho \times w_{HCl}}$$

3. standard problem

$$V_1 = \frac{0.100 \frac{\cancel{mole}}{\cancel{liter}} \times 5.00 \cancel{liter} \times 36.45 \frac{\cancel{g}}{\cancel{mole}}}{1.13 \frac{\cancel{g}}{ml} \times 0.255}$$

8. calculation and answer

EXECUTION OF ROUTINE OPERATIONS

$V_1 = 63.2\ ml$

9. check
a.
b. OK
c.

if necessary:
10. tracking down mistakes

Figure 4. Worksheet with a worked problem on it. (1) Experienced solvers of this type of problem (presumably all readers of *this Journal!*) will recognize this as a standard problem. Most probably, in their solution, they proceed immediately to phase 3. (2) At this point, another KR can be chosen; this implies a different transformation route.

in the analysis, in the relations or in their validity, in the transformation (especially specification and substitution mistakes), or in the calculations. Writing down that the answer is different from the one expected.

4.2. Checking if the answer is the correct answer for the question asked; e.g., the final temperature and not a temperature difference, energy supplied to the system and not energy delivered by the system.

4.3 Checking if all sub-problems have been solved.

4.4. Looking back at the way the problem has been solved to improve problem-solving skills, writing down conclusions.
   - Is the way the problem has been transformed to a standard problem useful in other cases?
   - What mistakes have been made? How could they be prevented next time?
   - Which (key) relations have been used? Should a relationship that has been used probably be incorporated into the list of key relationships because of its importance in solving this type of problems?

An example of a problem (a familiar calculation in the training of laboratory technicians) that has been worked out according to PAM, is given in Figure 3 and 4.

Part II: Learning Problem Solving in a Thermodynamics Course, will discuss how the instruction in this systematic approach to problem solving is organized.

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