COMPUTATIONAL INTEGRATED OPTICS FOR PHOTONIC STRUCTURES

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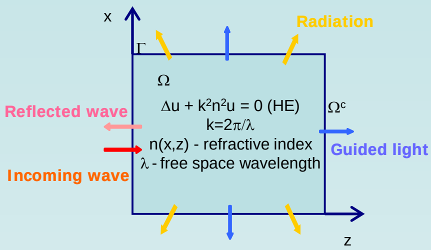
Project aims: Developing numerical programmes to model performance of small optical devices (such as being used on computer chips) ; the software will become a simulation tool for the design of actual new devices. The aims are to improve existing methods in various ways, most notably in the topics of the two sub-projects:

1. *TRANSPARENT-INFLUX BOUNDARY CONDITIONS* (TIBC's) .

MOTIVATION

Small optical devices manipulate light that is often fluxed into the device through waveguides that extend to 'infinity'. On the plane, when restricting to TE-polarization, the propagation of light is described by the Helmholtz Equation (HE). For efficient calculations, we want to enclose the device into a small computational window. The problem is that the boundary should be able to transfer prescribed influx and be transparent for unknown outflux (including radiation). Our aim is to use variants of the Dirichlet-to-Neumann operator, which basically requires the solution of the exterior problem. We will use variational methods to attack this problem.



CASE STUDIES FOR UNIFORM EXTERIOR

We considered a square domain, with uniform exterior, and designed:
A. TIBC's on the vertical sides, while Dirichlet b.c. at the horizontal lines.
B. TIBC's on 'all' sides (for computational purposes, the corner points still have zero Dirichlet values, which introduces small errors).

The DtN-operators were obtained using plane-wave (Fourier) expansion techniques. For instance, on the right vertical boundary, the non-local Fourier operators are

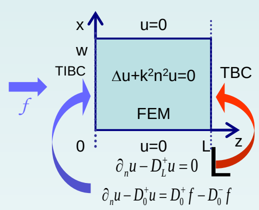
$$D_L^{\pm}(g) = \ ?$$

$$g(x) = \sum_m A_m \phi_m(x), \quad \phi_m(x) = \sin(\frac{m\pi x}{w})$$

$$u_{ext}(x,z) = \sum_m A_m \phi_m(x) e^{\pm i\beta_m (z-L)}$$

$$D_L^{\pm}(g)(x) = \partial_z u_{ext}|_L(x) = \sum_m \pm i\beta_m A_m \phi_m(x)$$

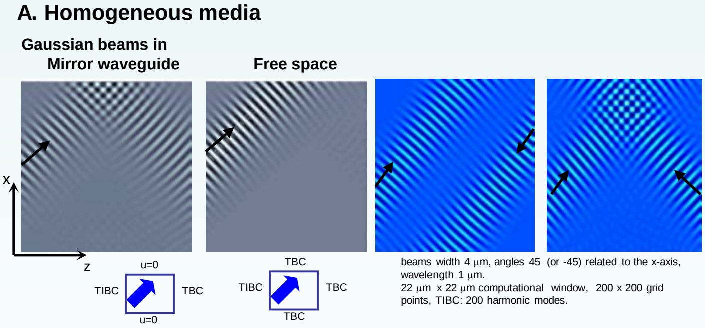
$$A_m = \frac{2}{w}\int_0^w g(x)\phi_m(x)dx \text{ and } \beta_m^2 + (\frac{m\pi}{w})^2 = k^2 n^2.$$



Such type of operators are included in a FEM-formulation that uses 1st order splines. Some results are shown below.

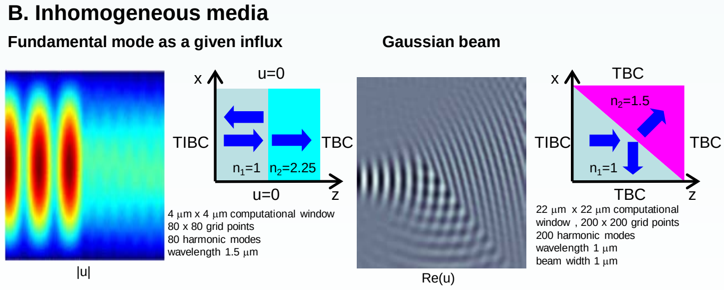
NUMERICAL RESULTS

A. Homogeneous media



beams width 4 μm, angles 45 (or -45) related to the x-axis, wavelength 1 μm.
22 μm x 22 μm computational window, 200 x 200 grid points, TIBC: 200 harmonic modes.

B. Inhomogeneous media



|u|: 4 μm x 4 μm computational window
80 x 80 grid points
80 harmonic modes
wavelength 1.5 μm

Re(u): 22 μm x 22 μm computational window , 200 x 200 grid points
200 harmonic modes
wavelength 1 μm
beam width 1 μm

2. DISCONTINUOUS GALERKIN FINITE ELEMENT METHOD THAT IS *HP*-ADAPTIVE, for time accurate, fully 3D applications.

Time dependent Maxwell's equations

$$\frac{\partial}{\partial t}(\varepsilon E) = \nabla \times H - J,$$

$$\frac{\partial}{\partial t}(\mu H) = -\nabla \times E,$$

$$\nabla \cdot (\varepsilon E) = \rho, \quad \nabla \cdot (\mu H) = 0.$$

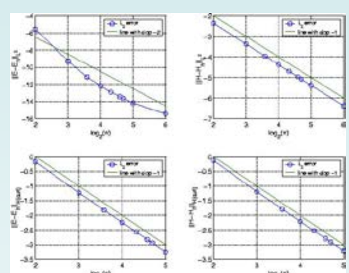
- E – electric field , H – magnetic field .
- To be solved in an unbounded domain (see left side of the poster).
- Permitivity ε and permeability μ (tensors) define the structure.

Numerical Results – 3D Simulations

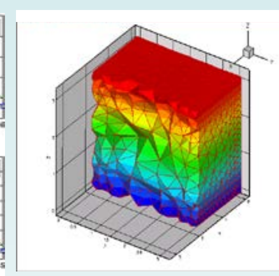
Recent years for electromagnetics problems more elegant *H(curl)* and *H(div)* conforming finite elements came into usage.

- *Nedelec* elements – edge, face elements.
- Provide right type continuity of the fields across element boundaries.
- Non-spurious solutions.
- Easy to impose the boundary conditions.
- ***Drawbacks*** – difficult to do *hp*-adaptation.

We test the method on the unit cube with perfectly conducting boundary conditions.



Typical convergence behavior of first order vector FEM.



Tessellation of the domain into tetrahedrons using Centaur mesh generator.

New numerical approach: *hp* – adaptive Discontinuous Galerkin Finite Element Methods (DGFEM)

DGFEM weak formulation

$$\partial_t \int_K \varepsilon E_h \cdot \varphi dx = \int_K H_h \cdot (\nabla \times \varphi) dx + \int_{\partial K} (n_k \times \hat{H}_k) \cdot \varphi ds,$$

$$-\int_K E_h \cdot \nabla u dx + \int_{\partial K} u \hat{E}_k \cdot n_k ds = 0,$$

$\hat{E}_k$ and $\hat{H}_k$ are numerical fluxes.

Why DGFEM?

- *hp*-DGFEM uses locally refined meshes (*h*-refinement) and polynomial approximations of varying degree in each element (*p*-refinement).
- *hp*-DGFEM uses completely discontinuous finite element spaces, hence we can easily deal with elements of various shape and order.
- These elements are being designed to satisfy the divergence constraints (locally and globally).
- *hp*-DGFEM is useful for problems with local singularities and rapidly changing or discontinuous material properties.
- The object oriented code will be written in C++ and will allow efficient parallel implementation.

Applied Analysis & Mathematical Physics (AAMP)
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