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Coordinating feeder and collector public transit lines for efficient MaaS services

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Abstract

Coordinating the schedules of feeder and collector public transit lines can reduce the passenger travel times. With advances in smart mobility, mobility-as-a-service (MaaS) schemes allow passengers to book a combined ticket for all their trip legs. This detailed information about the origins and destinations of door-to-door trips offers the opportunity to coordinate the schedules of public transport lines to reduce the passenger travel times. In this study, we model the coordination problem of feeder and collector lines by explicitly considering the regularity of the feeder lines and the transfer times of passengers. The coordination problem is modeled as a nonlinear non-convex problem and reformulated to an easy-to-solve convex optimization problem. Because of the travel times uncertainty, we also introduce a stochastic optimization formulation based on the sample average approximation approach. We test the performance of our approach in a case study with feeder and collector lines in Singapore showing an improvement potential of 5-10% in passenger travel times.

Key words: MaaS scheduling; feeder lines; last-mile; public transport; service synchronization

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1. Introduction

The emergence of disruptive ride-hailing services provided by transportation network companies (TNCs) offers an alternative to travelers and traditional public transport captives (see Shaheen and Cohen (2018)). This adds pressure to public transit to improve its operations and offer services with competitive door-to-door travel times (Cohen, 2018). By improving their services, public transport authorities and operators can reverse the modal shift from public transport to car-based services, resulting in major benefits on pollution, congestion, and travel safety (Rode et al., 2017). This is imperative because even in countries with excessive use of public transit and active modes the vast majority of kilometers traveled are by car (e.g., in the Netherlands 8,000 out of the 11,000 kilometers traveled annually by the average person are in a car (Statistics Netherlands, 2016)).

One of the main reasons for selecting car-based, door-to-door mobility services is their reduced travel times compared to public transit. Practical evidence from a metic-

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ulous investigation of the International Association of Public Transport (UITP) in sixty metropolitan areas suggests that the public transport ridership increases in line with the reduction of door-to-door travel times (UITP, 2015). This requires the coordination of first/last-mile feeder lines with collector lines to reduce the travel times of journeys conducted by public transit. This coordination can be supported by the emergence of Mobility-as-a-Service (MaaS) schemes that offer an opportunity to collect highly granular passenger demand data regarding the origin-destination trip(s) of passengers (Kamargianni and Matyas, 2017; Farahmand et al., 2021). In past literature, MaaS services are designed based on the isolated economic interests of the service providers (e.g., first/last-mile operators, public transport, para-transit) that do not consider the overall passenger travel times (Kliewer et al., 2006; Shaheen and Chan, 2016; Jittrapirom et al., 2017; Geurs et al., 2018). To bridge this gap, this study focuses on adjusting the schedules of feeder lines to the schedules of collector lines to reduce the travel times of passengers that use a combination of public transport modes in their journeys.

The remainder of this study is structured as follows. In section 2, we review the related studies on public transport scheduling and coordination of feeder/collector lines. In section 3, we formulate our problem of adjusting the schedule of a feeder line to the schedule of a main (collector) line as a nonlinear convex optimization problem and we offer a stochastic optimization formulation to account for the travel time uncertainties. Section 4 shows the improvement potential of our approach in a case study in Singapore that includes a feeder bus line and a collector mass rapid transit line. Concluding remarks and future research directions are presented in section 5.

2. Related Studies

Shared mobility has gained attention over the past years. Recently, Zha et al. (2016) provided an economic analysis of ride-sourcing markets, where private car owners drive their own vehicles for profit. Ride-sourcing is a prime example of a sharing economy that can change the future of city traffic (Jin et al., 2018). Although it is perceived as an improved taxi service, half of ride-sourcing trips have replaced modes other than taxi and it is considered to be a faster and more reliable mobility option (see the survey of Rayle et al. (2016) in San Francisco). Dias et al. (2017) developed behavioral choice choice models for ride-sourcing and car-sharing, showing that users of these services tend to be young, well-educated, higher-income, working individuals residing in higher-density areas. Ride-sourcing is part of the broader area of shared mobility that includes bike-sharing, car-sharing, micro-transit, and e-scooters. Shared mobility services can be seen as competitors to traditional public transit, but they can also create synergies if they are used to perform specific trip legs which are part of a public transport journey (i.e., its firs/last-mile).

Several works have examined the role of mobility-on-demand and the coupling of mass transit and last-mile services (see Basu et al. (2018)). Car-based services, such as Uber, Lyft, BlaBlaCar, and bus-based options that include smart bookable last-mile options are examples of services that can be integrated with mass transit (Hensher, 2017). Gambella et al. (2019) incorporated Internet of Things (IoT) components to prove the necessity of real-time data for the efficient and seamless integration of public transport services inside on-demand mobility systems. Further, Pandey et al. (2019) underscored the need for

MaaS platforms to improve integration among competing first/last-mile services. Methodological approaches, such as the work of Chen and Wang (2018), have developed nonlinear optimization models for designing first/last-mile services with the aim to maximize the social welfare. In addition, Liang et al. (2016) developed two integer programming models to optimize automated taxi systems that perform the last mile of train trips by considering that every last-mile trip request must be satisfied and taxis can accept or reject reservations according to a profit maximization strategy.

Despite the positive effect that can emerge from the integration of public transport services, traditional public transport planning typically seeks to optimize the in-vehicle passenger travel times and the waiting times at stops instead of the overall travel times of passengers (Ghoseiri et al., 2004; Yin et al., 2017; Gkiotsalitis and Cats, 2021; Liu et al., 2021). However, this myopic scheduling approach disregards the negative effects of long transfer times between feeder/collector lines in the case of poor synchronization. The schedules of feeder lines are typically planned in isolation with objectives related to improving productivity and reducing operating costs without considering the travel times of passengers that will need to transfer to a collector line (Kliewer et al., 2006; Wang, 2019). Thus, there is a lack of methodologies for the seamless integration of collector and feeder public transit lines for improving the passenger travel times. Most works model the scheduling of public transport lines as an integer or mixed-integer mathematical program where the decision variables are the dispatching times of the daily trips of the lines (Wong et al., 2008; Fügenschuh, 2009; Chu, 2018; Yin et al., 2021). Because of their discrete nature, such programs cannot be easily solved and many scheduling approaches try to produce robust schedules that can perform well in case of travel time and passenger demand disruptions to avoid numerous rescheduling(s) (Tang et al., 2019; Gkiotsalitis and Alesiani, 2019).

Using a baseline schedule developed at the tactical planning level, several works resort to dynamic rescheduling for adjusting to the travel time and passenger demand variations (Gkiotsalitis and Cats, 2020; Gkiotsalitis and Van Berkum, 2020). Bly (1976) used rescheduling on depleted bus services to provide equal headways for the buses in the schedule. Gkiotsalitis (2020) proposed a rescheduling strategy that holds vehicles at stops to maintain equal headways. Li et al. (2008) modeled and solved the single depot bus rescheduling problem in pseudo-polynomial time using a parallel auction algorithm. In a follow-up work, Li et al. (2009) showed that the bus rescheduling problem is NP-hard, and used a Lagrangian relaxation-based insertion heuristic for its solution.

There are also several works that synchronize the timetables of feeder lines which feed urban rail transit networks. Their objectives typically involve the minimization of transfer waiting times and passenger congestion. The problem of transfer synchronization between feeder and collector transit lines has been studied at the tactical planning level (Ceder et al., 2001), and at the operational level (Wu et al., 2016). Eikenbroek and Gkiotsalitis (2020) proposed a robust rescheduling and holding model for autonomous buses that feed collector transit lines. Ceder and Yim (2003) introduced a scheduling method for shuttle bus services to provide easy access to main haul transit services. In their work, the feeder shuttle service was coordinated with the Bay Area Rapid Transit (BART) service. Sivakumaran et al. (2012) investigated the usefulness of the coordination between feeder and collector lines, and showed that when the frequencies of collector and feeder services can be established jointly, coordination can be Pareto improving, meaning that operator and user costs both diminish. Almasi et al. (2015) developed a metaheuristic algorithm to optimize transit services with a feeder bus and a rail system. Several other studies have also focused explicitly on the scheduling problem of a feeder bus that needs to be synchronized with the schedule of a rail service (Chowdhury and I-Jy Chien, 2002; Wu et al., 2015; Yang et al., 2020).

Rescheduling has also been used for synchronizing services to reduce the transfer waiting times of passengers. Cevallos and Zhao (2006b) and Cevallos and Zhao (2006a) proposed simple perturbations that merely shift the pre-existing timetables to solve the aforementioned problem with a genetic algorithm. In addition, Coffey et al. (2012) treated the synchronization problem as a demand-supply matching problem. In their approach, they rescheduled the timetables of public transport modes by matching the passenger demand expressed via journey planners with the public transport supply to reduce missed connections. Ceder et al. (2001) and Gkiotsalitis et al. (2019) developed deterministic and robust optimization models for reducing the transfer times among public transit lines. Notwithstanding, the aforementioned works do not consider the overall travel times of passengers and merely focus on the reduction of transfer times.

There is also a specific line of research on bus bridging. Kepaptsoglou and Karlaftis (2009) proposed an integrated optimization procedure for planning and designing an efficient bus bridging network. Conducting experiments in Athens, Greece, they showed that up to 35% of passenger demand can be accommodated with a reasonable number of buses in the case of disruptions in the operation of a metro network. Jin et al. (2016) developed an optimization model that designs bus bridging services to respond to degradations of urban transit rail networks. Gu et al. (2018) proposed a plan-based flexible bus bridging operation strategy that assigns buses to bridging routes. For this, they developed a Weight Shortest Processing Time first (WSPT) rule-based heuristic algorithm. Bus bridging routes were also designed by Deng et al. (2018). Recently, Kang et al. (2019) proposed a joint last train timetabling optimization and bus bridging service management method for disrupted urban railway transit networks. Finally, Liang et al. (2019) considered the stochasticity of bus travel times when designing a bus bridging service resulting in a robust optimization model. Although bus bridging decides about the assignment of buses to routes in the case of train disruptions to mitigate their effects, our study is mostly concerned with the minimization of passenger travel times when using one or more public transport services.

To this end, this study models both the transfer times of passengers using multiple public transport modes and the waiting times of single-mode passengers to produce schedules that reduce the overall passenger travel times. In this pursuit, this study aims to provide an answer to the following research questions:

- how can we model the scheduling of feeder public transit lines to account for passengers' travel times?
- can we derive a model to synchronize the schedules of feeder lines to the schedules of the collector lines? If yes, which is its computational complexity and can it be solved within a reasonable time?
- which is the improvement potential in terms of passenger travel times and waiting times at stops when synchronizing the feeder line schedule to the schedule of a collector line?

By answering the aforementioned research questions, the main contributions of this study are the development of a novel, easy-to-solve model for coordinating the feeder and collector lines, the development of a stochastic optimization model that accounts for the travel time uncertainty of public transport vehicles, and the investigation of the improvement potential in a case study in Singapore.

3. Problem Formulation

To reduce the travel times of passengers, we adjust the feeder line schedules to the schedules of the collector lines. The schedule of a collector line is typically considered being fixed because it is difficult to adjust it (Liang et al., 2016). For instance, it is not trivial to adjust the schedules of train or metro lines, given the interconnections with other lines that use the same tracks (Şahin, 1999). On the contrary, more flexible feeder lines (e.g., bus or minibus lines) can adjust their schedules to reduce the travel times of passengers that use both feeder and collector lines as part of their trip legs. That is to say, the schedule adjustments of feeder modes is applied to find solutions that improve the service performance. In the schedule's adjustment, we allow the modification of the planned dispatching times of trips (e.g., trips can be dispatched earlier or later than originally planned).

The specific problem considered in this study is the problem of adjusting the baseline schedule of a feeder line to improve its synchronization with a collector line and reduce the travel times of passengers. In this context, we define a feeder line as a public transit line that transfers a fraction of its passengers to a specific collector line. Formally, we have limited fleet availability for each feeder line and these lines cannot always meet their ideal headways at each bus stop. Thus, we strive to use their limited vehicles in such a way that we minimize the irregularity of each line, while satisfying the layover constraints and the transfer synchronization constraints between feeder and collector lines.

This work makes a number of reasonable assumptions, such as:

(i) when rescheduling the feeder line, its baseline schedule can be modified by changing the dispatching times of its trips. Additional measures, such as introducing new trips, are not considered because this will affect the already planned vehicle and crew schedules resulting in requests for more vehicles and drivers that might not be available;

(ii) we do not allow bus trips from the feeder line to overtake one another by slowing down the following vehicle when an overtaking is about to occur (an assumption used in several seminal works, such as Xuan et al. (2011); Chen et al. (2013)).

(iii) the number of vehicles assigned to each line during the vehicle scheduling phase suffices to satisfy the expected passenger demand. This fleet size is determined with the use of optimal frequency setting methods at the tactical planning stage that precedes the rescheduling part of our work (see Hadas and Shnaiderman (2012); Ferguson et al. (2012)).

Before proceeding to the description of the feeder/collector line synchronization problem, we introduce the following notation.

NOMENCLATURE

Sets

$N_f = 1, 2, ..., n, ...$	is the set of all the planned daily trips of the feeder line
$N_m = 1, 2, ..., m, ...$	is the set of all the planned daily trips of the main line
$S_f = 1, 2, ..., s, ...$	is the ordered set of public transport stops of the feeder line
$S_m = 1, 2, ..., s, ...$	is the ordered set of public transport stops of the main line
$B = S_f \cap S_m$	vector denoting all transfer stops between the feeder and the main line

Parameters

h	the ideal headway of the feeder line that should be maintained at all bus stops for attaining a regular service. Note that this might differ at different times of the day (peak, off-peak)
$t_{n,s}$	the expected inter-station travel time of the feeder line trip $n \in N_f$ between stops $s-1$ and s, including the dwell time at stop $s-1$
δ_{min}	the pre-determined dispatching time of the first trip of the feeder line that prevents starting the daily operations before the bus drivers are available
δ_{max}	the pre-determined dispatching time of the last trip of the feeder line that prevents schedule sliding
ψ	the required layover time for the feeder line for performing two successive trips with the same vehicle
$\gamma_{n,s}$	the arrival time of trip $n \in N_m$ at stop s which cannot be rescheduled because it belongs to the main line
Y_{bnm}	0-1 parameter, where $Y_{bnm} = 1$ if trip $n \in N_f$ needs to synchronize its arrival times with trip $m \in N_m$ at stop $b \in B$, and 0 otherwise
$\Phi_{n,n'}$	0-1 parameter, where $\Phi_{n,n'} = 1$ if trips $n, n' \in N_f$ of the feeder line are operated by the same vehicle in a sequential order (e.g., n' after n), and 0 otherwise
M	a very large positive number

Variables

x_n	the (re)scheduled dispatching time of the n^{th} trip of the feeder line
$a_{n,s}$	arrival time of trip $n \in N_f$ at stop s
$h_{n,s}$	inter-arrival headway between successive feeder line trips $n-1$ and n at stop s

The decision variables of our problem are the rescheduled dispatching times of the feeder line trips, x_n. Changing the dispatching times of trips modifies their expected arrival times at stops as follows:

$$a_{n,s} := x_n + \sum_{z=2}^{s} t_{n,z} \quad (\forall n \in N_f, \ \forall s \in S_f \setminus \{1\}) \tag{1}$$

where $t_{n,z}$ is the expected travel time of trip n from stop $z-1$ to stop z, including the dwell time for boardings/alightings. We note that this travel time is stochastic in nature and in section 3.5 we offer a stochastic optimization formulation. Because overtaking is not allowed, the arrival time of each feeder line trip $n \in N_f \setminus \{1\}$ at stop s should be greater than the arrival time of its previously dispatched trip $a_{n-1,s}$. That is to say, the dispatching order of trips is maintained upon their arrival at all stops and Eq.(1) is modified to:

$$a_{n,s} := \max \left\{ a_{n-1,s}\ ;\ x_n + \sum_{z=2}^{s} t_{n,z} \right\} \qquad (n \in N_f \setminus \{1\},\ \ s \in S_f \setminus \{1\}) \tag{2}$$

which ensures that the dispatching order of feeder line trips is maintained. We further introduce a boundary condition for the first trip of the day which does not have a preceding trip,

$$a_{1,s} := x_1 + \sum_{z=2}^{s} t_{1,z} \qquad (s \in S_f \setminus \{1\}) \tag{3}$$

This results in the varying arrival times at stops depicted in Fig.1. Note that in Fig.1 we only show the varying arrival times of the trip of the feeder line, $a_{n,s}$, because we cannot modify the arrival times of the trip of the main collector line. Those arrival times vary subject to the rescheduled dispatching time, x_n, and the stochastic travel and dwell times $t_{n,s}$.

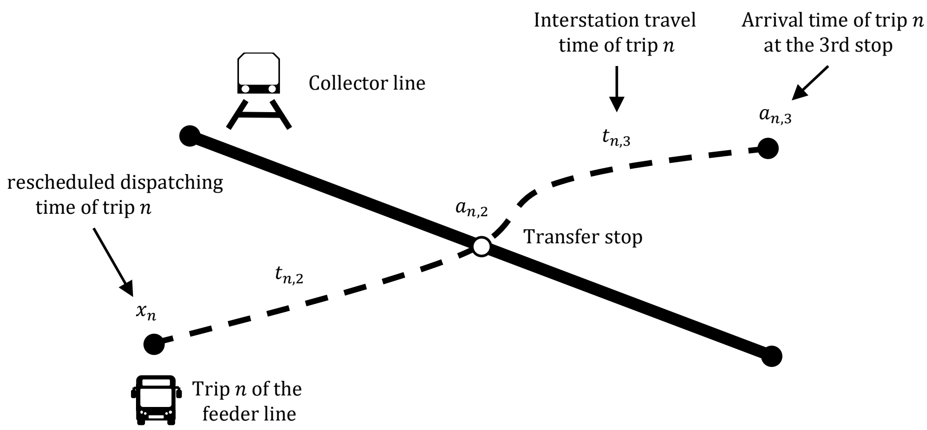

Figure 1: Illustration of the varying arrival times of trip n that belongs to the feeder line based on the dispatching time decision x_n.

One of the main modeling contributions of our work is that we seek to satisfy the synchronization requirements for transferring passengers from the feeder to the collector line while, at the same time, we seek to minimize the excessive waiting times of passengers at the *no transfer* stops of the feeder line. This implies that we seek to improve the regularity of the feeder line at stops with no transfers subject to synchronizing the feeder with the collector line at transfer stops.

To increase the regularity of a public transport service, the actual inter-arrival time headways[2] at bus stops should be as close as possible to their ideal values, h . Although the inter-arrival headways of the *collector* lines do not depend on our decisions, the inter-arrival headways of the *feeder* lines are linked to the rescheduling of their trip dispatching times as follows:

$$h_{n,s} := a_{n,s} - a_{n-1,s} \qquad (\forall n \in N_f \setminus \{1\}, \ \forall s \in S_f \setminus \{1\}) \tag{4}$$

3.1. Rescheduling objectives

The aim of rescheduling the feeder line is to reduce the travel times of public transit passengers that might use only the feeder line or a combination of the feeder and a collector line. Namely, passengers may use only one line or the combination of a feeder and a collector line to complete their origin-destination trip. Note that we do not consider origin-destination trips where passengers need to perform more than one transfer to arrive to their final destination because this is highly uncommon (see Kujala et al. (2018)). To account for the passengers' travel times when scheduling the feeder line, we aim to minimize:

- the excessive waiting times when waiting at the regular[3] feeder line stops;
- and the transferring waiting times from the feeder to the collector line.

That is, the aforementioned waiting times are used as proxies for the estimation of passenger travel times since the in-vehicle travel times cannot be changed significantly with rescheduling (e.g., they depend on external factors such as congestion at peak hours, roadworks and accidents). More precisely, we seek to reduce the excessive passenger waiting times at regular stops while synchronizing transfers from the feeder to the collector line.

The excessive passenger waiting times are the main key performance indicator in high-frequency feeder line services in London, Singapore, and many other densely populated areas and are used to monitor the regularity of services (Randall et al., 2007). Note that in high-frequency services, the waiting time of a passenger of trip $n \in N_f$ at stop s is half the inter-arrival headway between trip n and trip $n-1$, $\frac{h_{n,s}}{2}$, because the passenger arrivals at stops are random (Welding, 1957; Randall et al., 2007). The reason behind this is that the high-frequency headways are so small that passengers do not plan their arrival times at stops based on the arrival times of buses/trains (Muñoz et al., 2013).

To reduce the excessive waiting times of the feeder line passengers, one should minimize the sum of the squared difference between the actual and the ideal headways:

$$f(h) := \sum_{s \in S_f \setminus \{1\}} \sum_{n \in N_f \setminus \{1\}} \left(\frac{h_{n,s}}{2} - \frac{h}{2} \right)^2 \tag{5}$$

where $f(h)$ is the daily excessive waiting time of the feeder line passengers that indicates the service regularity and forms the *objective function* of our mathematical program

[2] the time headway between two consecutive trips $n, n+1$ at stop s is the headway between the front bumpers of the respective buses at the time of their arrival at stop s

[3] a regular stop of a feeder line is a stop with no transfers

that is formally expressed in Eqs.(8)-(14). Adopting this objective function, we assume that the last mile trips of the feeder line are within the catchment area of the collector line and the utility of the feeder line depends on its service reliability. It is also worth noting that the ideal headway h can vary across different time periods of the day according to the passenger demand and the availability of vehicles. For instance, h might receive lower values during peak-hours where we would need to deploy more vehicles and higher values during off-peaks.

Now, let us consider the waiting times of passengers when transferring from a feeder to a collector line. Reckon that B is the set with all transfer stops between the feeder and the collector line, where $Y_{bnm} = 1$ if trip $n \in N_f$ needs to synchronize its arrival time with trip $m \in N_m$ at the transfer stop $b \in B$, and $Y_{bnm} = 0$ otherwise. Ceder et al. (2001) considers a perfect synchronization when trip $n \in N_f$ arrives at the transfer stop $b \in B$ exactly at the same time as trip $m \in N_m$. In this way, the transfer times of passengers are minimized when $a_{n,b} - y_{m,b} = 0$, where $y_{m,b}$ is the arrival time of the trip of the collector line at stop s. Later studies by Eranki (2004) and Ibarra-Rojas and Rios-Solis (2012) proposed a more flexible scheme where the bus trip n is still considered synchronized if it arrives within a time range of $[0, \Delta t]$ seconds before the arrival of trip m at the transfer stop b. In our study, we follow the flexible synchronization scheme of Eranki (2004) and Ibarra-Rojas and Rios-Solis (2012) by modeling the required synchronizations at transfer stops as problem constraints:

$$0 \le Y_{bnm}(y_{m,b} - a_{n,b}) \le \Delta t \qquad (\forall n \in N_f \setminus \{1\}, \ \forall m \in N_m \setminus \{1\}, \ \forall b \in B) \tag{6}$$

Obviously, when $Y_{bnm} = 0$ the inequalities in Eq.(6) hold for any value of the arrival times $a_{n,b}$ and $y_{m,b}$ because we are not required to synchronize the arrival times of those two trips.

3.2. Layover constraints

Typically, if one vehicle operates two successive trips of the same line, there should be a layover time by the time the first trip is finished until the next trip is dispatched (e.g., because of union contractual agreements drivers may need to take a short break before starting their next trip – see Berrebi et al. (2018)). When rescheduling the dispatching times of our feeder line trips, one should factor in this layover time. This yields the inequality constraints:

$$\Phi_{n,n'}\left(x_{n'} - a_{n,|S_f|}\right) \ge \Phi_{n,n'}\psi \qquad (\forall n, n' \in N_f) \tag{7}$$

That is, if trip n' is the next trip of trip n that is operated by the same vehicle (e.g., $\Phi_{n,n'} = 1$), then the dispatching time of trip n', $x_{n'}$, should be greater than the arrival time of trip n at the last stop, $a_{n,|S_f|}$, plus the required layover time ψ. It is worth noting that if trips n, n' are not operated by the same vehicle in a successive order, inequality (7) is satisfied because $\Phi_{n,n'} = 0$.

3.3. Model formulation

The following mathematical program summarizes the proposed feeder line synchronization problem that explicitly considers (i) the excessive waiting times of the feeder line, and (ii) the transferring waiting times.

$$
\begin{aligned}
(Q): \quad & \min_{h} \quad f(h) && (8)\\
& \text{s.t.:} \quad (x,a,h) \in F(x,a,h) = \left\{ (x,a,h) \,\middle|\, (x,a,h) \text{ satisfy Eqs.(2)-(7)} \right\} && (9)\\
& x_1 \geq \delta_{min} && (10)\\
& x_{|N_f|} \leq \delta_{max} && (11)\\
& x_n \in \mathbb{R}_{\geq 0} && (12)\\
& a_{n,s} \in \mathbb{R}_{\geq 0} && (13)\\
& h_{n,s} \in \mathbb{R}_{\geq 0} && (14)
\end{aligned}
$$

Inequality constraints (10) and (11) ensure that (i) we dispatch the first feeder trip of the day when drivers and vehicles are available, and (ii) we dispatch the last trip of the day before the latest possible dispatching time, δ_{max}, to avoid schedule sliding. Program (Q) is a nonlinear (quadratic) mathematical program which is not convex because the constraint of Eq.(2) includes the non-convex term $a_{n,s} := \max(a_{n-1,s}; x_n + \sum_{z=2} t_{n,z})$. Hence, it cannot be solved to global optimality with numerical optimization methods and one should resort to heuristics. To rectify this, we propose a reformulation. Since the non-smooth term $a_{n,s} := \max(a_{n-1,s}; x_n + \sum_{z=2} t_{n,z})$ results in a non-convex mathematical program:

- we replace the equality constraint $a_{n,s} := \max(a_{n-1,s}; x_n + \sum_{z=2} t_{n,z})$ with the inequality constraints $a_{n,s} \geq a_{n-1,s}$ and $a_{n,s} \geq x_n + \sum_{z=2} t_{n,z}$
- we introduce the penalty term $M \left[a_{n,s} - (x_n + \sum_{z=2} t_{n,z}) \right]^2$ to the objective function which will force the value of $a_{n,s}$ to be equal to $\max(a_{n-1,s}; x_n + \sum_{z=2} t_{n,z})$ at the solution of the mathematical program. This is because $a_{n,s}$ is forced to be equal to $x_n + \sum_{z=2} t_{n,z}$ if $x_n + \sum_{z=2} t_{n,z} \geq a_{n-1,s}$ to avoid a major cost increase in our objective function because of the large value of M (for more details, refer to the Big M theory in Griva et al. (2009)).

With this transformation, program (Q) is reformulated to the following program ($\tilde{Q}$)

that has an equivalent solution:

$$
\begin{aligned}
(\tilde{Q}):\quad & \min_{h,a,x} \quad f(h) + \sum_{n \in N_f \setminus \{1\}} \sum_{s \in S_f \setminus \{1\}} M\left(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z})\right)^2 \\
\text{s.t.:}\quad & (x,a,h) \in F(x,a,h) = \left\{(x,a,h) \,\middle|\, (x,a,h) \text{ satisfy Eqs.(3)-(7)}\right\} \\
& a_{n,s} \geq a_{n-1,s}, \quad \forall n \in N_f \setminus \{1\}, \quad \forall s \in S_f \setminus \{1\} \\
& a_{n,s} \geq x_n + \sum_{z=2}^{s} t_{n,z}, \quad \forall n \in N_f \setminus \{1\}, \quad \forall s \in S_f \setminus \{1\} \\
& x_1 \geq \delta_{min} \\
& x_{|N_f|} \leq \delta_{max} \\
& x_n \in \mathbb{R}_{\geq 0}, \quad a_{n,s} \in \mathbb{R}_{\geq 0}, \quad h_{n,s} \in \mathbb{R}_{\geq 0}.
\end{aligned}
\tag{15}
$$

Note that the equality constraint (2) in program (Q) is replaced by the two new inequality constraints and the objective function penalty $\sum_{n \in N_f \setminus \{1\}} \sum_{s \in S_f \setminus \{1\}} M\left(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z})\right)^2$.

Proceeding further, we prove that our reformulated program $(\tilde{Q})$ can be solved to global optimality.

Theorem 3.1. *A local minimizer of $(\tilde{Q})$ is also a global minimizer.*

Proof. A local minimizer of $(\tilde{Q})$ is also a global minimizer if the objective function is convex and the feasible region is a convex set. The feasible region in $(\tilde{Q})$ is defined by linear inequalities and is a polyhedron (thus, it is also a convex set). Further, we prove that the objective function in $\tilde{Q}$ is convex with respect to h, a, x. Let $\tilde{f}(h,a,x) := \sum_{s \in S_f \setminus \{1\}} \sum_{n \in N_f \setminus \{1\}} \left(\frac{h_{n,s}}{2} - \frac{h}{2}\right)^2 + \sum_{n \in N_f \setminus \{1\}} \sum_{s \in S_f \setminus \{1\}} M\left(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z})\right)^2$. To prove the convexity of our objective function, $\tilde{f}(h,a,x)$, we introduce functions $g_{n,s}(h) := \left(\frac{h_{n,s}}{2} - \frac{h}{2}\right)^2$ and $k_{n,s}(a,x) := M(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z}))^2$. Then, $\tilde{f}(h,a,x)$ can be written as:

$$\tilde{f}(h,a,x) := \sum_{s \in S_f \setminus \{1\}} \sum_{n \in N_f \setminus \{1\}} g_{n,s}(h) + \sum_{n \in N_f \setminus \{1\}} \sum_{s \in S_f \setminus \{1\}} k_{n,s}(a,x)$$

Function $g_{n,s}(h)$ is convex with respect to h for any $n \in N_f \setminus \{1\}$, $s \in S_f \setminus \{1\}$ because

$$\frac{\partial^2 g_{n,s}}{\partial h^2} = \frac{1}{2} > 0 \qquad \forall n \in N_f \setminus \{1\}, \quad \forall s \in S_f \setminus \{1\}$$

Similarly, function $k_{n,s}(a,x)$ is convex with respect to (a,x) for any $n \in N_f \setminus \{1\}$, $s \in S_f \setminus \{1\}$ because its Hessian matrix is positive semi-definite. Namely, the Hessian of $k_{n,s}(a,x)$ is:

$$H = \begin{bmatrix} \frac{\partial^2 k}{\partial a^2} & \frac{\partial^2 k}{\partial a \partial x} \\ \frac{\partial^2 k}{\partial x \partial a} & \frac{\partial^2 k}{\partial x^2} \end{bmatrix} = \begin{bmatrix} 2M & -2M \\ -2M & 2M \end{bmatrix}$$

which is positive semi-definite since the leading principal minor det(H) = $4M - 4M = 0$ is non-negative. Therefore, $\tilde{f}(h, a, x)$ is convex as the sum of the convex functions $g_{n,s}(h)$, $k_{n,s}(a, x)$, and this completes our proof. ■

As a nonlinear convex optimization problem, program ($\tilde{Q}$) can be solved to global optimality with an interior-point method that uses a logarithmic barrier function to translate the constrained optimization problem in ($\tilde{Q}$) to an unconstrained one (Wright et al., 1999). Then, the unconstrained problem that preserves its convexity can be solved with line search methods (i.e., Newton's line search of BFSG) in polynomial time with quadratic or superlinear convergence rates.

3.4. Solution demonstration in a toy network

To demonstrate the solution of our mathematical program, consider an idealized network with a feeder and a collector line (Fig.2) performing two trips each. The parameter values in this toy network are presented in Table 1.

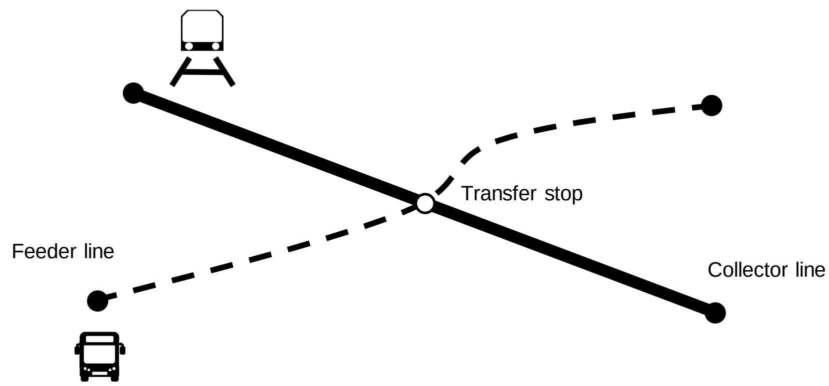

Figure 2: Toy network.

Table 1: Parameter values

Parameter	Value(s)	Parameter	Value(s)
N_f	1, 2	δ_{min}	0 s
N_m	1 , 2	δ_{max}	900 s
S_f	1, 2, 3	ψ	15 s
S_m	1 , 2, 3	$\gamma_{1,2}$	310 s
B	{2}	$\gamma_{2,2}$	610 s
h	300 s	$Y_{2,1,1}$	1
$t_{1,2}$	300 s	$Y_{2,2,2}$	1
$t_{1,3}$	300 s	$\Phi_{1,2}$	0
$t_{2,2}$	320 s	M	10E+6
$t_{2,3}$	290 s	Δt	20 s

Note that from Table 1, $\Phi_{1,2} = 0$. This means that different vehicles operate trips 1,2 of the feeder line and there is no layover time requirement. In addition, Table 1 reports the expected values of the inter-station travel times and dwell times $t_{n,s}$. Those expected values can be easily replaced by stochastic values via sampling from a probability distribution if one wants to solve the problem as a stochastic optimization one - see the sample average approximation method in Kleywegt et al. (2002).

This scenario is programmed in Python 3.6 and solved by the commercial solver Gurobi (8.1.1). Solving program $(\tilde{Q})$ with Gurobi (8.1.1) requires 11 iterations and the computational cost is just 0.01 s (see Table 2).

Table 2: Solution of program $(\tilde{Q})$ with Gurobi 8.1.1

Iteration	Primal	Dual	Time
0	5.35E+12	4.74E+11	0.00 s
1	2.38E+11	4.49E+10	0.00 s
2	1.34E+10	2.60E+09	0.00 s
3	3.14E+07	6.08E+06	0.00 s
4	1.37E+05	2.65E+04	0.00 s
5	2.13E+01	7.37E+00	0.00 s
7	7.89E+01	1.25E+01	0.00 s
8	1.69E+03	1.25E+01	0.00 s
9	1.26E+01	1.25E+01	0.00 s
10	1.26E+01	1.25E+01	0.01 s
11	1.25E+01	1.25E+01	0.01 s

The resulting values of the variables (dispatching times, arrival times, headways) when our problem is solved are:

$$(x_1, x_2) = (5, 290) \text{ s}$$
$$\begin{pmatrix} a_{1,2} & a_{1,3} \\ a_{2,2} & a_{2,3} \end{pmatrix} = \begin{pmatrix} 305 \text{ s} & 605 \text{ s} \\ 610 \text{ s} & 900 \text{ s} \end{pmatrix}$$
$$(h_{2,2}, h_{2,3}) = (305, 295) \text{ s}$$

Note that the above solution:

- synchronizes the transfers of the feeder with the collector line because $0 \leq y_{1,2} - a_{1,2} \leq \Delta t$ and $0 \leq y_{2,2} - a_{2,2} \leq \Delta t$ (see the synchronization range in Fig.3);
- minimizes the squared deviation between the actual headways (305 and 295 s) and the ideal headway (300 s) of the feeder line.

Fig.3 presents the trajectories of the feeder line trips based on their rescheduled dispatching times $(x_1, x_2) = (5, 290)$ s. The synchronization range indicates the time period within which trips 1 and 2 should arrive at the transfer stop to transfer passengers to the collector line. Note that both trips arrive at the transfer stop within that time period.

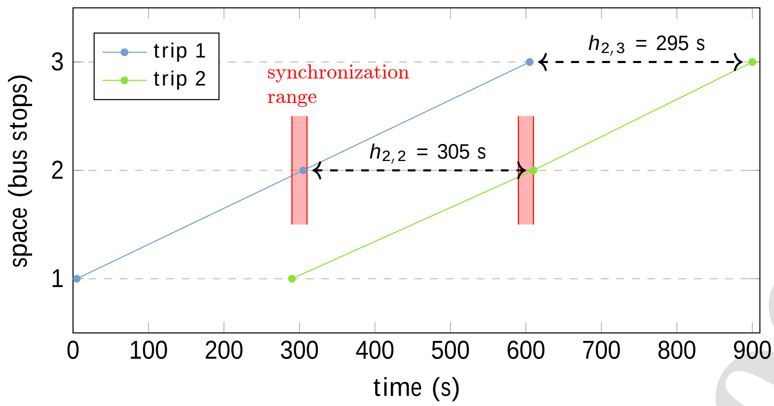

Figure 3: Trajectories of trips $N_f = \{1, 2\}$ of the feeder line based on the rescheduled dispatching times.

3.5. Stochastic optimization model

Our convex mathematical program $(\tilde{Q})$ uses the expected values of the inter-station travel times and dwell times of feeder trips $t_{n,s}$ $\forall n \in N_f, s \in S_f \setminus \{1\}$. During the daily operations, however, these travel times can vary significantly from their expected values. To incorporate this effect in the optimization model, these travel times can be perceived as *uncertain* parameters.

To cope with parameters with uncertain values, one could search for a solution that optimizes the *expected* value of the objective function. That is, we seek to minimize:

$$\mathbb{E}\left[\sum_{s \in S_f \setminus \{1\}} \sum_{n \in N_f \setminus \{1\}} \left(\left(\frac{h_{n,s}}{2} - \frac{h}{2}\right)^2 + M\left(a_{n,s} - (x_n + \sum_{z=2}^{s} t_{n,z})\right)^2 \right)\right] \tag{16}$$

This expected value can be estimated by the sample average approximation (SAA) method that provides a sample average estimate of the objective function derived from a random sample of Monte Carlo simulations (Kleywegt et al., 2002; Verweij et al., 2003). In more detail, instead of considering the average values of $t_{n,s}$, we can consider a large sample of inter-station travel time and dwell time realizations $t_{i,n,z}$ where $i = \{1, 2, ..., I\}$ are these realizations. Then, Eq.(16) becomes:

$$\frac{1}{I}\sum_{i=1}^{I}\left[\sum_{s \in S_f \setminus \{1\}} \sum_{n \in N_f \setminus \{1\}} \left(\left(\frac{h_{i,n,s}}{2} - \frac{h}{2}\right)^2 + M\left(a_{i,n,s} - (x_n + \sum_{z=2}^{s} t_{i,n,z})\right)^2 \right)\right] \tag{17}$$

The combined inter-station travel time and dwell time realizations, $t_{i,n,z}$, appear also in the constraints of our optimization problem and can result in infeasibilities (i.e., missed synchronizations). For this reason, the synchronization constraint that seeks to ensure that feeder bus n arrives within the time range of $[0, \Delta t]$ seconds before the arrival of collector trip m at a transfer stop b can be relaxed to allow a small number of missed synchronizations instead of requesting to always meet this constraint. To achieve this, constraint (6) is relaxed as:

$$0 \le Y_{bnm}\,(y_{m,b} - a_{i,n,b})\,\zeta_{i,n,b} \le \Delta t \qquad (\forall i \in I,\ n \in N_f \setminus \{1\}, m \in N_m \setminus \{1\}, b \in B) \tag{18}$$

$$\sum_{i=1}^{I} \sum_{n \in N_f \setminus \{1\}} \sum_{b \in B} \zeta_{i,n,b} \ge \theta(|N_f| - 1)|B| \tag{19}$$

$$\zeta_{i,n,b} \in \{0,1\} \qquad (\forall i \in I, n \in N_f \setminus \{1\}, b \in B) \tag{20}$$

where $a_{i,n,b}$ is the i-th arrival time realization of trip n at stop b which is affected by the combined inter-station travel time and dwell time realizations; $\zeta_{i,n,b}$ a binary variable indicating whether trip n should meet the synchronization constraint at transfer stop b during the i-th travel time realization; and θ a parameter indicating the minimum number of realizations for which we should satisfy the synchronization constraints. In particular, parameter θ can receive values in the range $[0, I]$ and θ/I can be perceived as the minimum percentage of realizations for which we require a synchronization between the feeder and collector line(s). In the extreme case where $\theta = 1$ we demand that every trip n is always synchronized with the trip of the collector line at stop b (e.g., for any realization $i \in \{1, 2, ..., I\}$). In the other extreme where $\theta = 0$ we ignore completely the transfer synchronization constraints.

To summarize, the *stochastic* optimization model that considers inter-station travel time and dwell time uncertainties is cast as:

$$(\tilde{P}): \quad \min \ \frac{1}{I} \sum_{i=1}^{I} \sum_{s \in S_f \setminus \{1\}} \sum_{n \in N_f \setminus \{1\}} \left(\left(\frac{h_{i,n,s}}{2} - \frac{h}{2} \right)^2 + M \left(a_{i,n,s} - \Big(x_n + \sum_{z=2}^{s} t_{i,n,z}\Big) \right)^2 \right)$$

$$\begin{aligned}
\text{s.t.:} \quad & a_{i,1,s} = x_1 + \sum_{z=2}^{s} t_{i,1,z} && \forall i \in I, s \in S_f \setminus \{1\} \\
& a_{i,n,s} \ge a_{i,n-1,s} && \forall i \in I, n \in N_f \setminus \{1\}, s \in S_f \setminus \{1\} \\
& a_{i,n,s} \ge x_n + \sum_{z=2}^{s} t_{i,n,z} && \forall i \in I, n \in N_f \setminus \{1\}, s \in S_f \setminus \{1\} \\
& h_{i,n,s} = a_{i,n,s} - a_{i,n-1,s} && \forall i \in I, n \in N_f \setminus \{1\}, s \in S_f \setminus \{1\} \\
& 0 \le Y_{bnm}\,(y_{m,b} - a_{i,n,b})\,\zeta_{i,n,b} \le \Delta t && \forall i \in I, n \in N_f \setminus \{1\}, m \in N_m \setminus \{1\}, b \in B \\
& \sum_{i=1}^{I} \sum_{n \in N_f \setminus \{1\}} \sum_{b \in B} \zeta_{i,n,b} \ge \theta(|N_f| - 1)|B| \\
& \Phi_{n,n'} \left(x_{n'} - a_{i,n,|S_f|} \right) \ge \Phi_{n,n'}\,\psi && \forall i \in I, n \in N_f, n' \in N_f \\
& x_1 \ge \delta_{min} \\
& x_{|N_f|} \le \delta_{max} \\
& x_n \in \mathbb{R}_{\ge 0},\ a_{i,n,s} \in \mathbb{R}_{\ge 0},\ h_{i,n,s} \in \mathbb{R}_{\ge 0},\ \zeta_{i,n,b} \in \{0,1\}.
\end{aligned} \tag{21}$$

4. Case Study

4.1. Current status of the operations and theoretical improvement potential

Our case study is feeder bus line 302 in Singapore operating from Choa Chu Kang Interchange and looping at Choa Chu Kang Crescent. It serves Choa Chu Kang Way, Choa Chu Kang Street and Yew Tee MRT. The feeder bus line is connected with the collector Mass Rapid Transit (MRT) North-South line at station Yew Tee MRT (NS5). Hence, the arrival times of the feeder line trips at stop Yew Tee MRT should be adjusted to the arrival times of the main MRT North-South line, while minimizing the excessive waiting times of feeder line passengers.

Feeder line 302 serves residential blocks, schools and public amenities along Choa Chu Kang Way, Choa Chu Kang St, Choa Chu Kang Dr, Choa Chu Kang North and Choa Chu Kang Cres, connecting them to Choa Chu Kang Town Centre and Yew Tee MRT. Its primary area of service is Choa Chu Kang Neighbourhoods 5 and 6. The route serves schools such as Kranji Secondary School, De La Salle School and Unity Primary School. It also serves community amenities such as Limbang Shopping Centre, Yew Tee Point and Yew Tee Community Centre. High capacity articulated buses are deployed on a daily basis because of high demand from residents. It serves $S_f = 1, 2, ..., 22$ stops and is a circular line (Fig.4).

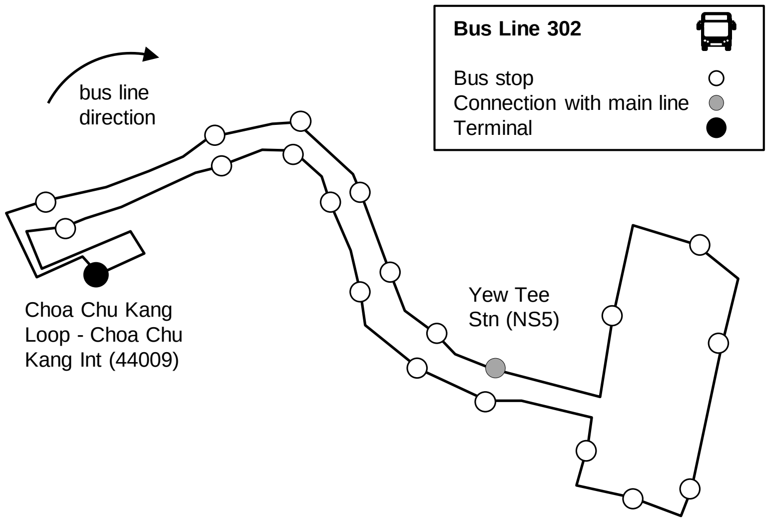

Figure 4: Topology of feeder bus line 302 and location of the transfer stop Yee Tee MRT (NS5) that connects it to the main MRT north-south line.

Feeder line 302 operates 245 bus trips on a typical weekday. Its route length is 8.1km and its total trip travel time ranges between 35 to 40 minutes among peak/off-peak hours. Bus line 302 is selected as our main case study because:

- it is a feeder line connected to a collector MRT line;
- it is one of the seven high-frequency bus lines in Singapore monitored in terms of excessive passenger waiting times (Leong et al., 2016). Hence, maintaining its regularity is important because regular operations receive monetary incentives from

the transport authority (up to 3000$ for every 0.1 minute improvement in regularity at the end of each month).

The ideal headways of feeder line 302 di er across peak/o -peak hours, as presented in Table 3.

Table 3: Ideal headway of feeder bus line 302 at di erent times of the day

Period	Ideal (target) Headway
05:30-08:30	4 min
08:30-19:00	5 min
After 19:00	8 min

Additionally, Fig.5 presents the expected inter-departure travel times of the trips of the feeder bus line based on historical data.

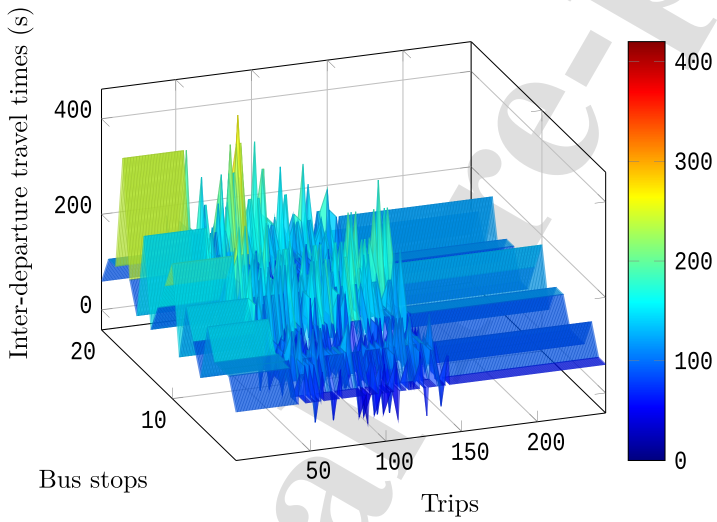

Figure 5: Expected inter-departure travel times between bus stops for the 245 daily trips of the feeder line.

The original schedule that includes the dispatching times of all 245 daily trips of the feeder line is presented in Fig.6.

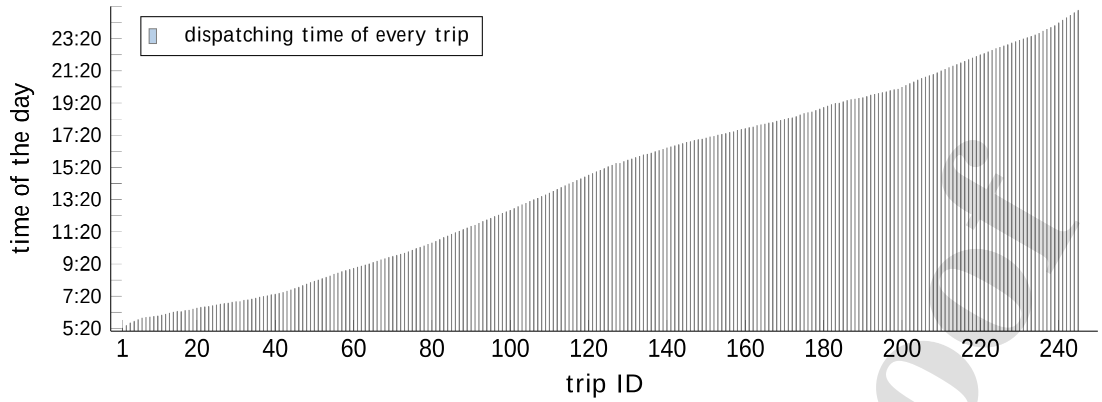

Figure 6: Dispatching times of the original schedule of the feeder line for the 245 trips of the day.

Considering the collector MRT line, the north-south MRT line operates 45km with 27 stations and is a high capacity line with 6-car fully automatic trains. We present its topology in Fig.7, including the transfer station with the feeder line (Yew Tee Stn). Train headways are 2 to 3 minutes during the peak hours of 7am to 9am and about 5 to 7 minutes during off-peak hours.

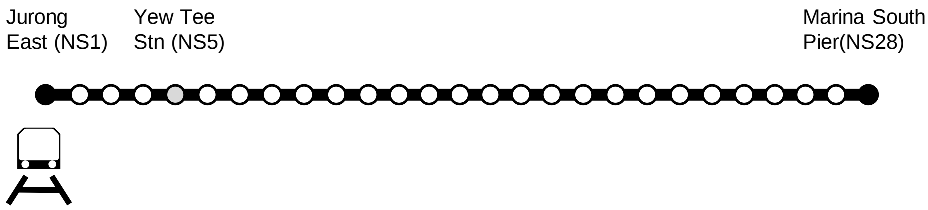

Figure 7: Topology of the North-South MRT line

Passengers transferring from the feeder bus line 302 to the main MRT North-South line at stop NS5 have the following average transfer times at different times of the day, according to the original schedules of the operations (see Fig.8).

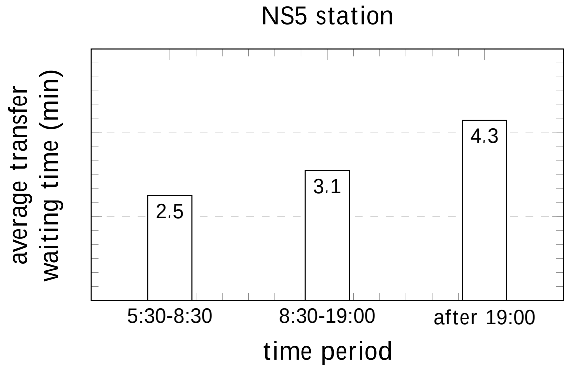

Figure 8: Average transfer waiting times from the feeder bus line 302 to the North-South MRT line at different time periods of the day.

In addition, the passenger excessive waiting times of bus line 302 are measured at twelve control point stops (see Fig.9). Note that the excessive waiting time is the mean value of the square root of Eq.(5) that computes the squared deviation among actual and ideal headways.

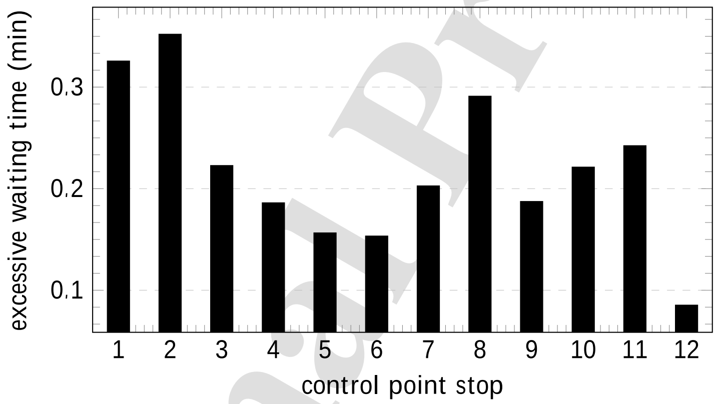

Figure 9: Excessive waiting times of feeder bus line 302 at different control point stops during the daily operations. The overall excessive waiting times after aggregating the regularity from all control point stops is 0.21 min.

With our proposed method, one can compute improved passenger travel times by solving the reformulated convex program ($\tilde{Q}$). The convex program can be solved with numerical optimization methods, such as sequential quadratic programming or the interior point methods (Boggs and Tolle, 1995; Mehrotra, 1992). In this work, we program ($\tilde{Q}$) in Python 3.6 and solve it to global optimality with the commercial optimization solver Gurobi (8.1.1). Because of the convex formulation of ($\tilde{Q}$), the optimization solver is able to converge to a globally optimal solution within 0.29 seconds.

The rescheduled dispatching times of the feeder line resulting from the solution of our program ($\tilde{Q}$) are presented in Fig.10. Fig.10 shows the dispatching time change of every daily trip w.r.t. the dispatching time of the original schedule presented in Fig.6. The rescheduled trips modify their dispatching times in the range -1 min to +4 min. That is, we have trips that are dispatched 1 minute earlier than originally scheduled and trips that are dispatched up to 4 minutes later than originally scheduled.

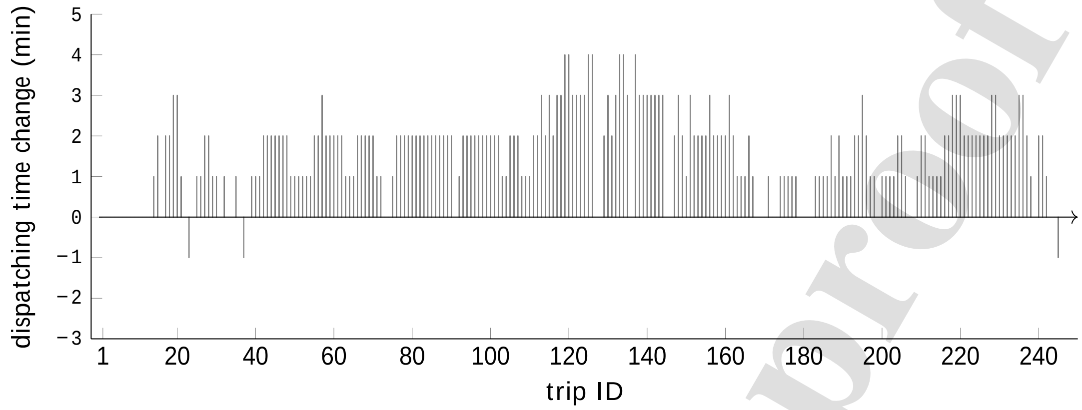

Figure 10: Dispatching time changes of the feeder line trips compared to the orginal schedule.

After rescheduling the dispatching times of the feeder line based on our globally optimal solution, the average transfer waiting times are presented in Fig.11.

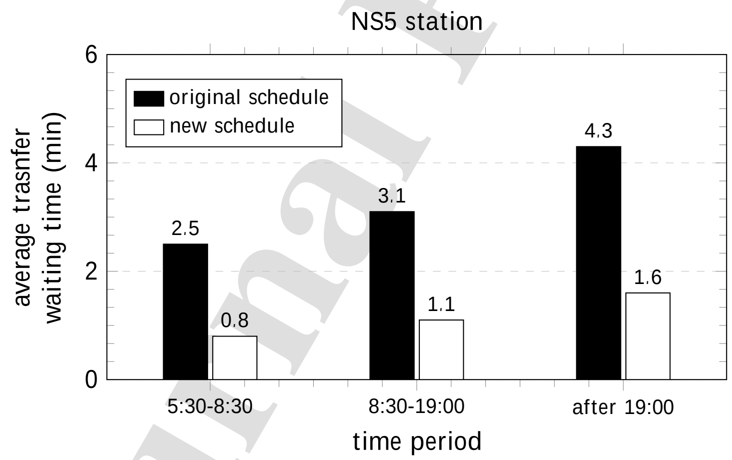

Figure 11: Average transfer waiting times from the feeder bus line 302 to the North-South MRT line with the original schedule and after rescheduling.

Additionally, the excessive waiting times of the feeder line passengers are presented in Fig.12.

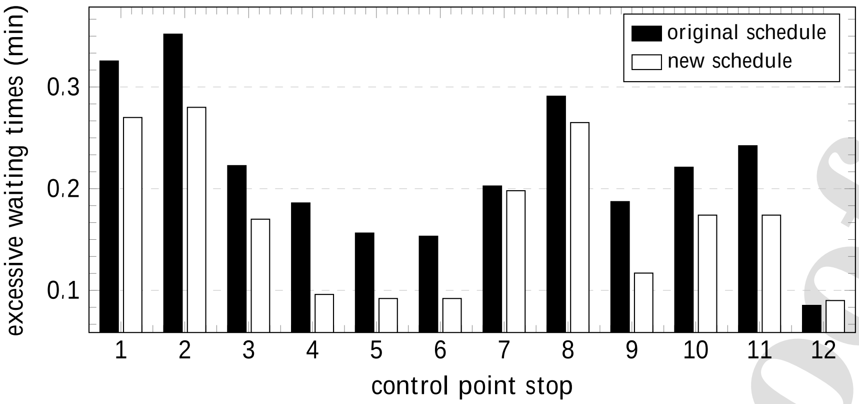

Figure 12: Excessive waiting times of feeder bus line 302 at different control point stops with the original and the new schedule.

As it can be seen from Fig.12, the overall theoretical improvement after applying the new schedule is at the level of 25% demonstrating a significant improvement potential.

4.2. Simulation-based evaluation

After showing the theoretical gain based on the expected travel times, we perform a simulation-based evaluation to compare the performance of our proposed coordination-based schedule (CS) presented in Fig.10 against the following baseline strategies:

- Original schedule (OS). OS refers to applying the original schedule of the feeder line presented in Fig.6. The original schedule is not optimized in terms of regularity or coordination between the feeder and collector line.
- Regularity-based strategy (RB). RB refers to the case where the schedule of the feeder line is adjusted to improve its regularity, without considering the transfer synchronization and the waiting times at transfer stops (see Cats (2014)). To derive such schedule, one should solve program $(\tilde{Q})$ without considering the synchronization constraint (6).

The selected evaluation metrics assess the travel times of passengers and the regularity of the feeder mode. This study uses the following three evaluation metrics:

- Average transfer waiting times of passengers, which indicate the coordination level between the feeder and the main public transport line;
- Excessive waiting times at the regular bus stops of the feeder line;
- The overall travel times of passengers in the public transport system.

The values of the aforementioned evaluation metrics are calculated after applying the OS schedule, the RB schedule, and our proposed CS schedule in 500 Monte Carlo simulation experiments.

To perform realistic simulations, we use as a basis the average values of inter-station travel times and passenger demand observations. Based on these historical averages, in each Monte Carlo simulation we sample the inter-departure travel times and the passenger demand from probability distributions fitted to the inter-station travel time and passenger demand observations. This procedure is described in Fig.13.

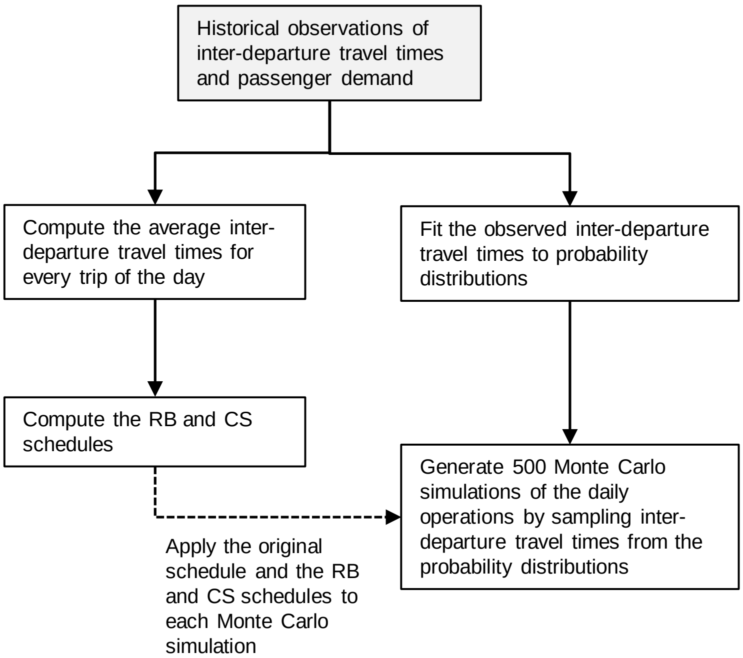

Figure 13: Process of generating Monte Carlo simulations from historical data observations.

Using the simulations, the inter-departure travel times of trips when traveling from one stop to the next can take any value by sampling them from the fitted probability distributions. This enables us to investigate the potential of schedule synchronization in realistic conditions with inter-station travel time and dwell time variations.

After applying the three schedules to 500 simulated daily operations with different inter-station travel times and passenger demand patterns, the results of the daily average transfer waiting times are presented in Fig.14. It is worth noting that the RB and CS schedules are static, in the sense that these schedules are computed considering the average inter-departure travel time values that were presented in Fig.5. These derived schedules are then evaluated in simulation scenarios where the inter-station travel times and dwell times might take different values from their expected ones.

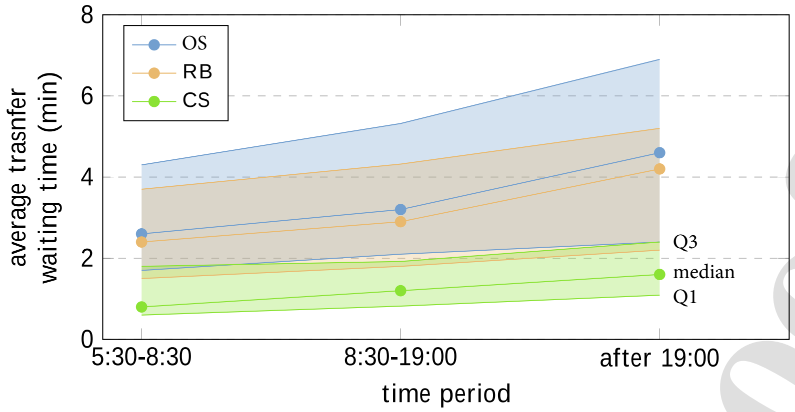

Figure 14: Median and interquartile range of the average transfer waiting time when using the OS, RB, and CS schedules in 500 Monte Carlo simulations of the daily operations.

The dots in Fig.14 present the median value of the average passenger transfer waiting times in the 500 simulations for the three di￿erent time periods of the day. The lower and upper lines present the interquartile range (IQR=$Q_3 - Q_1$) between the 25th and the 75th percentile of the average transfer waiting times over the 500 days. Note that there is significant improvement on the average transfer waiting times when using our CS schedule. To investigate how this improvement is translated into a reduction of passengers travel times, in Fig.15 we present the average passenger travel times after 500 simulations together with the excessive waiting times and transfer times.

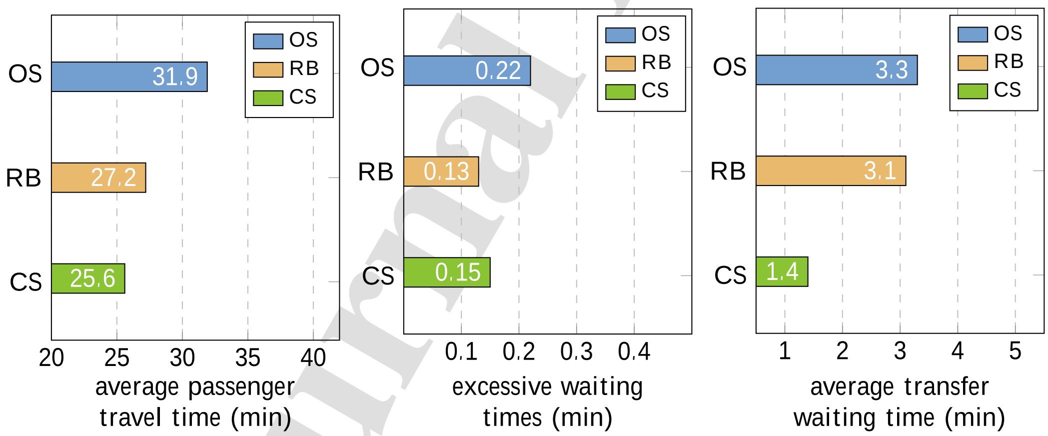

Figure 15: Median of the service regularity of the feeder bus line when using the OS, RB, and CS schedules in 500 Monte Carlo simulations of the daily operations.

From Fig.15, one can note that our proposed schedule (CS) improved the average transfer waiting times by 54% when applied in 500 simulated daily operations. However, the improvement of the overall passenger travel times when applying our schedule is only 6%. This is the case because (i) the transfer times are a small part of the total passenger travel times, and (ii) not all passengers are transferring from the feeder to the collector line. We finally note that the RB schedule, which does not consider the transfer times from the feeder to the collector line, results in a 13% improvement on the excessive travel times of *feeder* line passengers.

4.3. Evaluation of stochastic vs deterministic optimization

In §4.2 the computed RB and CS schedules were deterministic because we considered the average inter-departure travel times from the historical inter-departure travel time observations. These average values were presented in Fig.5. In this section, we further investigate the performance of deterministic schedules by comparing them against a stochastic CS schedule that is derived after solving the stochastic optimization problem $(\tilde{P})$.

To solve the stochastic optimization problem, instead of using the average inter-station travel times in Fig.5 we use directly the historical data observations as input in the optimization process. Each observed day offers another realization of inter-departure travel times and we seek a solution that minimizes the expected cost across all observed days. We note here that when considering multiple days with varying inter-station travel times and dwell times it is not always possible to ensure that there is a feasible schedule that satisfies the synchronization constraints between the feeder and collector lines(s) at all days. That is, there might be some days for which we have missed connections. For this reason, we compute two stochastic optimization schedules:

- CS stochastic 90% which is the stochastic CS schedule that meets 90% of the synchronization constraints across all days with historical data observations
- CS stochastic 95% which is the stochastic CS schedule that meets 95% of the synchronizations constraints across all days with historical data observations

These two stochastic schedules are compared against the deterministic schedule that considers the average inter-departure times. In these comparisons, the two stochastic schedules and the deterministic schedule are applied to 500 Monte Carlo simulations for evaluating their performance, as presented in Fig.16. We note that the computational times of computing the CS stochastic 90% and CS stochastic 95% schedules are 219 sec and 204 sec, respectively. These times are significantly higher than the 0.29 sec required to solve the deterministic problem.

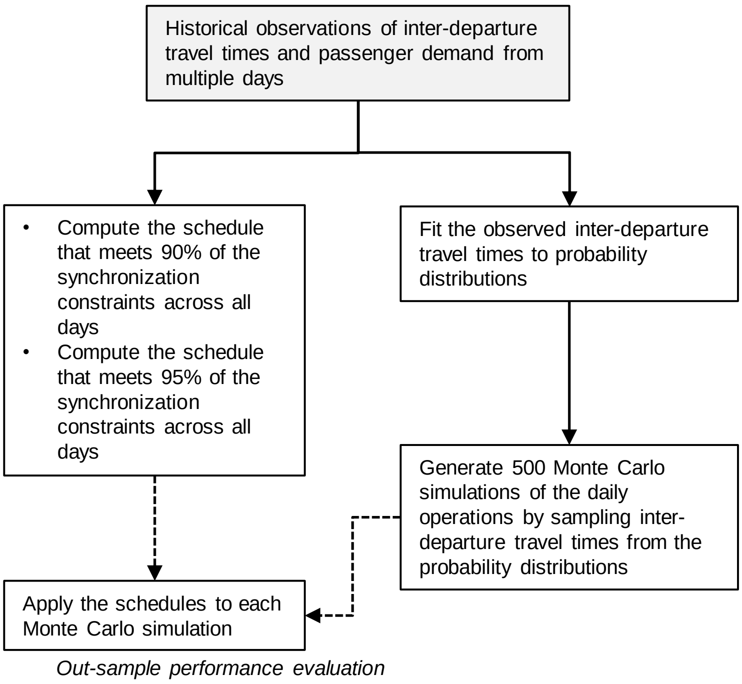

Figure 16: Process of computing the stochastic schedules from historical data and evaluating the performance with the use of newly generated data via Monte Carlo simulations.

That is, we use 500 out-sample Monte Carlo simulations for testing the e□ciency of the derived schedules. The results are presented in Table 4 in terms of the following three key performance indicators: (i) the average travel time of passengers (ATT); the average excessive waiting times at regular stops (EWT); and the average transfer waiting time (TWT).

Table 4: Performance of the coordination-based deterministic and stochastic schedules in 500 simulations

Schedule	ATT (min)	EWT (min)	TWT (min)
CS: deterministic	25.6	0.150	1.4
CS: stochastic 90%	25.5	0.158	1.1
CS: stochastic 95%	25.3	0.162	0.9

From Table 4 one can note that the average transfer waiting times decrease when using a stochastic schedule. This was partly expected because stochastic schedules request to satisfy the synchronization requirements for 90% and 95% of the cases, thus resulting in more reliable transfers. This comes with a cost, however, since guaranteeing more reliable transfers requires to compromise in terms of minimizing the excessive waiting times of passengers at regular stops. This can be explained based on the attributes of the stochastic and deterministic optimization models. The stochastic optimization models are more strict w.r.t. satisfying the transfer synchronization constraints, whereas in a deterministic optimization it is su□cient to satisfy the transfer synchronization constraints in the case of average inter-station travel times. For this reason, the stochastic optimization models have to search on a restricted feasible region when trying to minimize the objective function and this results in increased excessive waiting times at regular stops. In total, the stochastic schedules decrease the average passenger travel time, but only slightly since they do not improve the excessive waiting times at regular stops.

5. Concluding Remarks

This study introduced a mathematical model for reducing the travel times of passengers that use up to two public transit modes to complete their origin-destination journey. This model reschedules the dispatching times of trips that belong to the feeder line and, after its proposed reformulation, it can be solved to global optimality. To reduce the passenger travel times when demand is uncertain, we use the excessive waiting times at regular stops and the waiting times at transfer stops as passenger travel time indicators. To evaluate the performance of our approach in realistic operations, we used a case study in Singapore with a feeder and collector line showing an improvement potential of 6% in terms of passenger travel times. This improvement can increase if there is more demand for transferring from the feeder to the collector line, or reduce if there is limited demand for transfers. For this, the public transit operators might consider relaxing the synchronization constraint at time periods when the demand for transfers is low, and vice versa.

To facilitate the reproduction and extension of our work, we hereby report its main limitations:

(a) our work can be applied when the number of vehicles assigned to the feeder and collector lines at the tactical planning stage su□ces to satisfy the passenger demand;

(b) our work can be applied to reduce the travel times of passengers that use up to two public transit modes for completing their origin-destination trips.

Our model provides a first step towards coordinating feeder and collector lines to improve the e□ciency of MaaS schemes. One can extend it further with the use of highly granular transfer demand data derived from combined public transit tickets purchased via MaaS providers (Kamargianni and Matyas, 2017). In addition, it can be expanded outside the realm of public transit by considering bike-sharing or car-sharing services as feeder modes. This expansion can be further examined in future research where the transferring demand can also be part of the problem formulation.

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- A novel, easy-to-solve model for the coordination of feeder and collector transit lines
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- Improvement potential of 5-10% even under uncertain demand conditions

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