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Cyclic graphs

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Abstract

A subclass of the class of circulant graphs is considered. It is shown that in this subclass, isomorphism is equivalent to Adam-isomorphism. Various results are obtained for the chromatic number, line-transitivity and the diameter.

Keywords: Circulant, Adam isomorphism, line transitive, diameter.

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1 Introduction and summary

We study a subset of the class of circulant graphs. A circulant graph can be defined as follows. Let n be a natural number and let $S = \{k_1, \dots, k_r\}$ with $1 \le k_1 < k_2 < \cdots < k_r \le \frac{n}{2}$. Then the point set of the circulant graph $G(n, S)$ is $\{0, 1, \dots, n-1\}$ and the set of neighbors of the point p is $\{(p \pm k_j) \bmod n | j = 1, \dots, r\}$.
Circulant graphs have been extensively studied, see e.g. Elspach and Turner [1], Davis [2]. The special case we consider, the cyclic graphs $C(n, k)$ have point set $\{0, 1, \dots, n-1\}$ and lines $\{i, i+1\} (\bmod n)$ and $\{i, i+k\} (\bmod n) (i = 1, \dots, n)$ where k is an integer with $2 \le k \le n-2$. So $C(n, k) \simeq G(n, S)$ with $S = \{1, \min\{k, n-k\}\}$.
The graphs $C(n, k)$ are point-transitive, 3-regular if $n = 2k$ and 4-regular otherwise.

In section 2 we identify some well-known graphs of the form $C(n, k)$ and we consider isomorphism between its members.
In section 3 we consider the chromatic number, in section 4 line-transitivity, and in section 5 the diameter of $C(n, k)$.

2 Special graphs and isomorphism

In Table 1 below we list some members of the $C(n,k)$-family.

n	k	graph
4	2	K_4
5	2	K_5
$2m$	2	m-sided antiprism ($m \geq 3$)
6	3	$K_{3,3}$
8	3	$K_{4,4}$
$2m$	m	Möbius-ladders ($m \geq 3$)
10	3	$K_{5,5}\backslash$ perfect matching

Table 1

Note that $C(5,2) \simeq C(5,3)$. More generally, $C(n,k) \simeq C(n,n-k)$ and also if $kk' \equiv \pm 1 (\mathrm{mod}\, n)$, then $C(n,k) \simeq C(n,k')$. The notion of Ádám-isomorphism for circulant graphs (see e.g. Boesch and Tindell [3]) reduces to just these two cases for cyclic graphs. For circulant graphs, pairs of isomorphic graphs are known that are not Ádám-isomorphic. The first example $G(16,\{1,2,7\}) \simeq G(16,\{2,3,5\})$ was found by Elspas and Turner [1]. However the following result says that for cyclic graphs, Ádám isomorphism is equivalent to isomorphism. It is convenient to use the term "sides" for lines of the form $\{i,i+1\}$, and "chords" for lines of the form $\{i,i+k\}$.

Theorem 2.1 *If $k' \neq k, k' \neq n-k$, and $k'k \not\equiv \pm 1 (\mathrm{mod}\, n)$, then $C(n,k) \not\simeq C(n,k')$.*

Proof. For $n \leq 8$ the result can be verified directly. Let $n > 8$. If an isomorphism maps all sides to sides, then $k' = k$ or $k' = n-k$. And if an isomorphism maps all sides to chords, then $kk' \equiv \pm 1 (\mathrm{mod}\, n)$.
Let ϕ be an isomorphism that maps neither all sides to sides nor all sides to chords. Then without loss of generality, the side $\{1,2\}$ is mapped to a side and the side $\{2,3\}$ to a chord; see Figure 1.

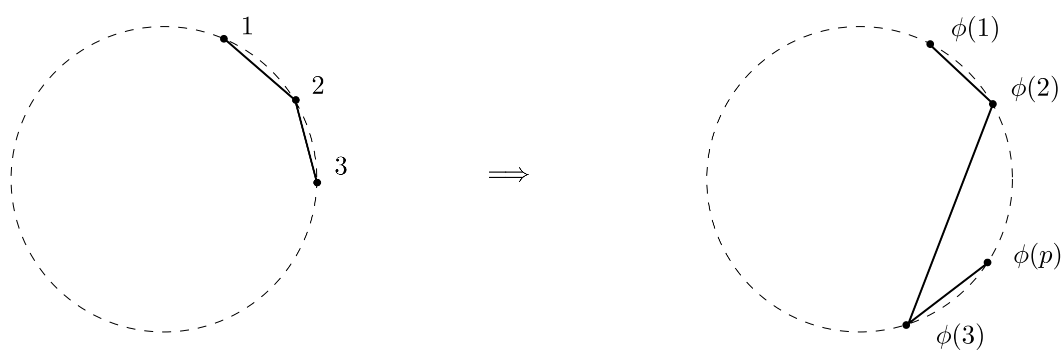

Fig. 1

The predecessor of $\phi(3)$ in the image graph is the image $\phi(p)$ of a point $p \neq 2$. Now p is adjacent to both 1 and 3, hence either $k = 3$ or $k = \frac{n-2}{2}$; see Figure 2.

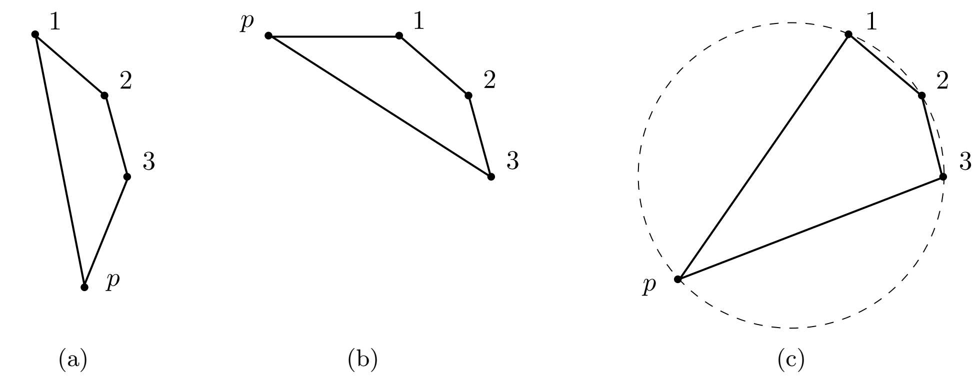

Fig. 2

Since $k = k'$ is excluded, at least one of k and k' is 3. We assume w.l.o.g. that $k = 3, k' = \frac{n-2}{2}$.
In $C(n, 3)$ we now have three possibilities for the sides $\{0, 1\}$ and $\{2, 3\}$.

1. $\{0, 1\}$ is mapped to a side,
2. $\{0, 1\}$ and $\{2, 3\}$ are mapped to intersecting chords,
3. $\{0, 1\}$ and $\{2, 3\}$ are mapped to parallel chords.

In case 1, $\phi(3)$ is adjacent to $\phi(4)$ and $\phi(6)$, hence $\phi(1)$ is also adjacent to $\phi(4)$ and $\phi(6)$; see Figure 3. It follows that 1 is adjacent to 4 and 6, so $n = 8$: a contradiction.

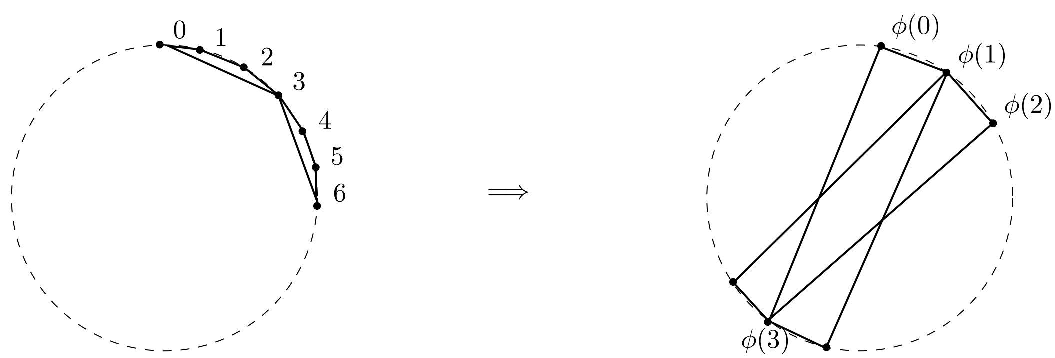

Fig. 3

In cases 2 and 3 the argument is exactly the same. Figure 4 gives the situation in case 2. □

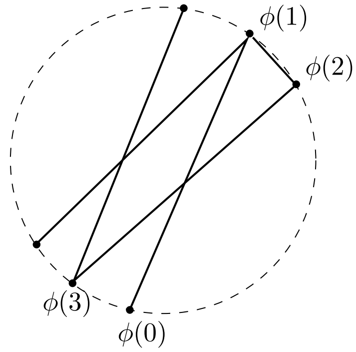

Fig. 4

In view of $C(n-k) \simeq C(n, n-k)$ we restrict our attention to pairs (n, k) with $n \geq 2k$.

3 The chromatic number

We denote the chromatic number of $C(n, k)$ by $x(n, k)$. For the colors we use red, white, blue, abbreviated R, W, B.

Proposition 3.1

a) n *even and* k *odd* $\Longleftrightarrow x(n, k) = 2$.

b) $n = 5 \Longleftrightarrow x(n, k) = 5$.

c) If $3|n$ *then* $x(n, 2) = 3$,
if $3 \nmid n$ *and* $n \neq 5$ *then* $x(n, 2) = 4$.

d) If k *even and* $n = 2k \geq 8$, *then* $x(n, k) = 3$.

Proof. Trivial. □

We now present several sub-families of graphs, each with a 3-coloring.

Proposition 3.2
If n *and* k *are odd and* $n \geq 3k$, *then* $x(n, k) = 3$.

Proof. Let n and k be odd and $n \geq 3k$. Color the points $1, 2, \ldots, k$ alternatingly B and R, the points $k+1, \ldots, 2k$ alternatingly R and W, and the rest alternatingly W and B. This is a proper coloring as is easily verified. □

Proposition 3.3

$$\begin{aligned} x(n,k) \leq 3 \quad & \textit{if} \quad n \equiv 0, \quad k \equiv 1 \textit{ or } 2, \\ & \textit{or} \quad n \equiv 2, \quad k \equiv 1, \qquad n \geq 8, \\ & \textit{or} \quad n \equiv 1, \quad k \equiv 1, \qquad n \geq 2k+4, \end{aligned}$$

where all congruences are modulo 3.

Proof. If $n \equiv 0$ and $k \equiv 1$ or 2, one simply repeats the pattern RWB. It works locally since $3 \nmid k$, and globally since $3|n$.
If $n \equiv 2, k \equiv 1$, the pattern RWB can again be used, now followed by a "tail" of two points which are colored RW. The case $n = 5$ has to be excluded.
If $n \equiv 1, k \equiv 1$, then write $n = n_1 + n_2$ where $n_i \equiv 2$ and $n_1 - n_2 = 0$ or 3. Then apply the pattern RWB and a tail RW to the points $1, 2, \ldots, n_1$, and the same for the remaining points. □

Proposition 3.4

$$x(n,k) \leq 3 \quad \textit{if} \quad n \equiv 0, \quad k \equiv \pm 1, \\ \textit{or} \quad n \equiv 2, \quad k \equiv 1,$$

where all congruences are modulo 5.

Proof. Repeat the pattern $RWBRW$; cut it off when $n \equiv 2$. □

Proposition 3.5

$$x(n,k) \leq 3 \quad \textit{if} \quad n \equiv 0, \quad k \equiv \pm 1, \\ \textit{or} \quad n \equiv 2, \quad k \equiv 1, \\ \textit{or} \quad n \equiv 5, \quad k \equiv -1$$

where all congruences are modulo 7.

Proof. Repeat the pattern $RWBRWRW$; cut it off when $n \not\equiv 0$. □

Proposition 3.6

$$x(n,k) \leq 3 \quad \textit{if} \quad n \equiv 0, \quad k \equiv 1, 3, 6, \ \textit{or} \ 8, \\ n \equiv 2, \quad k \equiv 1, 3, \ \textit{or} \ 8, \\ n \equiv 4, \quad k \equiv 1 \ \textit{or} \ 3, \\ n \equiv 5, \quad k \equiv 6 \ \textit{or} \ 8, \\ n \equiv 7, \quad k \equiv 1, 6, \ \textit{or} \ 8,$$

where all congruences are modulo 9.

Proof. Repeat the pattern $RWRWBW\,BRB$; cut if off when $n \not\equiv 0$. □

The cases covered by Propositions 3.1a,3.2,. . .,3.6 do by no means form a set of covering congruences. The smallest case not covered is $C(13, 5)$.

Proposition 3.7 $x(13, 5) = 4$.

Proof. Suppose a 3-coloring of $C(13, 5)$ exists. We distinguish two cases.

1. There exists an i such that the points i and $i+3$ have the same color. Without loss of generality assume that the points $0, 1, 2, 3$ have the colors R, W, B, R, respectively, and that point 8 has the color W. Then the colors of the points $7, 6, 5, 4, 9$ are successively forced, and point 10 then has 3 differently colored neighbors.

2. For all i, the points i and $i+3$ have different colors. Then without loss of generality the points $0, 5, 10$ have the colors R, W, B, respectively. Each following point in the sequence $10+5i \pmod{13}$ then has a forced color, and at the last point, a fourth color is necessary.

In both cases we have a contradiction, hence $x(13, 5) = 4$. □

4 Line-transitivity

It is easily seen that of the "known" graphs in Table 1, $C(n, k)$ is line-transitive for the pairs $(n, k) = (4, 2), (5, 2), (6, 2), (6, 3), (8, 3)$, and $(10, 3)$. But there are others.

Proposition 4.1
If $k^2 \equiv \pm 1 \pmod{n}$, then $C(n, k)$ is line-transitive.

Proof. $f(i) = ki \pmod{n}$ is an automorphism. □

Proposition 4.2
If $n = 2k + 2$, then $C(n, k)$ is line-transitive.

Proof. If k is odd, say $k = 2v + 1$, then $k^2 = 4v^2 + 4v + 1 \equiv 1 \pmod{n}$, so these cases are covered by Proposition 4.1. If k is even, consider the map f defined by

$$f(i) = \begin{cases} ki & \text{for} \quad 0 \le i \le k, \\ ki + k + 1 & \text{for} \quad k+1 \le i \le 2k+1. \end{cases}$$

We claim that f is an automorphism.

First consider a pair of the form $\{i, i+1\}$. There are 3 cases.

1. $\mathbf{0 \le i < i + 1 \le k}$. $f(i) = ki, f(i+1) = (k+1)i$, and the difference is k.

2. $\mathbf{i = k}$. $f(i) = f(k) = k^2, f(i+1) = k(k+1) + k + 1$, and the difference is $2k + 1 \equiv -1 \pmod{n}$.

3. $\mathbf{i = 2k + 1}$. $f(i) = k(2k+1) + k + 1, f(i+1) = f(0) = 0$, and the difference is $2k^2 + 2k + 1 \equiv 1 \pmod{n}$.

Next consider a pair of the form $\{i, i+k\}$. There are 4 cases.

4. $\mathbf{i = 0}$. $f(i) = 0, f(i+k) = f(k) = k^2$, and the difference is $k^2 = \frac{k}{2}(2k+2) - k \equiv -k (\bmod n)$.

5. $\mathbf{1 \leq i \leq k}$, hence $k+1 \leq i+k \leq 2k.f(i) = ki, f(i+k) = k(i+k)+k+1$, and the difference is $k^2+k+1 \equiv 1(\bmod n)$.

6. $\mathbf{i = k+1}$. $f(i) = k(k+1)+k+1, f(i+k) = k(2k+1)+k+1$, and the difference is $k^2 \equiv -k(\bmod n)$. (See 4).

7. $\mathbf{k+2 \leq i \leq 2k+1}$. $f(i) = ki+k+1, f(i+k) = k(i+k)$; the difference is $k^2-k-1 \equiv 1(\bmod n)$.

Note that in the cases 4,5,6,7 we must have k even.
In all 7 cases, a line is mapped to a line. □

We now present some cyclic graphs that are not line-transitive. One way to prove results of this kind is as follows. A priori there are 2 kinds of lines: sides and chords. Now if a side belongs to more triangles, say, than a chord, then obviously the graph is not line-transitive. If triangles do not work, we can take some other graph. In the sequel we use a somewhat more convenient way of counting: if all subgraphs of a specified form together contain more sides than chords, then the graph is not line-transitive (assuming $n \neq 2k$).

Proposition 4.3
If $n > k(k+1)$, then $C(n,k)$ is not line-transitive.

Proof. Consider subgraphs of the form C_{k+1}. Let $i_0, i_1, \ldots, i_k$ with subscripts mod$(k+1)$ be the points of some C_{k+1}. Then $\forall_j : v_j := i_{j+1} - i_j \in \{1, -1, k, -k\}$. Also

$$(*) \quad (*) \sum_0^k v_j \equiv 0(\bmod n).$$

¿From the condition $n > k(k+1)$ it follows that $\sum_0^k v_j = 0$ is the only way to satisfy (*). Let v_j assume the values $\pm k$ t times. If t is odd then the terms $\pm k$ cannot cancel, and hence $t = 1$. Since the number of sides (terms ± 1) is larger than the number of chords (terms $\pm k$) in each C_{k+1}, it follows that $C(n,k)$ is not line-transitive when $n > k(k+1)$ and t odd.
If t is even then the terms $\pm k$ can cancel, leading to other ways of forming C_{k+1}'s. These may have a surplus of chords, which might compensate the abundance of $1's$ in the previous type of C_{k+1}. But to each C_{k+1} with a surplus of $\pm k's$, a different C_{k+1} corresponds which annihilates this effect. E.g. when $k = 5$, we have $5+5+1-5-5-1 = 0$, but also, interchanging $1's$ and $5's : 1+1+5-1-1-5 = 0$. Hence, also when t is even the number of sides is larger than the number of chords in all C_{k+1}'s together. Again, $C(n,k)$ is not line-transitive. □

Proposition 4.4

For all pairs (λ, k) with $\lambda = 2, k \geq 4$ or $\lambda \geq 3, k \geq 3$, $C(\lambda k, k)$ is not line-transitive.

Proof. For $\lambda = 2, k \geq 4$, consider subgraphs of the form C_4. These are all of the form $k + 1 - k - 1$, with equal numbers of sides and chords. However, in these graphs (Möbius ladders) the number of chords is only half the number of sides.
For $\lambda = 3, k \geq 3$, consider subgraphs of the form C_3. These are all of the form $k+k+k \equiv 0$.
For $\lambda = 4, k = 3$, consider $C_4's$. There are three types; see the table below.

type of C_4	number	A	B
3-1-1-1	12	36	12
3-1-3+1	12	24	24
3+3+3+3	3	0	12

Under the heading A we find the total numbers of sides per type, under the heading B the total numbers of chords. Since the column totals of A and B are different, $C(12, 3)$ is not line-transitive.
For $\lambda = 4, k \geq 4$ we consider $C_4's$ again, and finally for $\lambda \geq 5, k \geq 3$ we consider $C_\lambda's$. The details for these two cases are left to the reader. □

5 Diameter

We denote the diameter of $C(n, k)$ by $d(n, k)$. The diameter has a very irregular behaviour, e.g. $d(n, k)$ is not monotone in n for fixed k, and even $\min_k d(n, k)$ is not monotone in n. But sharp upper and lower bounds will be given.

Proposition 5.1 $$d(n, k) \leq d(n, 2) = \lfloor \frac{n+2}{4} \rfloor.$$

Proof. $d(n, 2) = \lfloor \frac{n+2}{4} \rfloor$ follows easily from the fact that from 0 the points $2i - 1$ and $2i$ are at distance i for $0 \leq i \leq \frac{1}{2} \lfloor \frac{n}{2} \rfloor = \lfloor \frac{n+2}{4} \rfloor$.

To prove the inequality, it will suffice to prove that in $C(n, k)$ every point v, where $1 \leq v \leq \lfloor \frac{n}{2} \rfloor$, can be reached from 0 in $d(n, 2)$ steps or less. For $k = 3$, this can be verified directly.

Let $k \geq 4$ and let $v \leq \lfloor \frac{n}{2} \rfloor$. Choose r such that $rk \leq v < (r + 1)k$. Then v can be reached in at most $r + k - 1$ steps. If $r \geq 4$, then $(r - 2)(k - 2) \geq 4$ hence $r + k - 1 \leq \frac{rk}{2} - 5 \leq \frac{v}{2} - 5 < \lfloor \frac{n+2}{4} \rfloor$.

For the remaining cases, i.e. when $v \leq 4k - 1$ and $k \geq 4$, the inequality can be easily verified. □

Proposition 5.2 $\min_k d(n,k) \geq \frac{\sqrt{2n-1}-1}{2}$ *with equality if (and only if) the right hand side is an integer.*

Proof. To prove the inequality, start at point 0. If p is a point at distance L from 0, then $p = \alpha + \beta k \pmod n$ with $\alpha, \beta \in \mathbb{Z}$ and $|\alpha| + |\beta| = L$. For the value of α there are $2L+1$ possibilities. If $|\alpha| = L$, then β does not occur in the expression $\alpha + \beta k$, so here we have 2 possibilities. If $|\alpha| < L$, there are 2 possibilities for β, yielding $4L - 2$ possibilities for the pair (α, β). Hence the total number of possibilities for the pair (α, β) is $4L$. It follows that

$$(*) \quad (*) n \leq 1 + 4 + 8 + 12 + \ldots + 4d(n,k),$$

which is equivalent to $d(n,k) \geq (\sqrt{2n-1} - 1)/2$.

If $(\sqrt{2n-1} - 1)/2$ is an integer, say D, then $n = 2D^2 + 2D + 1$. Take $k = 2D + 1$. It is sufficient to show that each point in $C(n,k)$ is at distance $\leq D$ from 0, or alternatively that for all x, there exist $\alpha, \beta \in \mathbb{Z}$ such that $|\alpha| + |\beta| \leq D$ and

$$x \equiv (\alpha + \beta(2D+1)) \bmod n.$$

We have seen above that there are $1 + 4(1 + 2 + \ldots + D) = 2D^2 + 2D + 1$ possibilities for the pair (α, β). Is is sufficient to show that all residue classes are different.

Suppose $\lambda + \mu(2D+1) \equiv \alpha + \beta(2D+1)$ with $|\alpha| + |\beta \leq D$, $|\lambda| + |\mu| \leq D$. Then $\lambda - \alpha + (\mu - \beta)(2D+1)$ is a multiple of $2D^2 + 2D + 1$. Taking absolute values, it is clear that this multiple can only be 0 or $\pm(2D^2 + 2D + 1)$.

If it is 0, then $\lambda = \alpha$, $\mu = \beta$, and we are through. So suppose

$$\lambda - \alpha + (\mu - \beta)(2D+1) = 2D^2 + 2D + 1.$$

(the minus sign runs similarly.)

Then $\lambda - \alpha + (\mu - \beta)(2D+1) = D(2D+1) + D + 1$, hence $D + 1 + \alpha - \lambda$ is a multiple of $2D+1$. This multiple can only be 0 or $\pm(2D+1)$. If $D + 1 + \alpha - \lambda = 0$, then $\mu - \beta = D$, hence $\lambda = \alpha + D - 1$, $\mu = \beta + D$. But $|\lambda| + |\mu| \leq D$ and we have a contradiction. If $D + 1 + \alpha - \lambda = 2D + 1$ then $\alpha - \lambda = D$, hence $\mu - \beta = D + 1$, and again we have a contradiction. The case $D + 1 + \alpha - \lambda = -(2D+1)$ is similar. □

Proposition 5.3 *If* $d(n,k) \leq \frac{n}{k}$, *then* $d(n+2k, k) = 1 + d(n,k)$.

Proof. Clearly $d(n,k) < d(n+2k,k)$; in a shortest path from 0 to $k+i$, $i = 1, \ldots, n-1$, in $C(n+2k,k)$, leave out first chord to obtain a path from 0 to i in $C(n,k)$.

Also since $d(n,k) \leq \frac{n}{k}$, every shortest path from 0 to i in $C(n,k)$, $i = 1, \ldots, n-1$ can be made to a path from 0 to $i+k$ in $C(n+2k,k)$ by adding one chord.

Clearly the points $1, \ldots, k$ and $n+k, \ldots, n+2k-1$ can be reached in (either $d(n,k)$ or) $d(n,k)+1$ steps. □

Corollary *If* $n \geq k^2$, *then* $d(n+2k,k) = 1 + d(n,k)$.

The condition $n \geq k^2$ may seem strong, but it cannot be omitted. The smallest counterexample is $d(19,8) = 3$, $d(35,8) = 5$. The smallest counterexample with difference 3 is $d(31,14) = 5$, $d(59,14) = 8$.

Proposition 5.4 *For every $M > 0$ there are n and k such that $d(n+2k, k) > d(n, k) + M$.*

Proof. Take $n = 2k+3$, $k = 6M+2$. In $C(n, k)$ the points 3λ can be reached from 0 in 2λ steps by using chords ($\lambda = 1, 2, \ldots, M$) and hence the point $3M+1$ in $2M+1$ steps. Again by using just chords, the point $k - 3\lambda$ can be reached in $2\lambda + 1$ steps ($\lambda = 1, 2, \ldots, M$). It follows that $d(n, k) \leq 2M + 1$.

Now consider $C(n + 2k, k)$. To advance 3 places by using chords only, 4 chords are required. Hence here it is in no case optimal to use more than 3 chords in a path. To get from 0 to $k + \frac{1}{2}k + 1$, one may use either 2 or 3 chords. In the first case, the number of sides required is $\frac{1}{2}k - 1$, in the second case it is $\frac{1}{2}k - 2$. Hence the distance from 0 to $k + \frac{1}{2}k + 1$ is $\frac{1}{2}k + 1 = 3M + 2$. It follows that $d(n + 2k, k) \geq 3M + 2 > d(n, k) + M$. □

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