Absolute Position Detection in 7-Phase Sensorless Electric Stepper Motor

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Abstract—Absolute position detection in sensorless electric stepper motors potentially allows for higher space efficiency, improved shock resistance, simplified installation, reduced number of parts and lowered cost.

A prototype is demonstrated measuring $42 \times 42 \times 34 \text{mm}^3$ with seven coils arranged in a star configuration. The rotor is $25.8 \times 12.5 \text{mm}^2$ and has 51 teeth which are irregularly spaced. At the driver side, the coil currents are measured during motion in order to reconstruct the absolute position of the motor. Calibration and smoothing techniques are used to reduce systematic and stochastic measurement errors, respectively.

The motor is able to detect and correct its position after externally-induced stalls at the tested motor speeds from 40 rpm to 108 rpm. The holding torque is 0.23 Nm at an armature current of 1 A; on average the torque is 7% lower than that of a reference bipolar stepper motor with the same dimensions.

The results show that dynamic position sensing and correction are possible for a range of velocities, but not at standstill. The driver requires seven current sensors and sufficient computational power, and proper calibration of motor intrinsics is required beforehand. The presented technology could make existing 3-D printers and other machines with open-loop stepper motors more robust and increase the range of operating speeds and accelerations, without the adverse side-effects of increased complexity and cost associated with dedicated position sensors.

I. INTRODUCTION

Electric motors are commonly used to actuate robotic systems and other machines [1]. Permanent magnet stepper motors are popular in many applications thanks to the high torque-to-volume ratio which allows to directly drive the joints of e.g. a 3-D printer without need for a gearbox.

In many systems it is essential that the position of each joint is known with high accuracy. This can be done by a combination of calibration (measuring the position at a specific moment) and tracking (recording all angular displacements since the moment of calibration). Dedicated components may be needed for calibration and tracking, and re-calibration may be required when the tracking phase is disrupted due to e.g. a power cycle or a stalled motor (skipping of steps).

Calibration can be performed by a range of techniques. Most of these utilize the concept of a uniquely-identifiable home position using e.g. a limit switch, photointerrupter, Hall-effect sensor, multi-turn encoder, physical constraint, etcetera. Tracking can be done by counting commanded angular displacements in feed-forward control, or by measuring incremental displacements using photointerrupters, Hall-effect or back-emf sensors, etcetera [2]. Two digital sensors can be arranged in a quadrature encoding scheme to incorporate angular distance and direction [3], and the use of singular analog sensors such as a potentiometer, electromagnetic resolver [4] or grayscale photointerrupter [5] allows for absolute position measurements which can also be used in the calibration part.

The inclusion of physical sensors for position detection and/or calibration requires space (and additional cabling) and therefore reduces the torque-per-volume ratio of the motor assembly and adds complexity and cost to the system. Low-cost 3-D printers are increasingly affordable [6], [7] with commercial prices well under €200 for certain popular models. Being equipped with four or five stepper motors the budget per actuator is relatively low, and therefore such a 3-D printer is normally actuated by NEMA17-sized feed-forward stepper motors without dedicated position sensors.

An example stepper motor driver often used in 3-D printers is the DRV8825 (Texas Instruments, Dallas, USA). It uses STEP/DIR control signals in which the rising edge on the STEP signal corresponds to a predefined angular rotation in the direction given by the DIR control signal. Unexpected spikes in motor load due to e.g. high incidental friction may lead to stalls and incorrect positioning, resulting in a failed print. More advanced stepper motor drivers may allow to

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senses these stalls by current/back-EMF measurements, but recovery from such events requires re-calibration (i.e., executing the homing procedure) which is often inconvenient.

A different type of electric motor is the brushless DC motor (BLDC). Its stator windings are generally constructed in a three-phase star topology, as opposed to the two-phase disconnected topology in most stepper motors. Back-emf sensing in the driver is usually employed in BLDC systems to determine the right moments for switching the driver output phases, in order to operate the motor at the maximum speed for a given supply voltage and load. A BLDC motor which is controlled without physical encoder on the motor shaft is commonly called a sensorless BLDC motor, even though voltage and/or current sensors may be present in the driver electronics.

This research presents a new type of electric stepper motor which incorporates the feature of full-rotation absolute position measurement by current sensing in the driver. To our knowledge this is the first implementation of this kind. The motor is evaluated in terms of absolute position sensing accuracy and the torque-speed curve characteristics compared to a similarly-sized conventional stepper motor.

A. State of art

Morris et al. developed a switched reluctance motor with irregular rotor [8]. In this motor the absolute rotor position can be estimated by activating pairs of stator phases and observing the magnetic reluctance. The motor cannot be actuated while the position is sensed, also the concept of switched reluctance motor has inferior torque-to-volume characteristics compared to motors with permanent magnet(s) in its rotor [1], [9].

Consumer-grade 3-D printers are generally equipped with NEMA17-size electric stepper motors driven by a suitable stepper motor driver [10]. Trinamic developed the TMC2130 (Trinamic Motion Control, Hamburg, Germany) which can also detect stalls by means of current sensing [11].

The authors of this paper earlier developed a three-axis stepper motor with shared stator [12]. It uses seven wires to actuate three rotors independently with a common set of stator coils. The discriminating feature is the number of magnetic pole pairs, which is different for the three rotors and therefore each rotor operates in a different harmonic mode. A rotor also generates back-emf signals within its harmonic mode, which can be measured using current sensors and signal processing techniques. The three-axis motor by itself does not have methods for absolute position sensing, but this changes when the three output shafts would be mechanically linked together which effectively transforms it into one physical rotor consisting of three logical rotors.

II. METHODS

A. Design and construction

The presented motor consists of a stator with seven coils, an axially magnetized rotor with 51 north poles and 51 south poles mounted on a shaft and two ball bearings in bearing holders. The angular spacing between adjacent poles is variable. Seven electrical wires connect the motor to the driver electronics which houses seven half H-bridges and current sensors and a microprocessor.

1) Stator: The stator core measures $42 \times 42 \times 16.5$ mm$^3$ and is constructed by laser-cutting parts from 0.5 mm thick electrical steel plates (LCP Laser-Cut-Processing GmbH, Hermsdorf, Germany) of grade M400-50A, stack bonded with backlack. The seven arms are evenly spaced around a full circle, and each arm has seven teeth with spacing $360^\circ / 49 = 7.06^\circ$ (duty cycle 40%) so there are 49 teeth in total. This multiple of 7 is as close as possible to the 48 stator teeth and 50 rotor pole pairs present in similarly-sized bipolar stepper motors such as the Trinamic QSH4218. The seven stator windings have 55 turns of 0.33 mm thick copper wire and are arranged in a star configuration. The bore is reamed to 25.90 mm. Aluminium bearing holders with bearings type 625-2Z are mounted on the stator core. Figure 3 shows a cross-section of the stator and Figure 2 shows the actual parts.

2) Rotor: The rotor measures $25.80 \times 12.5$ mm$^2$. The neodymium permanent ring magnet (Eagle Magnet, Xiamen, China) has grade 50M and measures $23.8 \times 1.5$ mm$^2$ with a $13.1$ mm bore. The two rotor halves each have 5.5 mm thickness and are constructed from the same electrical steel plates as the stator. Figure 2 includes a picture of the rotor. The number of (irregularly spaced) physical rotor pole pairs $n_r$ must be in the form $7k \pm 2$ and equals 51 for our motor;
The maxima and minima of the $B$ are the mathematical analysis of absolute position reconstruction. This specific representation significantly simplifies the physical, irregular rotor can be seen as a superposition of three logical, regular distribution of pole pairs. The physical, irregular rotor can be approximately 1.5 T.

$B(\alpha_r)$ is an odd function, i.e. $B(\alpha_r) = -B(-\alpha_r)$. This is convenient for production as the rotor’s top half is identical to the bottom half positioned upside-down.

The seven stator arms with its coils are numbered $c = 0...6$ and are regularly spaced, so the angular positions of the arm midpoints are given by $\alpha_w(c) = 2\pi c/7$.

Using $\alpha_r(c) = \alpha_w(c) - \theta = 2\pi c/7 - \theta$ and working modulo $2\pi$ we can describe the total magnetic flux from the rotor’s permanent magnet through each coil $c$ at a given rotor position $\theta$:

$$
\Phi_B(\theta, c) = \Phi_0 \sum_{r=1}^{3} w_r \cos \left(2\pi n_r c/7 - n_r \theta + \gamma_r\right)
$$

### 3) Driver electronics:

The driver consists of a microcontroller, H-bridges and current sensors. The microcontroller is of type STM32H745 (STMicroelectronics, Geneva, Switzerland) and has two cores of which one is used: the Arm Cortex-M7 core with 480 MHz clock speed. There are fourteen half H-bridges of which seven are used in this research, of type DRV8874 (Texas Instruments, Dallas, TX, US). The H-bridges are pulse-width modulated at a frequency of 32 kHz with a duty cycle between 0% and 100% in 7500 distinct steps, resulting in 4.2 ns resolution.

The output currents are measured using integrated current sensors of type INA240 (Texas Instruments) and sampled by the microcontroller at a sampling rate of 49.7 kHz per channel with 12-bit precision, without PWM synchronization. The samples are transferred to a cyclic buffer using direct memory access (DMA).

The microcontroller runs a periodic task at a frequency of 5 kHz for input signal processing, discrete Fourier transform calculation, absolute position estimation and output waveform computation.

Communication with the host computer is done over two serial-over-USB interfaces. The high-speed interface is used for data transfer and the low-speed interface for debugging purposes.

### B. Control and sensing

The wires of the seven windings are arranged in a star configuration with the central point being the common virtual ground, i.e. with potential zero, of which we assume that this is halfway the positive and negative rails. Sinusoidal waveforms are used for driving the rotor with the sum of output voltages (relative to the virtual ground) always being equal to zero. If the coils were modelled as identical resistors then the definition of the virtual ground is consistent with Kirchhoff’s current law, and we assume that changes in the virtual ground level due to e.g. pulse with modulation, coil inductance, back-emf and other effects can be neglected within the context of this research for reasons that are explained later.
The currents through the seven wires are denoted $I_0...I_6$. Kirchhoff’s law states that the sum of currents must be zero, so there are six independent degrees of freedom (DOFs). The resulting 6-dimensional hyperplane can be conveniently represented by three orthogonal complex planes, or complex "power ports", which we call the 50-plane, 51-plane and 52-plane, with each plane having a complex value (phase and amplitude).

Two DOFs are used for actuation of the rotor, called the 51-phase and 51-amplitude. Two other DOFs, the 50-phase $\phi_{50}$ and 52-phase $\phi_{52}$, are used for absolute position reconstruction. The remaining two DOFs, the 50-amplitude and 52-amplitude, are not explicitly used in this research, although these could be utilized for more advanced sensing and control techniques, e.g. involving dynamic torque measurements.

The physical rotor angle is $\theta$. The driver does not know the exact value of $\theta$ but keeps track of an estimated value called $\hat{\theta}$. The difference is the position estimation error, $\varepsilon = \theta - \hat{\theta}$.

The commanded position, or setpoint angle, is $\theta_{\text{cmd}}$ which is externally controlled using a STEP/DIR interface with step angle 0.1125$\degree$.

The driver calculates the necessary motion profile to reach $\theta_{\text{cmd}}$, taking the maximum velocity and acceleration into account, and computes the angle of minimum energy $\theta_{\text{ctrl}}$ to apply torque on the rotor. A list of relevant symbols is included in Table [III].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_w$</td>
<td>rad</td>
<td>Angle in world coordinate frame</td>
</tr>
<tr>
<td>$\alpha_r$</td>
<td>rad</td>
<td>Angle in rotor coordinate frame, $\alpha_w - \theta$</td>
</tr>
<tr>
<td>$n_r$</td>
<td>-</td>
<td>Teeth count of logical rotor $r$</td>
</tr>
<tr>
<td>$w_r$</td>
<td>-</td>
<td>Relative weight of logical rotor $r$</td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>rad</td>
<td>Relative phase angle of logical rotor $r$</td>
</tr>
<tr>
<td>$R$</td>
<td>$\Omega$</td>
<td>Phase resistance</td>
</tr>
<tr>
<td>$L$</td>
<td>H</td>
<td>Phase inductance</td>
</tr>
<tr>
<td>$N_{\text{turns}}$</td>
<td>-</td>
<td>Number of turns per coil</td>
</tr>
<tr>
<td>$\Phi_B$</td>
<td>Wb</td>
<td>Magnetic flux</td>
</tr>
<tr>
<td>$V_{\text{nom}}$</td>
<td>V</td>
<td>Nominal coil voltage</td>
</tr>
<tr>
<td>$V_c$</td>
<td>V</td>
<td>Voltage across phase $c$</td>
</tr>
<tr>
<td>$I_c$</td>
<td>A</td>
<td>Current through phase $c$</td>
</tr>
<tr>
<td>$A_r$</td>
<td>-</td>
<td>Discrete Fourier transform, $\hat{A} = \mathcal{F}(\hat{T})$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>rad</td>
<td>Physical rotor angle</td>
</tr>
<tr>
<td>$\theta_{\text{cmd}}$</td>
<td>rad</td>
<td>Commanded rotor angle (setpoint)</td>
</tr>
<tr>
<td>$\theta_{\text{ctrl}}$</td>
<td>rad</td>
<td>Driver control angle</td>
</tr>
<tr>
<td>$\hat{\theta}$</td>
<td>rad</td>
<td>Estimated rotor angle</td>
</tr>
<tr>
<td>$\phi_{50}$</td>
<td>rad</td>
<td>Estimated 50-phase, $50\hat{\theta}$</td>
</tr>
<tr>
<td>$\phi_{52}$</td>
<td>rad</td>
<td>Estimated 52-phase, $52\hat{\theta}$</td>
</tr>
<tr>
<td>$\phi_{50}'$</td>
<td>rad</td>
<td>Measured 50-phase (from $\hat{A}$)</td>
</tr>
<tr>
<td>$\phi_{52}'$</td>
<td>rad</td>
<td>Measured 52-phase (from $\hat{A}$)</td>
</tr>
<tr>
<td>$\varepsilon$, $\varepsilon_{\text{smooth}}$</td>
<td>rad</td>
<td>Estimation error (smoothed), $\theta - \hat{\theta}$</td>
</tr>
<tr>
<td>$E_c$, $E_{\text{smooth}}$</td>
<td>-</td>
<td>Complex (smoothed) est. error, $\varepsilon + ic$</td>
</tr>
<tr>
<td>$\alpha_{\text{smooth}}$</td>
<td>rad</td>
<td>Angle constant of est. smoothing error</td>
</tr>
<tr>
<td>$N_c$</td>
<td>-</td>
<td>Macro position error number, $[\frac{\varepsilon}{2\pi} + \phi_{50}]$</td>
</tr>
<tr>
<td>$\Delta_c$</td>
<td>rad</td>
<td>Macro position error angle, $\frac{\varepsilon}{2\pi}N_c$</td>
</tr>
</tbody>
</table>

The macro position error is identified by $N_c$ and $\Delta_c$. This refers to the 51 possible macro states that the rotor can be in when the motor is powered, because the rotor has 51 teeth. Each macro state corresponds with one "valley" of the magnetic energy potential. In a rotor with regularly-spaced teeth there is no way to distinguish between these 51 states (without history information, physical sensor or other asymmetric features). The macro position error number is $N_c = \lfloor \frac{\varepsilon}{2\pi} \rfloor$ (mod 51), with $N_c \in [-25,25]$. The macro position error angle is $\Delta_c = \frac{\varepsilon}{\pi}N_c$.

Example: $\theta = 0.55$ rad and $\hat{\theta} = 0.3$ rad. The total position error is $\varepsilon = 0.55 - 0.3 = 0.25$ rad. The macro position error number is $N_c = [2.029] = 2$, the macro position error angle is $\Delta_c = 0.246$ rad.

In this research we primarily focus on controlling the macro position error number $N_c$ to zero, by employing a method which can more or less directly measure this number during motion so that the driver can adjust $\theta$ accordingly.

1) Actuation: The rotor is controlled by continuously adjusting $\theta_{\text{ctrl}}$ defining the set of minimum energy angles; torque is applied to the rotor whenever $\theta \neq \theta_{\text{ctrl}}$ (mod $\frac{2\pi}{51}$).

Given the control angle $\theta_{\text{ctrl}}$ and nominal coil voltage $V_{\text{nom}}$ the voltages for each coil $c = 0..6$ are calculated as:

$$V_c = V_{\text{nom}} \cos \left( \frac{2\pi c}{7} - \theta_{\text{ctrl}} + \gamma_2 \right)$$  (3)

The nominal coil voltage $V_{\text{nom}}$ depends on the desired output torque, current angular speed and system parameters such as coil resistance and back-emf constant. In practice $V_{\text{nom}}$ is adjusted such that the root-mean-square (RMS) current through the coils, or the total power flowing into the motor, is at the desired level. Given the supply voltage $V_{\text{supply}}$ it is now possible to calculate the pulse width of each half H-bridge to match the voltage for the respective coils.

The applied voltage across each phase equals the sum of resistor voltage, inductor voltage and back-emf voltage. For sake of simplicity we ignore the coil self-inductance:

$$V_c = I_cR + L \frac{dI_c}{dt} + N_{\text{turns}} \frac{d}{dt} \Phi_B(\theta, c)$$  (4)

$$\approx I_cR + N_{\text{turns}} \frac{d}{dt} \Phi_B(\theta, c)$$  (5)

Solving Eq. [5] for $I_c$ gives:

$$I_c = \frac{V_c}{R} - \frac{N_{\text{turns}}}{R} \frac{d}{dt} \Phi_B(\theta, c)$$  (6)

We get $\frac{d}{dt} \Phi_B(\theta, c)$ from Eq. [2] as follows:
\[ \frac{d}{dt} \Phi_B(\theta, c) = \frac{d}{dt} \Phi_0 \sum_{r=1}^{3} w_r \cos \left( 2\pi n_r c/7 - n_r \theta + \gamma_r \right) \]
\[ = \Phi_0 \hat{\theta} \sum_{r=1}^{3} w_r n_r \sin \left( 2\pi r c / 7 - n_r \theta + \gamma_r \right) \]
\[ = \Phi_0 \hat{\theta} (7.5 \cos (2\pi c / 7 - 50\theta) - 35.7 \cos (4\pi c / 7 - 51\theta) - 7.8 \cos (6\pi c / 7 - 52\theta)) \]
\[ \theta \text{ vanishes.} \]
\[ A_r = \sum_{c=0}^{6} I_c e^{-i2\pi c r / 7} \]
\[ \theta \text{ is a measured quantity.} \]
\[ A_0 = \sum_{c=0}^{6} I_c = 0 \text{ (up to measurement error), due to Kirchhoff's current law.} \]
\[ A_1 = \sum_{c=0}^{6} I_c e^{-i2\pi c / 7} = \sum_{c=0}^{6} \left( \frac{V_c}{R} - \frac{N_{\text{turns}}}{R} \frac{d}{dt} \Phi_B(\theta, c) \right) e^{-i2\pi c / 7} \]
\[ = -\frac{N_{\text{turns}}}{R} \sum_{c=0}^{6} e^{-i2\pi c / 7} \Phi_0 \hat{\theta} w_1 n_1 \sin (2\pi c / 7 - n_1 \theta + \gamma_1) \]
\[ = -\frac{N_{\text{turns}}}{R} \Phi_0 \hat{\theta} w_1 n_1 \sum_{c=0}^{6} e^{-i2\pi c / 7} \]
\[ = -\frac{7N_{\text{turns}}}{2R} \Phi_0 \hat{\theta} w_1 n_1 e^{i(-n_1 \theta + \gamma_1)} \]
\[ \text{We do not use } A_2 \text{ because the term with } V_c \text{ no longer vanishes.} \]
\[ A_3 = \sum_{c=0}^{6} I_c e^{-i6\pi c / 7} = \frac{7N_{\text{turns}}}{2R} \Phi_0 \hat{\theta} w_3 n_3 \]
\[ = e^{i(-n_3 \theta + \gamma_3 - \pi/2)} \]
For \( r = 4.6 \) we have \( A_{7-r} = A_r^* \), so \( A_{4.6} \) gives no new information.

Numerical evaluation of known quantities in \( A_1 \) and \( A_3 \) leads to:
\[ A_1 = 995.7 \Phi_0 \hat{\theta} e^{-509i} \]
\[ A_3 = 1035.5 \Phi_0 \hat{\theta} e^{-520i} \]

We define \( \phi_{50} = -\angle A_1 \) for \( \hat{\theta}_{\text{ctrl}} > 0 \) and \( \phi_{50} = -\angle A_1 + \pi \) for \( \hat{\theta}_{\text{ctrl}} < 0 \), resulting in \( \phi_{50} = 50\theta \) for nonzero \( \hat{\theta}_{\text{ctrl}} \).

Likewise, we define \( \phi_{52} = -\angle A_3 + \pi \) for \( \hat{\theta}_{\text{ctrl}} > 0 \) and \( \phi_{52} = -\angle A_3 \) for \( \hat{\theta}_{\text{ctrl}} < 0 \) so that we obtain \( \phi_{52} = 52\theta \) for nonzero \( \hat{\theta} \).

The fact that \( \phi_{50} \) and \( \phi_{52} \) are independent of \( \hat{\theta} \), \( N_{\text{turns}} \), \( \Phi_0 \), \( R \) and exerted torque is of significant importance in absolute position detection. This also stays the case when self-inductance is taken into account, as this acts only in the 51-plane. The measurement of signal-to-noise ratio (SNR) is proportional to \( |\theta| \), though.

Non-linear effects may result in systematic errors in \( \phi_{50} \) and \( \phi_{52} \) measurements. In particular the interaction between the rotor and stator with its specific geometries results in magnetic fluxes through coils that are different from those described in Eq. [2]. Furthermore, saturation of electrical steel plates, hysteresis, chopping by PWM, current sensor characteristics, changes in temperature and various mechanical inaccuracies such as stator/rotor eccentricity lead to additional sources of systematic and/or stochastic measurement errors. The systematic errors are mitigated by acquiring and storing reference waveform shapes for a set of representative velocities, and stochastic errors are reduced by sampling currents at high sampling rates (49.7 kHz per channel) which are subsequently smoothed out.

3) Absolute position reconstruction: Absolute position measurement is possible while controlling the motor at nonzero rotor velocity. A single measurement of the 7-element current vector \( I \) in theory suffices to estimate \( N_e \) and therefore the absolute rotor position. In reality a high number of measurements are generally needed due to the stochastic errors, especially at low speeds. The reconstruction algorithm steps are as follows, with all angles and phases modulo \( 2\pi \) (unless otherwise stated):

1) Calculate \( \hat{\phi}_{50} = 50\theta \) and \( \hat{\phi}_{52} = 52\theta \).
2) Measure \( \hat{I} \) and compute discrete Fourier transform \( \hat{A} = \mathcal{F}(\hat{I}) \) which yields \( \hat{\phi}_{50} = 50\theta \) and \( \hat{\phi}_{52} = 52\theta \) (up to measurement error), compensate for systematic errors using pre-computed correction functions and smoothen (in the complex plane for continuity) using a low-pass filter with time constant \( 0.3 \text{ rad}/\hat{\theta} \).
3) Calculate \( \delta_{50} = \phi_{50} - \hat{\phi}_{50} = 50(\theta - \hat{\theta}) \) and \( \delta_{52} = \phi_{52} - \hat{\phi}_{52} = 52(\theta - \hat{\theta}) \), with \( \delta_{50}, \delta_{52} \in [-\pi..\pi] \).
4) Calculate \( \varepsilon = (\delta_{52} - \delta_{50}) / 2 = \theta - \hat{\theta} \mod \pi \).
5) Add \( \pi \) to \( \varepsilon \) if appropriate (i.e. if \( |\delta_{51} + |\delta_{52}| > \pi \)), to obtain \( \varepsilon = \theta - \hat{\theta} \mod 2\pi \) with \( \varepsilon \in [-\pi..\pi] \).
6) Smoothen \( \varepsilon \) using another low-pass filter (again in the complex plane for continuity) with time constant \( 0.3 / \hat{\theta} \), to obtain \( \varepsilon_{\text{smooth}} \).
7) Calculate \( N_e = [51 \varepsilon_{\text{smooth}} / (2\pi)] \).
If \( N_c \) equals zero then the estimated macro position conforms to the physical macro position. If \( N_c \) is nonzero then \( \hat{\theta} \) (and also \( \epsilon_{\text{smooth}} \)) is adjusted in steps of \( \frac{2\pi}{51} \) to remove the discrepancy.

C. Validation

Two performance indicators are evaluated: the absolute position sensing capabilities and the torque/speed curve.

1) Absolute position sensing capabilities: The absolute position sensing capabilities are tested by manually holding and then releasing the dial on the output shaft when running at different speeds, and checking whether the system is able to recover to reach the commanded position. Speeds are controlled using a potentiometer in conjunction with up/down buttons.

For validation purposes the actual position is measured with an incremental optical encoder with a resolution of 0.18°, and the steady-state difference with the commanded position is quantified. For visualization purposes the commanded position is indicated with a dial on a secondary stepper motor. Figure 4 shows the setup for this experiment.

2) Torque versus speed: The torque/speed curve is acquired using a different setup. The presented motor is coupled to an electromagnetic hysteresis brake of type HB-140M-2 (Magtrol, Inc., Buffalo, NY, USA). A torque sensor of type DR2412-R/M150 (Lorenz Messtechnik GmbH, Alfdorf, Germany) is positioned between the motor and the brake. The motor is set at different velocities between 0 and 1000 rpm, starting at a coil current of 1 A (rms). The brake is gradually energized while keeping track of the motor torque and input motor power until the motor stalls.

A reference motor of type Trinamic QSH4218-35-026 is evaluated in the same way for comparison. Its dimensions and input motor power until the motor stalls.

III. Results and Discussion

A. Absolute position sensing capabilities

Fig. 5 shows the behaviour of the presented motor during the position sensing experiment. In the top graph the commanded, estimated and actual positions and position error are plotted over a period of nine seconds. Three stages with nonzero commanded speed can be distinguished which are 40 rpm, 74 rpm (reversed) and 108 rpm, respectively. It can be observed that during each stage the shaft was practically blocked for approximately 0.9 s although a small oscillation with an amplitude of approx. 0.02 rad can still be observed. The estimated angle also differs from the true angle by approx. 0.7 rad. As soon as the shaft is released it rotates at an elevated speed of 206 rpm and catches up with the commanded position, resulting in steady-state errors of 0.0012 rad to 0.0091 rad at standstill, or 0.52° at most.

The bottom graph of Fig. 5 shows the filtered \( \phi_{50} \) and \( \phi_{72} \) signals, and \( \epsilon \) being half its difference. It can be observed that whenever the moment the shaft is blocked, within 0.2 s the \( \epsilon \) signal makes a significant jump (to \( \pm 0.7 \text{ rad} \)) which repeatedly triggers the position correction algorithm until \( \epsilon \) is back to (approximately) zero again.

In the accompanying video a second stepper motor is used to visualize the commanded position next to the presented stepper motor. Both motors are running synchronously at various speeds and it can be observed that whenever the shaft of the presented motor is temporarily blocked or the motor is electrically disconnected, it is able to recover the
commanded position when it can rotate again.

B. Torque versus speed

Figure 6 shows the torque of the presented and reference motors at eight different velocities. The input motor power ranges from 16 W to 23 W. At 1000 rpm the presented motor showed noticeable oscillations which made a reliable measurement difficult. The torque of the reference motor is consistently higher than that of the presented motor by approximately 7% on average, excluding the 1000 rpm discrepancy. For both motors the torque remains relatively stable at speeds up to 300 rpm after which both decay at approximately the same rate. The holding torques are included in Table I.

C. Discussion

The results show that the presented stepper motor is capable of recovering its position after a temporary blockage of the shaft, under the tested conditions. The graphs show the successful position recovery procedure at three different speeds while the accompanying video shows many more. It appears that even during blockage the system is still able to estimate a relatively crude error \( \epsilon \), presumably due to the small oscillations which also generate measurable back-emf signals in the 50-plane and 52-plane.

After successful recovery the steady-state position error of 0.52° at standstill could be attributed to friction in the motor, imperfections in the system and/or the resolution of the optical encoder of 0.18°.

The quality of the absolute position sensing algorithms depends on an accurate calibration of the systematic errors in \( \hat{\phi}_{50} \) and \( \hat{\phi}_{52} \). A complicating factor is that the systematic errors not only depend on intrinsic properties and known parameters such as motor current and speed, but also on unknown parameters such as motor temperature and momentary load. This leads to additional fluctuations which must be taken into account. For example, the coil windings could be pre-heated to the average working temperature prior to calibration while the smoothing factors could be increased to iron out the remaining fluctuations at the cost of a slower response.

The torque measurements show that the presented motor is almost as strong as the reference motor, despite not yet taking the irregular distribution of pole pairs into account in the actuation system. Given that the presented motor is the first prototype with seven stator arms there is plenty of room for optimizations. For example, an optimized stator design could accommodate more turns per coil resulting in improved performance. The difference of 7% for velocities up to 750 rpm can also be compensated by the absolute position sensing capabilities as the torque margins could in theory be kept much smaller. Lastly, the outlier at 1000 rpm should be resolved by identifying and eliminating the resonance issue.

Contrary to most state-of-art position measurement mechanisms, the presented method requires no additional components in the motor itself. This in theory allows to construct motors with position sensing capabilities at approximately the same production cost as common bipolar stepper motors.

Besides the presented driver the TMC2130 stepper motor driver is also able to detect stalls when driving sensorless stepper motors, but it cannot recover the tracked position by itself so that it requires re-calibration of the motor position.

The controller uses separate integrated circuits for the H-bridges, current sensors and microprocessor, resulting in a relatively large footprint and high cost for the driver. If the system is to be used in low-cost 3-D printers then a small footprint is required, such as a dedicated integrated circuit similar to the Trinamic TMC2130.

1) Outlook: The torque of the presented motor could be improved with a newer stator design with more space for the coils. Preliminary experiments on such an optimized stator accommodating 80 turns per coil (instead of 55) show that it can indeed match the reference motor in torque. The presented motor could eventually be used in consumer-grade 3-D printers as replacement for the bipolar stepper motors. An important next step in this research is therefore to install such motors in certain axes of a 3-D printer and evaluate its performance during typical 3-D printing tasks.

The position detection capability also opens the door to new applications of stepper motors. For example, position sensing enables some form of field oriented control which in turn allows to operate the stepper motor like a brushless DC motor (BLDC). In such a configuration the rotational speed is automatically being optimized for the given load, yielding maximum output power for a given input current.

IV. CONCLUSION

A novel stepper motor and controller with absolute position sensing capabilities is presented. The motor uses the same type of components and has identical outer dimensions as a similarly-sized reference motor. The torque is 7% lower on average and it has seven wires instead of four.

Once properly calibrated, the presented motor is able to detect and subsequently correct discrepancies between the commanded and actual motor position whenever the motor
is in motion. After recovery the maximum steady-state error at standstill is approximately 0.5°.

ACKNOWLEDGMENTS

This research is partially funded by TTT-plan Thematic Knowledge Transfer Plan Smart Industries, grant no. TTT19003, voucher no. 04, and also by NWO Take-off Phase 1 grant no. 19318.

The authors would like to sincerely thank Pieter Post and André Eppingbroek for producing the mechanical parts.

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