Data-Driven Methods for Efficient Operation of District Heating Systems

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Abstract In this chapter, data-driven methods for the efficient operation of DHSs are described. DHSs are inherently non-linear and time-varying systems as the heating demand is highly influenced by non-linear dependencies on the weather conditions as well as the occupancy behaviour. Furthermore, the dependency on flow and temperature in delivering the needed heat demand using the district heating network gives a non-linear dependency on these two signals. This chapter presents several data-driven models to handle the non-linear and time-varying phenomena in order to ensure an efficient operation. First, we introduce forecasts that are used to reach an optimal operation as forecasts are needed for both control and production planning, e.g. heat demand and electricity price forecasts. Second, temperature control of a DHN will be introduced with a focus on how the physical characteristics of the

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network can be incorporated into a control scheme. A special focus will be on how to ensure that the temperatures in the network are high enough to ensure the needed heat supply for the attached buildings in the entire district heating network is met. We shall also briefly look at the role of smart buildings integrated into a DHN that can be used to enhance the efficiency and flexibility of a DHS.

Keywords Data-driven methods · Adaptive control · Heat load forecasting · Temperature optimization · Sector coupling · Renewable energy · Flexibility

1 Introduction

The transition to a low-carbon society calls for fundamental changes in the energy system. Today energy systems are operated and planned such that the production follows the demand. However, an efficient implementation of a low-carbon society calls for the exact opposite approach, namely systems where demand follows production which will be dominated by renewables. This highlights a need for new methods for planning and operation of energy systems. Most importantly the flexibility at virtually all types of end-users on all aggregation levels has to be unlocked. The typical variation in time of the energy produced by wind and solar power implies a need for flexibility that can be offered by well designed district heating and district cooling systems, and with the increased sizing of the district heating systems even seasonal energy storage solutions often become feasible.

The examples used in this chapter originate from Denmark. The history of Danish district heating is more than 100 years old and today about 65% of the households in Denmark are supplied with district heating. What started as a way of getting rid of waste in an efficient way is today a billion-dollar business and a cornerstone in the Danish energy system. In Denmark in 2020 more than 50% of the electricity load was covered by wind power, and this implies a further need for flexibility which effectively can be offered by the district heating system. The methods and results described in this chapter originates from a large number of national research and innovation projects like HEAT 4.0 [1], CITIES [2], FED [3] and IDASC [4]. We refer to the homepage of these projects for further information.

In the first generation of energy systems, central power plants were established to deliver electricity, leading to huge energy losses in the form of waste heat. In the next step, this 'waste heat' was utilised in the form of district heating by introducing combined heat and power plants (CHPs), an environmental leap step towards a sustainable future. District heating developed since to a 4th generation, where intermediate renewable energy sources and waste heat from industrial and other sources are utilised. To accommodate the mentioned sources efficiently, the district heating operation has to be adjusted to *low-temperature* operation, improving the efficiency of the whole system.

Diversity in production, large district heating networks, advanced distribution and time-varying demand side characteristics, result in increased complexity.

Digitalisation is proposed to facilitate the transformation from a rather simple to a highly complex system. Digitalisation introduces new possibilities and hereby complexities, naming wireless monitoring with Internet of Things (IoT), increased connectivity and communication, Artificial Intelligence (AI) and big-data analytics, systems-of-systems, and distributed system layouts with cloud-, fog- and edge computing. Digitalisation and the increased possibility of getting frequent sensor and meter data open up for the next generation of data-driven methods for the operation of district heating systems.

Common for the data-driven methods is, that the methods involve dynamical modelling based on grey-box and data-driven digital twin techniques, which again leads to new methods for real-time forecasting, control and optimization. The methods are adaptive, i.e. that they are automatically adapted to observed changes which can be deduced based on the received data from the system. Adaptive methods are crucial in order to handle the inherently non-linear and time-varying characteristics of heat loads and the district heating system.

State-of-the-art methods for forecasting are crucial for efficient operation of district heating systems. This is partly due to the fact that often the heat has to be produced several hours (or days) before the heat is delivered in the houses. Methods for forecasting are outlined in Sect. 2.

Obviously, methods for heat load forecasting have to be based on methods for weather forecasting supplied by meteorological weather forecast services. Standard Meteorological (MET) forecasts are targeting rural areas whereas district heating systems most often are seen in highly populated areas, and in Denmark, all the largest cities are supplied with district heating. Since the city weather often is rather different from the weather in nearby rural areas methods for forecasting city weather have to be considered, and such methods are briefly discussed in Sect. 3.

The heating is supplied by controlling the supply temperature at all plants and by controlling the flow in the network. In general, efficient operations of district heating networks call for data-driven methods for keeping the network temperature as low as possible, yet the temperature must be high enough to ensure that the heating systems of the supplied buildings are able to ensure a reasonable indoor temperature and a minimum temperature of the hot water. The temperature level is decisive for the efficient utilisation of renewable energy sources via e.g. heat pumps and for effective use of waste heat from low-temperature sources, e.g. from the cooling of data centres. Methods for temperature optimization are described in Sect. 4.

Section 5 briefly presents methods for the operation of buildings connected to a district heating network. The term *Smart buildings* is applied to highlight that it is necessary to be able to control the buildings if they have to play an active role in the optimisation and control of the overall district heating or power system. For smart buildings, the heat load can be partly controlled by the operator of the district heating network, and hereby we will be able to avoid or reduce peak demands and solve issues with bottlenecks in the district heating or the local power network.

Most of the examples in this chapter originates from studies conducted in HEAT 4.0 project funded by IFD using data from the district heating system in Brønderslev, Denmark. This heating system is servicing approx. 5,000 customers in 3 subnets.

The heat production is characterised by several different units such as CHP units, gas boiler, electric boiler, Organic Rankine Cycle (ORC), heat pumps, and concentrating, tracking solar thermal units and a large thermal storage capacity.

Some of the methods presented in this chapter links to Chap. 8 which considers a method for optimal production planning and power market bidding.

2 Forecasting for DHS

Forecasts are needed for optimal operation of DHS, e.g. for preparing the system for the future load by investigating future scenarios and selecting the appropriate strategy, e.g. what heating units to produce the heat, and what supply temperature to push into the DHN. Three forecasts usually are needed: heat demand, electricity price and weather. Heat demand forecasts play a crucial role in enhancing the operation of DHS as it is needed in both production and network. Weather forecasts are used as inputs for the heat demand model therefore accuracy of them in predicting the future weather inside cities is crucial. Electricity price forecasts are used for bidding strategy into the electricity market, hence an important task when scheduling the production and creating bids for the day-ahead and balancing electricity markets. The focus of this section will be on methods for heat load forecasting, while electricity price forecasting will be briefly considered.

Section 2.1 will introduce what drives heat consumption. This will be used to identify a heat load model and to introduce a suitable framework for producing online heat load forecasts. Section 2.1.1 introduces the forecasting toolbox and framework of producing an online forecast for heat demand using a linear regression model that adapts over time to handle the non-stationary. Section 2.1.2 describes the identification of important model elements using physical knowledge of the system, and it describes how this knowledge can be used in grey-box modelling procedure for identifying an optimal model for heat load forecasting. Finally, an example of a heat demand forecast is introduced in Sect. 2.1.3 to demonstrate the benefits of understanding the underlying physics of the dynamics.

District heating is mostly applied in cities where the climate is quite different to rural areas. A procedure to localize Numerical Weather Prediction (NWP) to the climate in cities is proposed in a subsequent section. Section 2.2 introduces briefly the need for them and a short introduction to the paper for further reading. In addition, Sect. 2.3 discusses other frameworks to produce heat demand forecasts.

2.1 Heat Load Forecast

Heat load in district heating consists of two components at the demand side; space heating and domestic hot water, plus related heat losses in the DHS. Due to the nature of the heat load, it varies during the day (the diurnal profile), week, and year creating a time-varying process. Gadd and Werner [5] splits the heat load into two categories, physical heat load and social heat load. The physical heat load depends on the climate and heat losses in the system, while the social heat load depends on the occupancy behaviour of the consumers and includes the usage of domestic hot water. Furthermore, the variation in heat load is described in [5] as follows:

– *Seasonal Heat Load Variation*: The physical heat load follows the increase and decrease in ambient air temperature throughout the year with a negative correlation. The social heat load also has a seasonal component, since, e.g., people tend to stay outside or away for longer periods during summer while people stay inside and consume more hot water during winter.
– *Daily Heat Load Variation*: The magnitude of the heat load during the day follows the ambient air temperature however the shape of it depends on the social behaviour. Social heat demand variations are explained by individual and collective social heat behaviour, e.g., harmonised working hours. The daily variation usually has two peaks, in the morning and the late afternoon.

Regional climate affects the individual district heating case, and the heat load related social behaviour can vary between systems. For example, a coastal locality is influenced by the damping thermal inertia of the sea. Taking into account the climate is very important to achieve efficient operation of DHS. Since DHSs are mostly used in urban areas, we will introduce the climatic characteristics inside cities and highlight the effects of climate variables on heat consumption in Sect. 3.

Thus, when developing a model to describe the dynamics of a system, it is important to understand the underlying process that drives the system. For DHS systems it is the physical description of how the weather influences the heat consumption through the buildings properties, e.g. walls and windows. Also, social heat consumption usually drives the peak consumption, e.g. in the mornings and evenings. A clear understanding of what drives the consumption from physics and social components will improve the models' performance in forecasting the future load of the system.

We will focus on utilizing the grey-box methodology which is using the physical understanding of the system, along with statistical methods to identify and estimate parameters of the model to establish an adequate forecasting model to predict the heating load. First, the heat load forecast framework for producing the predictions needs to be robust and simple to update the model and the predictions when new information is available. It needs to be stable and handle the non-stationary variation in the heat demand due to the weather-driven consumption of space heating. Also, being able to translate the nonlinear relationship between the effects of the weather on the consumption. This is addressed in Sect. 2.1.1 where the first part introduces a linear regression model where the parameters of the model are updated recursively and past information is exponentially down-weighted, i.e. addresses the non-stationarity. The second part proposes using a two-stage modelling procedure to be able to transform inputs that have a non-linearity relationship to the heat demand such that the coefficients of the linear model can be estimated. The second is the identification of the optimal model to be used in the forecasting framework. In Sect. 2.1.2, we introduce a model with a full physical representation of the heat consumption in a DHS. However, the full physical model is too computational heavy and would probably not be able to produce the forecasts before they become invalid. Therefore, an adequate heat load forecast model that uses parts of the physical insights of the consumption and what variables are known to influence the demand (e.g. ambient air temperature, wind, and solar radiation). Then combine this knowledge with statistical methods to identify the optimal forecasting model. Hence, it is preferable to use a model, which gives physically inspired descriptions of known relationships between climate variables and historical data to forecast the heat load. The coefficients are then estimated using data-driven methods based on a time series of observations from the system where the model will be used. However, in order to formulate such a data-driven digital twin model, where the parameters can be assimilated based on observed time series, the model cannot be too complex. Basically, the model has to be structurally identifiable.

2.1.1 Framework for Heat Load Forecasts

We will present here the framework of establishing heat load forecasts that can handle the non-stationary and non-linearity that comes with the heat load. This type of forecasting framework has been proven accurate and able to perform in online operations [6, 7]. The framework allows the forecasting model to adapt to the changes in the parameters of the model for heat load forecasting, like the transitioning from cold (winter) to warm periods (summer). This adaptive method has shown adequate results in multiple research fields, especially in energy applications that are greatly dependent on physical behaviour. See e.g. [7–9] for heat load forecasting, [10] for solar power forecasting, and [11, 12] for wind power forecasting.

The model used in the framework is a linear regression model that can be recursively updated when new information arrives and exponentially down-weights past information. The model is also able to handle dynamical and non-linear effects, e.g. the ambient air temperature influence on heat load. There will also be an emphasis on creating individual models for each k-step prediction horizon since the lagged values in the model have to be tailored to the horizon, i.e. a unique model with its own coefficients is created for each k-step forecast.

Linear regression models are well known and frequently used for statistics and forecasting. Regression models as defined in Madsen [13] are used to describe a static relationship between a dependent variable Y_t and p independent variables, $\boldsymbol{x}_t = \{x_{1,t}, \ldots, x_{p,t}\}$. Therefore, an index t is introduced to denote the variable at time t. A regression model can be written as

$$Y_t = f(\boldsymbol{x}_t, t; \boldsymbol{\theta}) + \epsilon_t \tag{1}$$

where $f(\boldsymbol{x}_t, t; \boldsymbol{\theta})$ is a known mathematical function of the $p + 1$ independent variables and t, but with unknown parameters $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_m)^T$. ϵ_t is a random variable with mean $\mathbf{E}[\epsilon_t] = 0$ and variance $\text{Var}[\epsilon_t] = \sigma_t^2$. In many applications the function f is linear, and the models describe how the parameters $\boldsymbol{\theta}$, are defining a linear combination of the independent variables, $\boldsymbol{x}_\text{t}$. Given N observations of the dependent variable and the p explanatory variable, we are able to estimate the model parameters.

Estimation of the parameters of a linear regression model is usually done with either Least Squares (LS) or Weighted Least Squares (WLS) methods. Given the estimated parameters $\hat{\boldsymbol{\theta}}$, we are able to predict a future value $Y_{\text{t+k|t}}$ given the independent variable $\boldsymbol{x}_{\text{t+k}}$. However, this type of forecasting model will not perform well for heat loads as it lacks adaptive properties to update when the system changes. This is necessary for heat load forecast due to the time-varying and non-linear characteristics of the heat load. Therefore, the model needs to be extended allowing for time-varying parameters that can adapt to changes and transformations of the input variables.

Hence, an ordinary linear regression model will not perform well for heat load forecast as it lacks adaptation as the heat load changes over time. Ljung and Söderström [14] propose an adaptive method that uses exponential weights using a forgetting factor, λ to discount old information. For auto-regressive models with exogenous input, this is called Recursive Least Squares (RLS) method with exponential weighting. It allows the model parameters to adapt over time when new information becomes available, therefore making it feasible to handle time-varying phenomena. The algorithm updates the parameters every time new information becomes available and uses the forgetting factor to discount old information and thereby increase the relative importance of the most recent observations. That is, the coefficients are recursively updated by an LS estimation with the weights exponentially decaying over time. The rate of decay is determined by the forgetting factor. The forgetting factor can change over time, but usually, it is set constant. The forgetting factor is between 0 and 1 and controls the level of adaptivity, where values close to 1 implies equal weight on both new and older observations. While values of the forgetting factor close to 0 emphasise more on newer observations in the estimation. The optimal λ is found by minimizing the Root Mean Square Error (RMSE) as shown in [7]. This proposed method of allowing the parameters to update as new information becomes available and discount old observation is desired for online operations, as recursive methods offer relatively simple and few computations to estimate the parameters for every iteration.

Apart from the time-varying dynamics of the heat load, the non-linear dependency between the heat load and suggested input variables need to be considered and combined with the recursive update. For this, a two-stage modelling procedure can be used as proposed in Nielsen et al. [15], Rasmussen et al. [16]. Before estimating the model, the observed input variables are used either by mapping input variables by using some function (e.g. splines) or by using them directly (instant effect of the independent variables) in a so-called *transformation stage*. After the transformation, it is possible to create a linear model of the transformed data to predict the dependent variable by using the RLS scheme to recursively estimate the parameters in the so-called *regression stage*.

In the *transformation stage*, transfer functions (e.g., low-pass filter), basis splines, and Fourier harmonic series can be used to transform the non-linear relationship between the independent and dependent variables, to a linear relationship. Other functions can also be used to create the regression vector, e.g., kernels. The low-pass filter has proven a useful tool for explaining the effect of climate variables on the heat dynamics of buildings. As buildings are insulated and have thermal mass, they do not react instantaneously to changes in ambient air temperature. Hence, low-pass filtering of the ambient air temperature leads to a better description of the temperature effect on the heat consumption. Climate variables are typically transformed using rational transfer functions to model their effect on heat demand adequately [7, 8]. Low-pass filters can be created from rational transfer functions with a stationary gain equal to one. For example, the simple first-order transfer function,

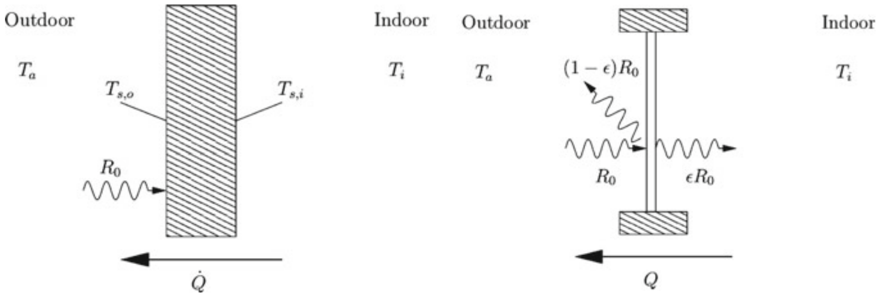
$$H_{a_{\mathrm{T}}}(q) = \frac{1-a}{1-aq^{-1}}. \tag{2}$$

where q^{-1} is the backward shift operator, i.e. $q^{-1}x_t = x_{t-1}$, and $a \in [0, 1]$ is the time constant, can be used to describe the dynamics between the dependent variable and independent variable that is being filtered. For instance, in a model describing heat transfer between indoor air temperature in a building and ambient air temperature, a high time constant would mean that the building has a high thermal mass and good insulation.

2.1.2 Model Identifications Procedure

Model identification of a system is a tedious process, as many things need to be considered. For instance, for model identification of heat consumption, questions arise on how to model the diurnal variation, is there a weekly tendency that needs to be modelled specifically, which climate variables are significant, how to translate their relationship to the heat consumption, and how to consider the physical representation of heat consumption to keep the desired indoor temperature in buildings. Here, we will first investigate the heat consumption from a physical point of view of the system, then we will demonstrate the use of statistical methods to enhance the model.

During winter in cold regions, the heat load is dominated by the demand for space heating, i.e. keeping the indoor temperature at the desired level to satisfy the thermal comfort of the consumers. Therefore, the physical model of heat demand can be viewed according to the heat loss characteristics of a building, the passive heat loss through the construction and the active contributions due to ventilation. These characteristics can then be used to suggest a forecasting model of a total heat load for a district heating network where other heat losses are added, e.g., losses related to the transportation of heat from production to consumer. This is the first step of establishing a grey-box model, identifying a model from physical knowledge. For instance, Madsen and Holst [17] demonstrate a grey-box modelling approach for the heat dynamics of a building using a continuous-time model based on stochastic



(a) Heat transfer through a wall. (b) Solar Radiation through a window.

Fig. 1 The figures demonstrate the stationary heat transfer through wall and window of a building. Missing is the effect of the ventilation. The source of these figures are found in [7]

differential equations. The final model is validated by both simulation and forecasting of the indoor temperature. In Nielsen and Madsen [18] this analysis is extended by considering the physical knowledge not just on a single building level but also the thermal characteristic of the entire DHS to create a model of the total consumption in the DHS. Climate measurements of ambient air temperature, wind speed, and global radiation are used to create an appropriate model. We will present the results from Nielsen and Madsen [18] on creating a heat load model of a DHS, first from a purely physical derived model then introduce their proposal of a model where the parameters of the model can be estimated by statistical methods. Thus, utilizing the Grey-box methodology of using physical knowledge and statistical information embedded in data to reduce the model-space of a purely physical model to describe the dynamics of the system.

Nielsen and Madsen [7] uses Fig. 1 to illustrate the heat dynamics of houses by describing the heat transfer through a wall and a window. The figure demonstrates the stationary heat transfer of a building without considering ventilation. Plot (a) shows the energy exchange through a wall as the heat flux, $\dot{Q}$ which is found from the stationary relation between the outdoor, wall and indoor environment,

$$\text{Outdoor: } \dot{Q} = h_{\mathrm{o}}(T_{\mathrm{s,o}} - T_{\mathrm{a}}) - \epsilon R_0, \tag{3}$$

$$\text{Wall: } \dot{Q} = U_{\mathrm{w}}(T_{\mathrm{s,i}} - T_{\mathrm{s,o}}), \tag{4}$$

$$\text{Indoor: } \dot{Q} = h_{\mathrm{i}}(T_{\mathrm{i}} - T_{\mathrm{s,i}}), \tag{5}$$

$$\text{Overall: } \dot{Q} = U(T_{\mathrm{i}} - T_{\mathrm{a}}) - U\frac{\epsilon}{h_{\mathrm{o}}}R_0, \tag{6}$$

where T_{a} is the ambient air temperature, T_{i} is the indoor temperature, $T_{\mathrm{s,o}}$ is the temperature of the outdoor surface of the wall, $T_{\mathrm{s,i}}$ is the temperature of the indoor surface of the wall, R_0 is the solar radiation orthogonal to the wall, and $U = (1/U_{\mathrm{w}} + 1/h_{\mathrm{i}} + 1/h_0)^{-1}$. The heat is transferred as convection, thus from warmer to colder areas and the h coefficients are the convection heat coefficient of the outside of the wall and inside the wall. They describe the impact of a boundary layers on the outer and inner wall surface where the former is influenced by wind speed and wind direction while the latter can be assumed constant due to the constant indoor environment. U_{w} is the thermal conductivity of the wall divided by the wall thickness. Hence, the overall stationary heat flux through the wall is described from the stationary relation between the ambient air temperature, indoor temperature, the solar radiation orthogonal to the wall by convection.

The energy transfer through the window (Fig. 1b) consists of both convection and conduction. Thus,

$$\dot{Q} = -\epsilon R_0 + U(T_{\mathrm{i}} - T_{\mathrm{a}}) \tag{7}$$

where ϵ is the fraction of solar radiation orthogonal to the window (R_0) is not reflected by the window and the $U(T_{\mathrm{i}} - T_{\mathrm{a}})$ describes the energy conduction through the window between the ambient air temperature and indoor temperature, and $U = (1/U_{\mathrm{win}} + 1/h_{\mathrm{i}} + 1/h_0)^{-1}$. U_{win} is the thermal conductivity of the window divided by the window thickness.

Ventilation, where the warm air from the buildings is replaced by the cold air gradually, also needs to be considered. The heat flux of the ventilation is

$$\dot{Q} = \mathcal{C}\dot{V}(T_{\mathrm{i}} - T_{\mathrm{a}}) \tag{8}$$

where $\mathcal{C}$ is the product of the specific heat capacity of the air and the mass density of the air and $\dot{V}$ is the flow of air through the building.

Heat load of an area can therefore be assumed to be the heat loss of the heat transfer through walls, and windows and by ventilation, thus $Q_{L,t} = Q_{Wall,t} + Q_{Window,t} + Q_{Ventilation,t}$ plus the energy needed for domestic hot water usage, $Q_{W,t}$ for all buildings in the area. There is also "free" heat $Q_{F,t}$ that contributes to the indoor temperature coming from e.g. electrical equipment but also humans. Thus, energy needed for space heating $Q_{H,t}$ can be expressed as,

$$Q_{H,t} = [Q_{L,t} - Q_{F,t}]_+. \tag{9}$$

The truncation of negative values is used since when the quantity inside the squared brackets gets negative the indoor temperature will increase or ventilation will be used to prevent this. Considering the total heat consumption as

$$Q_t = \delta_{c,t}[Q_{L,t} - \delta_{p,t} Q_{F,t}]_+ + \delta_{p,t} Q_{W,t}, \tag{10}$$

where $\delta_{c,t}$ is the fraction of the consumer heat load reacting to the climate. $\delta_{p,t}$ is the fraction of the potential consumption active at time t. This means $\delta_{p,t}$ accounts for holidays and $\delta_{c,t}$ accounts for the fact that during the summer almost no consumers react to the climate and that they do not all start/stop reacting on the climate at the same time of year. The dependence on holidays might be negligible since it will to a large extent only affect the demand for domestic hot water.

In [18] it is argued that these quantities of the model above can not be estimated using available measurement, i.e., the total heat consumption, ambient air temperature, wind speed and global radiation. Therefore, it is suggested to create a model structure, inspired by the physical quantities of the heat transfer qualities, that can be estimated only from data. Detail explanation of how to go from the physical orientated equation in Eq. (10) to the model in the equation below is found in [7],

$$\begin{aligned} Q_t &= \mu(h_t^{24}, \Upsilon_t) + a_{20} H_2(q) R_t \\ &+ a_{111} H_1(q) W_t + a_{120} H_1(q) T_{a,t} + a_{121} H_1(q) W_t H_1(q) T_{a,t} \\ &+ a_{100} H_1(q) R_t + a_{101} H_1(q) W_t H_1(q) R_t \\ &+ a_{211,0} W_t + a_{211,1} W_{t-1} + a_{220,0} T_{a,t} + a_{220,1} T_{a,t-1} + e_t, \end{aligned} \tag{11}$$

the filters $H_1(q)$ and $H_2(q)$ are found to be

$$H_1(q) = \frac{0.066}{1 - 0.934q^{-1}}, \tag{12}$$

$$H_2(q) = \frac{0.350 + 0.612q^{-1} - 0.226q^{-2}}{1 - 1.703q^{-1} + 0.739q^{-2}}. \tag{13}$$

The function $\mu(\cdot, \cdot)$ models the diurnal variation, R_t is the solar radiation on a square pillar (see [7]), W_t is the wind speed, T_a is the ambient air temperature, e_t is the model error (iid and $\mathbf{N}(0, \sigma^2)$), $a_{\cdot}$ are the coefficients of the model. The filters $H_1(q)$ and $H_2(q)$ are rational transfer functions and are proposed to filter the climate variables to model their effect on them to heat load. Nielsen and Madsen [7] argues for instance that constant indoor temperature will effectively eliminate the heat storage capacity of the floor and internal walls, it seems therefore reasonable to use a rational transfer function when filtering the climate variables for heat demand modelling. Also, low-pass filtering is ideal as the dynamics at the boundary layers of the wall can be neglected however the conduction through the wall needs to be modelled. The inertia due to thermal mass indicates a slow response which again suggests that low-pass filtering of the climate variables is preferable.

This model is proposed to be used to predict the future heat consumption and is demonstrated to perform adequately, however it was shown that there was auto-correlation in the errors. A noise model was included in the model to remove the correlation using an auto-regressive model on the errors using lagged error on the past; 1, 2, 3, 23, and 24 lagged errors. The model including auto-regression on the noise showed that the auto-correlation of residuals was removed. When comparing the result between the model in Eq. (11) and the same model using the noise model, it showed that for online applications the model including auto-regressive terms performed significantly better for short-term horizons while the model without auto-regressive terms performed better for longer horizons. Hence, using information from the prediction errors of the model can improve the predictions on shorter horizons.

The model in Eq. (11) was used for demonstrating online prediction and adaptive RLS update on parameters, it showed high accuracy for forecasting the heat consumption for a large area where the different methods of modelling diurnal profile and different forgetting factors are compared.

Model identification of a heat load model can also be done using traditional statistical approaches, thus finding the optimal structure of the forecasting model by finding which independent variables that describe the heat load adequately. Also, how those independent variables enter the model, i.e. how they are mapped in the *transformation stage*, here the physical understanding of the system will be helpful. Model building procedure is described in detail in Madsen [13, Chap. 6] along with the estimation of parameters and validating the model (model checking). Model building is an iterative process where the model is changed until it is adequate for its purpose, e.g. forecasting. The initial step of the identification procedure for heat load is to find a suitable initial model describing the response variable, the heat load. For instance, a model that includes an intercept and some factor that models the social effect of the heat load, i.e. the diurnal profile. It is recommended not to include climate variables in the initial model so that one can use the initial model to identify which climate variables are significant. Modelling the social effect in the initial model is necessary as it is common knowledge that the heat demand has a diurnal profile. The profile can be modelled using Fourier harmonic functions [7, 8], splines or even using the hour of the day as input [19]. After modelling the social component, it is necessary to identify which climate variables influence the heat load and how to include them. The cross-correlation function can be used to identify what climate variable to include by computing the cross-correlation between the prediction residuals (e.g. one-step-ahead prediction) from the initial model and the climate variable time series. The climate variable with the most significant correlation is then added first to the initial model. Selecting what transfer function to use can be done by computing an error metric score that is suitable, for instance, the RMSE score. The model extensions are then compared to find which function (or the instant effect, i.e. no mapping function) describes the dynamics between the climate variable and heat load adequately. After finding the optimal function, these steps are iteratively repeated until the final model accuracy can not be improved. This model identification process is demonstrated in [8] where the climate variables are used to find the ideal model of the heat load of single-family houses. This procedure is data-driven as the variables are added or transformed and then the results are investigated to see if an adequate model has been found, this is repeated until the result is satisfactory. Here, the understanding of the physical nature of the system dynamics is an advantage, as it gives insight on what variables to be included and how they affect the system, e.g. low-pass filter of the ambient temperature to describe the heat consumption. We will present an example of forecasting performance by including a low-pass filter of the ambient temperature in Sect. 2.1.3. Additionally, the performance of a state-of-the-art forecasting model is also illustrated.

2.1.3 Demo: Brønderslev

In this section, we will demonstrate the performance of a forecasting model that is created based on the methodologies discussed earlier in a real district heating case. We will highlight how the model can be improved by implementing physical knowledge of the relation between heat consumption and weather. A state-of-the-art forecasting model supplied by HeatFor™ [20] will be compared with the simple model proposed here.

A first attempt to forecast the heat load in Brønderslev would be to include an intercept, Fourier harmonics series to describe the diurnal variations and NWP of the ambient air temperature. Hence, an initial model is

$$\hat{Y}_{t+k|t} = \theta_{0,k} + \mu(t, n_{\text{har}}, \alpha_{\text{diu}}) + \theta_{1,k} T^{\text{a,NWP}}_{t+k|t}, \tag{14}$$

where the subscripts t is the time and k is the prediction horizon. This model will be referenced as *Model 1*. The estimation and forecast of this model is done using the `R` package, `onlineforecast` [21]. The package gives the opportunity to create forecasting models with estimated coefficients for each prediction horizon and estimates the coefficients adaptively using recursive least squares with an exponential forgetting factor as described in Sect. 2.1.1. Model 1 will be extended to model the physical knowledge on how changes in ambient air temperature affect the heat consumption's by using the low-pass filter $H_a(q)$,

$$\hat{Y}_{t+k|t} = \theta_{0,k} + \mu(t, n_{\text{har}}, \alpha_{\text{diu}}) + \theta_{1,k} H_a(q) T^{\text{a,NWP}}_{t+k|t}, \tag{15}$$

and will be referenced as *Model 2*. The results are shown in Fig. 2 where on the left plot, the RMSE for prediction horizons from one to 72 h ahead is illustrated. The improvement of including the low-pass filter is significant as the RMSE for all horizons has decreased. The right plot in Fig. 2 shows one realizations of the forecasts into the future made at 2021-01-10 12:00 (vertical grey dashed line). This

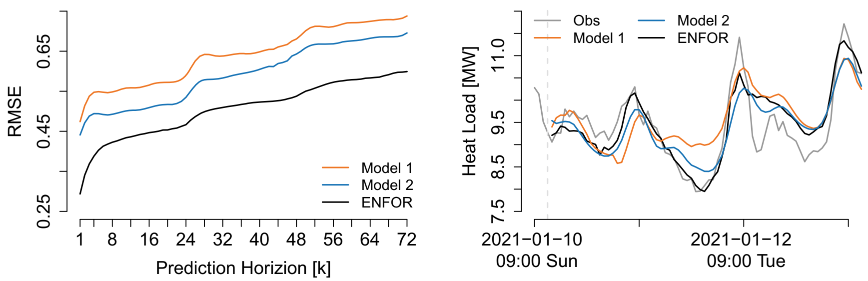


Fig. 2 Figure demonstrates performance of three heat demand forecast models. The left plot shows the RMSE for different k-step prediction horizons while on the right plot shows realizations of the forecast from the models

is only one realization and therefore should not be used to compare the accuracy between the models (use the RMSE plot). These plots clearly show that the improved model represents the dynamic changes in more detail. Thus, exploiting the physical dynamics will improve the forecast accuracy. The state-of-the-art performance is also visualized and a quite significant improvement in accuracy is clear when compared to the simple models where the most significant difference is in the short term forecast. High accuracy in the short term forecast is crucial for enhancing the temperature optimization of the network, i.e. lowering production cost and reducing heat losses by keeping the supply temperature low.

2.2 Electricity Price Forecast

Here, we will briefly describe the forecast of electricity prices to optimize the economic performance of district heating with respect to the production balancing of heat and electricity, see further information in Chap. 8. In the case of electricity markets dependent units such as CHP or heat pumps, successful trading on the electricity market is important for the district heating operator. Most of the electricity is usually traded on the day-head market or wholesale markets and prices are often quite volatile and not determined by physical laws. The volatility and complexity increases also with the increasing share of decentralized Renewable Energy System (RES). Therefore, electricity price forecasts are generally only based on data-driven models. Jónsson et al. [22] propose a two-step method to forecast the electricity price and its dependency on forecasts of load and wind power production. Uncertainty of the price forecast is not given in Jónsson et al. [22], however information on the uncertainty is often beneficial and needed e.g. for including the forecast into stochastic optimization methods [23–25]. There is a trend towards research on electricity price forecasting including uncertainties and probabilistic forecasts are gaining interest [26]. For an overview and more detailed information, we refer the reader to the review papers [27, 28] or [29].

2.3 Further Heat Load Forecast Models

We have proposed grey-box modelling in order to formulate a model for heat load forecasting and a framework to deliver online predictions of the heat load in this section. However, black-box models that are purely driven by data are often also suggested for heat load forecasting. For example, linear models where coefficients are estimated using ordinary least squares [19, 30]. AutoRegressive Integrated Moving Average (ARIMA) models are also a common method, often with a seasonal component. In Grosswindhager et al. [31] it is proposed to use seasonal ARIMA (SARIMA) models using a state-space framework. Using the state-space formulation implies that they are able to produce online predictions using the Kalman filter to update the forecasts and generate new forecasts when new information is available. These methods are easy to understand and computationally fast however they fail to describe the nonlinear dependency without applying transformations of the input. More advanced black-box models, e.g. machine learning methods, have demonstrated that they can capture the complex non-linear dependency that can be difficult to model using linear models.

In Sejling [32] it is proposed to use a feedforward neural network using two layers of neurons and using a sigmoid as the activation function. The neural network is applied without using any physical knowledge about the DHS. The results from the neural network are compared to a grey-box model where the nonlinear relationship between demand and weather variables are taken into account by transfer functions to create a linear regression model. Similar inputs are used for both models however the number of coefficients is significantly higher for the neural network with two layers of five and two neurons. In this case, the grey-box model has a better performance in the out-of-sample comparison.

Dahl et al. [33] compares a linear model, a feed-forward neural network, and Support Vector Regression (SVR) where the input variables are also investigated. They find that the SVR has the highest accuracy. Additionally, the results show that including holidays and calendar data as inputs improved all models. However, this method has problems with the time-varying dynamics of the heat load. Recurrent Neural Network (RNN) have been proposed to solve the time-varying issue as demonstrated in Kato et al. [34] where it is shown that RNN handles trends in heat demand data while a feedforward neural network cannot.

3 City Weather Forecasting

Both heat load forecast and temperature control are heavily dependent on the most recent weather observations, in particular, the ambient temperature is important for the operation. The climate where the district heating system is located needs to be analyzed to operate the network in an optimal setting as mentioned at the beginning of this section. Madsen et al. [6], Nielsen and Madsen [7] suggest that the climate variables: ambient temperature, solar radiation, and wind speed (including direction) have the largest effect on the heating demand (in order of decreasing importance). Nielsen and Madsen [7] gives a detailed description of how these climate variables influence heat consumption based on physical and stationary considerations:

– *Ambient Temperature*: The ambient temperature affects the indoor climate through heat conduction in the outer walls and windows, but also through ventilation. It is shown that the outdoor temperature affects the indoor temperature through a low-pass filtered signal, and a simple transfer function model is suggested to describe how the variations in the outdoor temperature affect variations in the indoor temperature.

– *Solar Radiation*: The solar radiation affects the indoor climate based on the angle of beams hitting the building, where the orientation of the beams through the windows and the window area are most important. Basis functions are used to translate the non-linear dynamics of the solar radiation to its contribution to heating consumption.
– *Wind Speed*: Wind speed and wind direction affect the indoor climate as natural ventilation, the effect is depending on the tightness of the building. The wind speed also affects the convection heat coefficient on the outside of the buildings. It is therefore modelled using a low-pass filter for describing the contribution to the consumption.

Hence, in order to use these climate variables for enhancing the operational performance of the district heating system, both a clear understanding of how these variables affect the heat consumption and a forecast of them is needed. However, weather forecasts models are usually tuned for rural areas, not urban areas where district heating is applied. The difference between the climate in rural and urban areas is quite significant. Historically climate variables have been measured in rural areas. For instance, airports usually have climate stations and they are usually located outside the cities in an open area, where the only impact is from the natural environment including lack of woody vegetation and directly exposed to natural rain, sun, and wind. However, climate inside cities is different from rural areas and therefore variables like the air temperature measured at the airports may deviate from the temperature inside cities, where the air temperature is exposed to human activities and the built environment. Research shows that the ambient air temperature typically is higher in urban areas than in rural areas [35]. The effect is termed Urban Heat Island (UHI). A UHI is an urban area that is warmer than its surrounding rural areas due to human activities or build human infrastructures. The variation of outdoor air temperature data is both spatial and temporal. Several studies point towards a typical average difference in urban and rural temperatures of 2–3 K. For instance, Bergsteinsson et al. [4] investigates the UHI phenomenon in Copenhagen and demonstrate it by using three climate stations that are located inside the city, in the outskirts and in a rural area outside of the city. The hourly temperature average from the climates stations is computed and illustrates that the temperature measured at each station is different. The climate station inside the city always has the highest temperature, whereas the station in the rural area has the lowest. It is also shown that the spatial temperature has time-varying characteristics, both a diurnal and an annual variation is seen. For example, comparing the result during summer shows that the average temperatures in the mornings are very similar. However, later in the day, it differs, with higher temperature difference in the city, and during the night it gets colder at the rural side. Thus, the city does not lose heat as fast as the rural part. While during winter, there is quite a constant offset between the stations.

3.1 Localizing Numerical Weather Predictions

NWPs are obtained by a physical model of the atmosphere and ocean to predict climate variables. They are computed over a grid of the earth and are then interpolated to a specific location where weather predictions are needed. However, NWPs are designed for rural weather forecasting, and they often have problems adjusting to the local climate in cities due to the local climate phenomenon. The models seem to have trouble adjusting to local heat contributions, e.g., solar heat in the street, heat from buildings, etc. DHS relies heavily on NWP to operate their system efficiently, therefore it is important to correct the NWPs before using them as input e.g. models for heat load forecasting. Especially, for temperature control of the district heating system as it is done on a short-term horizon (between one and 24 h) and is heavily dependent on the current local climate. Using a local climate station to localize the NWP, corrects the short term NWP by adapting them to the climate using real-time climate measurement [7]. Hence, this yields an optimal weather forecast for a certain area that can be used to operate the temperature control in the most optimal setting. In Glahn and Lowry [36] it is proposed to use Model Output Statistics (MOS) to bind NWP to local climate stations observations, e.g., localize the forecasts. The MOS is a simple technique that uses linear regression where the observed climate variable is the response variable and predictors are the NWP variables which therefore bind the NWP to the local climate. It is a simple and frequently used method that will reduce systematic bias in the NWP if there is any. Crochet [37] propose using an adaptive method to reduce the systematic bias and lower the RMSE of the NWP.

4 Temperature Optimization

Efficient operation of district heating networks implies an objective to minimize production cost of the heat production and reduce heat waste without compromising the consumer comfort. For most district heating systems, this is achievable by minimizing the supply temperature at the production site while providing the total desired heat and fulfilling the requirement of minimum supply temperature at any time point at all points in the district heat network. Decreasing the supply temperature is also highly valuable for heat production sites where electricity is also produced (e.g. with CHP units) since it implies an increase in the ratio of power to heat output, and, as electricity is more valuable than heat, a more profitable operation is achieved.

Temperature Optimization aims at optimizing the supply temperature. Traditionally, the supply temperature has been either controlled using a reference curve schema or experience of the network operators [38]. A typical reference curve is illustrated in Fig. 3 where the supply temperature is determined as a function of the current ambient air temperature. The supply temperature is kept constant during high ambient temperature for providing solely heat for domestic hot water during non-heating periods. The temperature is set high enough to reduce any bacteria risk. As the ambi-

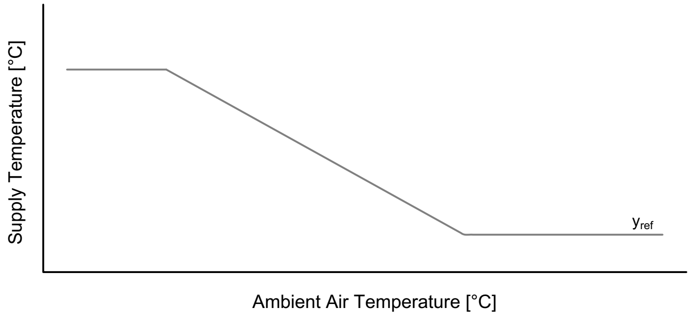


Fig. 3 Example of an reference curve for controlling the supply temperature at the production site

ent temperature decreases the supply temperature is increased until the maximum supply temperature of the system is reached. This control schema aims at ensuring that the consumers receive sufficient heat as it models the relationship between supply temperature and ambient air temperature as a worst-case scenario, i.e. the temperature should not be lower than the reference curve. The curve also considers that the supply temperature for the given ambient temperature will have sufficiently large safety margin for the flow to be adjust to satisfy the heat demand before it reaches the maximum limit of the system.

The use of a single reference curve at the plant usually results in a high supply temperature which is high enough to account for all other variables not taken into account. This results in higher costs and more heat losses in the system. Thus, the curve is not an optimal strategy as it does not consider other climate variables that are known to influence heat consumption like wind speed, wind direction and solar radiation. Furthermore, it ignores the social behaviour and the time-varying relationship to demand. One of the important time-varying relationships is the time-varying time delay between time of production and the time when the water reaches the end-users. It is also not a predictive controller, i.e. it does not look into the future while selecting the supply temperature. Even though this method is naive, it is more advantageous than operating the supply temperature at the maximum limit and allowing the flow to vary. However, the supply temperature is kept unnecessarily high when discarding other factors. When including all of the above factors into a control schema using predictive methods, new possibilities of lowering the supply temperature without violating any restrictions are possible. It is also important to point out that the hydraulics limitation of the DHN implies some restrictions on the minimal supply temperature in order to ensure that the flow is below the maximum limit with some safety margins. Thus, then the supply temperature should be increased to meet the demand thus a predictive method is needed to optimal select supply temperature before the flow reaches the maximum.

With the increase in both computational power and research in adaptive control during the 1980s, more sophisticated methods for DH have been developed. Adaptive controllers are able to operate time-varying and non-linear systems since they can adapt to changes based on the output feedback of a system (see, e.g., [39] for further reading). For DH, this improvements in research related to control theory was necessary as the system is inherently non-linear and non-stationary.

In this chapter, we will focus on temperature optimization based on adaptive control using feedback from critical points in the DHN. Our description is based on the research published in amongst others [6, 40–43]. The described method also resulted in commercial software that is used for temperature optimization in DHN in Denmark [44]. The method has proven to be able to provide significant reductions.

Section 4.1 will introduce and describe the characteristics of a DHN. Section 4.2 will build on the knowledge of the network characteristics to introduce temperature optimization and control. A demo case will be used in Sect. 4.3 to demonstrate the performance and savings. Finally, we give an overview of additional work on temperature optimization in Sect. 4.4.

4.1 DHN Characteristics for Control

The dynamics in a DHN are driven by the physical dynamics and the consumer consumption, i.e., the heat load. As discussed in Sect. 2, consumption is driven by intra-day variations, climate, and local social behaviour. Benonysson et al. [45] propose an extensive physical-simulation model of a DHN and describe the important physical factors to consider. The physical factors are *time delay*, *heat loss*, *pressure*, and *friction loss*. These factors are very important for accurate simulation and understanding of the DHN to maintain acceptable temperature and differential pressure in the network. Thus, they are used to operating the network efficiently by delivering the desired heat to the consumer while minimizing the operation cost. Heat loss from pipe to surroundings is determined by the time of being transported in the DHN from production to consumer (time delay), along with the temperature of the hot water and the resistance of the pipe insulation. The time delay or the transport time of sending hot water to a point in the network is determined by the flow that is controlled by pumps based on the differential pressure applied in the system. The pressure in the system controls how fast the hot water travels in the network. Hence, the temperature loss between consumers and the production plant depends on these variables. Pressure loss in the system depends on the fiction which is a function of flow rate and other pipe properties. The network is designed to maintain fixed differential pressure in the network which the pump control tries to maintain by adjusting the flow in the system. If the flow becomes too high then it becomes impossible to maintain the needed differential pressure due to limitation of the pump. In these cases, the differential pressure over the substations at the consumers will start to decrease, and the consumer will then not receive the desired heat. Thus, these four factors contribute to the two main components of delivering heat to the consumer; temperature and flow. They are jointly linked together in a nonlinear relationship. For instance, heat losses in the system are based on the temperature and the flow, hence, the temperatures in the network are determined by the temperature and the flow in the system.

The DHS usually also has restrictions based on these factors on the operation of the network due to physical limitations and additional constraints made by the utility. Nielsen [42] describes the following usual restrictions:

– *A maximum allowable flow rate in the system*: The restrictions in the flow rate are due to the (always) limited pumping capacity, the risk of cavitation in heat exchangers and difficulties maintaining a sufficiently high differential pressure in the remote parts of the network during periods with high flow rates.
– *A minimum guaranteed inlet temperature at the consumers*: This restriction is due to limitations in the consumer installations as well as minimum domestic water usage temperature requirements imposed by hygienic concerns.
– *A maximum allowable supply temperature*: This restriction ensures not to damage pipelines and consumer installations.
– *Limited short term variation in the supply temperature*: The stresses inflicted on the network by large and frequent fluctuations in the supply temperature dictate that the short term variations in supply temperature should be limited.
– *Maximum allowable diurnal variations of the supply temperature*: In some systems the size of the expansion tanks may impose limitations on the allowable diurnal variation of the supply temperature.

Hence, the framework of the temperature control has many constraints and physical limitations, thus operating a network consequently needs to consider multiple aspects to result in an optimal operation. Optimal operation of the network is achieved by minimizing the production cost without; compromising the safe operation of the system, adversely affecting the maintenance cost of the system, or sacrificing consumer satisfaction. Note that the physical description and system restriction listed here are not valid for every system. Each network is unique with different physical limitations or restrictions. However, they need to be considered before implementing the temperature control to reduce the risk of failure and achieve optimal operation of the network.

4.2 Controlling the Supply Temperature

In this section, we will introduce a control schema for the optimal operation of the DHN. To simplify the schema, we will only present a DHS that is supplied from a single station. We also assume that the return temperature and the diurnal peak load are not affected by the optimization. The operation costs can then be minimized by minimizing the supply temperature without violating the requirements discussed in previous sections. The methodology introduced here is a statistical approach of estimating the supply temperature by using adaptive estimation and transfer functions to model the network as initially proposed by [6, 41, 46] and extended in [42, 43]. Hence, we do not use a detailed physical model of the network, but statistical methods along with measurement data to describe the dynamics.

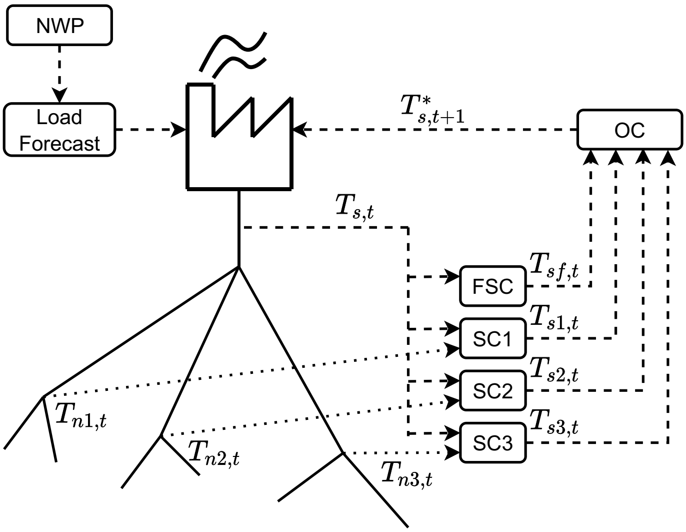


Fig. 4 Schematic view of the proposed controller: The supply temperature sub-controller (SC) which models the relationship between supply temperature, $T_{\mathrm{s,t}}$ at the production and netpoint temperature, $T_{\mathrm{n,t}}$. The flow sub-controller (FSC) that uses heat load and flow to estimate the supply temperature. Finally, the overall controller (OC) that select the highest supply temperature, $T^*_{\mathrm{s,t+1}}$ from the sub-controllers

Since the adaptive controller incorporates feedback information, the DHN needs to have some measurement wells in the network so that the controller gets feedback from the network. The placement of the wells should be located at points with the lowest temperature in the network, i.e. largest temperature loss. These wells are usually decided at the time of the design of the network or added later in the network when needed. Hence, if the supply temperature requirement is satisfied at those points in the network, we expect that the requirements of all consumers in these areas are satisfied. These measurement points will be referred to as *critical netpoints*. A more recent study outlines how to use frequent meter readings at the consumer as a cheaper and flexible alternative to measurement wells in the network [47].

The main concept of the proposed control strategy is illustrated schematically in Fig. 4. The overall controller (OC) selects the highest supply temperature from the flow sub-controller (FSC) and the supply temperature sub-controllers (SC) at the critical netpoints to be used in the next time step. The SCs estimate the lowest possible supply temperature from the plant using statistical identified transfer function without violating any restrictions at the consumers. The FSC computes the supply temperature without violating the maximum flow limit. The main principle of the proposed control schema is to keep the supply temperature as low as possible. The heat demand can be satisfied, by varying the mass flow through the network and by varying the supply temperature,

$$p_t = q_t c_w (T_{s,t} - T_{r,t}), \tag{16}$$

where p_t is the heat load, q_t is the flow, c_w is the specific heat constant of the water in [$Jkg^{-1°}C^{-1}$], $T_{s,t}$ is the supply temperature and $T_{r,t}$ is the return temperature.

The proposed control strategy here ensures the heat demand is met by varying the mass flow before increasing the supply temperature. Thus, the supply temperature is kept as low as possible while the flow is varied, the temperature is only increased when the flow is at the maximum value. The methodology behind the controllers and the transfer function between the production and network to establish a temperature controller is introduced briefly below. More detailed explanation can be found in [6, 32, 40–43, 48–50].

Flow Sub-Controller (FSC): The FSC is to perform an online control of the supply temperature in order to ensure that the flow rate q is kept close but below a maximum value q_{max}. The controller uses prediction models of the heat load to find the optimal supply temperature while keeping the flow as high as possible. The future control signal, $T_{s,t+1}$ is found by solving for the supply temperature in Eq. (16),

$$T_{s,t+1} = \hat{T}_{r,t+1|t} + \frac{\hat{p}_{t+1|t}}{c_w q^{ref}_{t+1|t}}, \tag{17}$$

where $\hat{T}_{r,t+1|t}$ is the forecasted return temperature, $\hat{p}_{t+1|t}$ is the forecasted heat load, $q^{ref}_{t+1|t}$ is the mass flow from a reference that is a prediction of the flow and it is below the maximum flow, q^{max} with a large probability, e.g. 99%. This simple controller only considers one-step ahead into the future however the change of supply temperature will affect the consumers after different time delays. Madsen et al. [48] propose to use weights on the j-step predictors to estimate the desired supply temperature, $T_{s,t+1}$. More advanced flow controllers are proposed in Madsen et al. [40], Nielsen [42] where the uncertainty from prediction of heat load and return temperature are considered.

The Transfer Function Model: An important aspect of the supply temperature sub-controllers based on a statistical approach is to identify the dynamic relationship between the temperature at the critical netpoints in the network and the supply temperature and flow rate at the plant for the supply temperature sub-controller. An accurate estimate of the network characteristics, including the time-delay is important for enhancing the controller. Hence, a model of the district heating network is needed for optimal control. A pure physical model of the network (white-box) is almost impossible to establish and would also be likely to be too complex for control purposes. The white-box models can be adjusted in operational settings when calibrating the models based on previous operational data however it can take too long due to the complexity of the model or the assumptions being wrong.

We therefore propose, a model of the network that is simple and is easily updated when new information or data is available about the network characteristics. The model is found by identifying a statistical transfer function of the network. It can be modelled using a single-input single-output AutoRegressive-eXtraneous (ARX) structure using a fixed time delay. An example of a model for a DHN is given here

$$T_{\mathrm{np,t}} = a_1 T_{\mathrm{np,t-1}} + b_{0,\mathrm{t}-\tau} T_{\mathrm{s,t}-\tau} + b_{1,\mathrm{t}-\tau-1} T_{\mathrm{s,t}-\tau-1} + b_{2,\mathrm{t}-\tau-2} T_{\mathrm{s,t}-\tau-2} + \epsilon_{\mathrm{t}}, \quad (18)$$

where T_{np} is the netpoint temperature, T_s is supply temperature, ϵ is the noise, and τ is the time-delay between the two temperatures (at the plant and the critical point). The coefficients; b_0, b_1, and b_2 describe the diurnal variation in the system, see [46] for further details. The model coefficients can be constants, but since DHS are non-stationary it is better to estimate them recursively. The model is formulated as a 1-step prediction model. The k-step predictions are then obtained by recursive use of the 1-step prediction model. Søgaard [46] propose different advanced methods to estimate the time-delay recursively, however [6, 42] propose a simple scheme to estimate the time-delay for the transfer function. The simple method estimate the time delay as the lag with the largest numerical value of the cross-covariance function between the time series of the supply temperature and the netpoint temperature. Pinson et al. [43] suggest more advanced methods to forecast the netpoint temperature and estimate the time-delay using conditional Finite Impulse Response (cFIR) as a transfer function of the supply and netpoint temperature. This allows for a nonlinear estimation of the time delay depending on the mass flow in the system.

Supply Temperature Sub-Controller (SC): The SC controller focus on creating a control schema to vary the supply temperature at the production without violating the requirements at the critical net points. It utilizes the transfer function model of the supply temperature and netpoint temperature, then it is possible to create a control scheme based on the network characteristics. In Palsson et al. [51, 52] an eXtended Generalized Predictive Controller (XGPC) is proposed to be used to control the supply temperature based on the transfer function between the supply and netpoint temperature. The Generalized Predictive Controller (GPC) is modified to handle non-stationary systems as it assumes that the predicted output can be expressed as a linear combination of present and future controls. Traditionally, this is obtained by solving the Diophantine equations as proposed in Clarke et al. [53] however these equations are formulated for time invariant systems, and hence modifications are needed. Palsson and Madsen [51] propose a modification of the GPC where the optimal prediction is the general conditional expectation of the system output. The GPC is only reasonable if the underlying process has a slow time-variation, while the XGPC can handle time-varying processes. This is important since the transfer functions that describes the network characteristics are inherently non-stationary due to the flow which results in a time-varying time-delay and different heat losses as a function of the hot water temperature and the flow.

The XGPC is based on an AutoRegressive Moving Average-eXtraneous (ARMAX) model with time-varying parameters, and the transfer function of the network is given by

$$A_{\mathrm{t}}(q^{-1}) y_{\mathrm{t}} = B_{\mathrm{t}}(q^{-1}) u_{\mathrm{t}} + C_{\mathrm{t}}(q^{-1}) e_{\mathrm{t}}, \quad (19)$$

where A_t, B_t and C_t are time-varying coefficients polynomials and q^{-1} is a back shift operator. The coefficients can be estimated adaptively using recursive methods. As the model is based on a time-varying process, the j-step predictor $\hat{y}_{t+j|t}$ is described by the conditional expectation of y_{t+j} given observations of the output up to time t

$$\hat{\boldsymbol{y}}_t = \boldsymbol{H}_t \boldsymbol{u}_t + \boldsymbol{v}_t, \tag{20}$$

where $\hat{\boldsymbol{y}}$ is the vector of predicted reference netpoint temperature, $\boldsymbol{u}$ is the vector of the future control signal (supply temperatures), $\boldsymbol{v}$ is the vector containing the expected response from the input free system, and $\boldsymbol{H}_t$ is a matrix of size $N \times N$ containing the time-varying impulse response of the system. The N is the maximum prediction horizon of the controller, $N \geq 1$. In relation to an ARMAX model and Eq. (20), it can be seen that h is the weight of the time-varying impulse function on future control values and v is the expected response from the past, see [42, 51] for detailed explanation. It is possible to use individual models for each j-step prediction. Then each row in $\boldsymbol{H}_t$ and $\boldsymbol{v}$ corresponds to the j-step prediction model.

The object of the controller is to minimize the difference between measured netpoint temperature and the reference temperature without violating the restriction with a certain probability. Additionally, the controllers object function contains a term with the purpose of minimizing the supply temperature fluctuations. A derivation of the optimization problem using the XGPC control law and model of the network as shown in Eq. (20) in detailed is provided in Nielsen [42]. The cost function is optimized over multi-step ahead horizons due to fact that the time-delay in the system varies, and hence a minimum and maximum time period is considered for the cost calculation. It is then shown that solving for the input vector, $\boldsymbol{u}_t$ (supply temperature) results in

$$u_t = -[\boldsymbol{H}_t^T \boldsymbol{H}_t + \boldsymbol{F}^T \boldsymbol{\Lambda}_t \boldsymbol{F}]^{-1} [\boldsymbol{H}_t^T (\boldsymbol{v}_t - \boldsymbol{y}_t^{\text{ref}}) + \boldsymbol{F}^T \boldsymbol{\Lambda}_t \boldsymbol{g}_t]. \tag{21}$$

At each timestep, the control signal is estimated, therefore only the first element of the control vector is implemented, i.e.,

$$u_t = [1, 0, \dots, 0] \boldsymbol{u}_t \tag{22}$$

In Madsen et al. [40] extended formulations of the XGPC controller are propose. This includes for instance equality constraints.

Overall Controller (OC): The OC selects the highest supply temperature, $T^*_{s,t+1}$ from the required supply temperatures computed from the FSC and the SC sub-controllers. This temperature is used as the supply temperature for the plant in the following hour; the model parameters, the predictions and the controller signals are updated each hour.

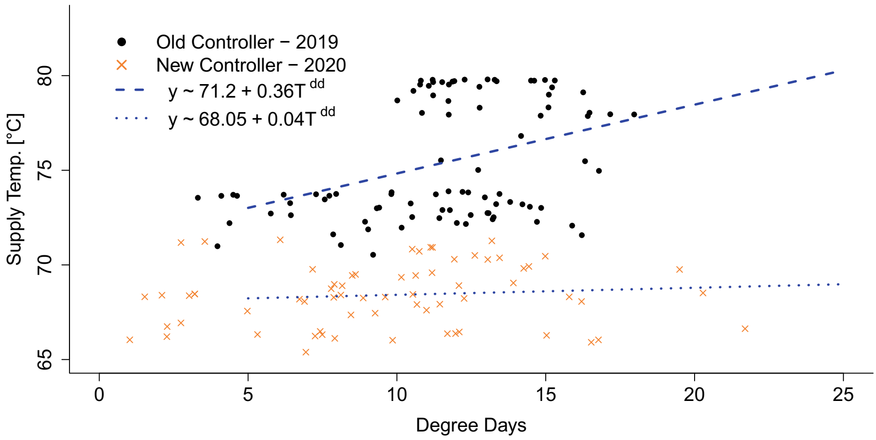


Fig. 5 The figure demonstrates the difference in performance using simple operations of controlling the supply temperature and using advanced data-driven method

4.3 Temperature Control: Demo Case

Supply temperature at Brønderslev is now being operated by data-driven temperature optimization in an on-line operation. It has been operating since beginning of 2020. Before the data-driven operation, the network was operated by a simple algorithm that did not receive online feedback from the network. The data-driven temperature optimization was done with the `HeatTO`™ [44]. Figure 5 illustrate the results from the previous operation and the current data-driven operation. The operations are compared using three months (February, March and April) in 2019 and 2020 for the old and new controller. The plot compares the performance of the controllers versus the degree days. Degree days are used to compare supply temperature between heating seasons when comparing different operations. Degree days, T^{dd}, are computed as the positive difference between the average ambient temperature ($\bar{T}_a$) over one day, and a cut-off of heating demand from buildings (we use 17 °C here), i.e.

$$T^{dd} = \max(0, 17 - \bar{T}_a). \tag{23}$$

The average supply temperature for the given day is then computed and plotted versus the degree day as seen in Fig. 5. To compare these operations, a regression model using Ordinary Least Squares to estimate the parameters of a model with an intercept and slope have been fitted to each operation as shown in Fig. 5 and the result is,

$$\text{New controller: } T_{supply} = 68.05 + 0.04\, T^{dd} \tag{24}$$

$$\text{Old controller: } T_{supply} = 71.2 + 0.36\, T^{dd} \tag{25}$$

We see that the supply temperature regression line is approximately 3 °C lower at y-intercept of for the new controller when $T^{dd} = 0$, and further the slope is lower for the new controller resulting in larger differences for higher T^{dd}. To compare the difference between the two methods, for instance, if we investigate a degree day of 10 then the difference is around 7 °C.

Decreasing the supply temperature for operation leads to an increase in savings for the utility. Madsen et al. [41] suggest a rule of thumb for savings resulting from lowering the supply temperature in CHP plant; For each degree lowered, the savings for the heat loss in the network is 0.5 % and the savings from more efficient production is 1 %, thus the savings can be compute as

$$\text{Savings} \; = (\text{Cost}_{\text{before}} * x \; [^{\circ}C] * 0.5\%) + \text{Shares}_{Production}(\text{Cost}_{\text{before}} * x \; [^{\circ}C] * 1\%) \tag{26}$$

where x is the supply temperature difference between operations to evaluate the savings.

Thus, the estimated savings from using data-driven temperature optimization would be roughly 7 °C and the savings would be around 10.5%. The equations are just a rule of thumb to demonstrate potential savings when sufficient production data is not available.

4.4 Further Temperature Optimization Methodologies

Grosswindhager et al. [54] propose similar temperature optimization approaches where statistical methods are used as describe above. They also take advantage of adaptive methods to model the time-varying behaviour. However, instead of the XGPC and cFIR, they use fuzzy modeling to describe the response characteristics of the network when supply temperature and flow are varied. They model the relationship between the supply temperature and network temperature using multiple local linear models that are only valid in certain regions, i.e., a fuzzy modeling to handle the non-linear dynamics of the system.

All above mentioned methods utilize statistical descriptions of the network to estimate the transfer function between two points (plant and netpoint temperatures) and derive controllers from there. The goal was keeping the supply temperature as low as possible to minimize the operational production cost for the system. However, other approached have been suggested in literature on how to estimate the network characteristics to ensure consumer are receiving the required temperature. For example, models derived from a purely physical representation of the network by modelling heat and mass transfer of the hot water. Benonysson et al. [45] propose a physical model of the network to describe the dynamics of it. This is done by predicting the network temperature and flows at nodes in the network, referred to as the node method. In this, the DHN is represented by a set of nodes by their physical description, e.g. heat capacities and pipe diameters. Therefore, it is possible to simulate the network characteristics and estimate suitable supply temperatures. This model gives promising results in offline settings where it optimize both the production and the network however for online purposes, the optimization is too computational heavy to deliver on time. Larsen et al. [55] suggest a simplified physical representation of the network by reducing the number of pipes into a tree structure without a circular loop. They demonstrate this in a case study where pipe system is reduced from 1079 nodes to approximately 10 nodes without sacrificing any accuracy and reducing the computation time by 99 %. In Sandou et al. [56, 57] also physical models are derived to operate a DHS using predictive control where the time-delay and temperature are estimated from a physical simulation model. Additional research where both production and network or only the network are modelled using physical derivation of the network has been carried out (e.g. frequently, the node-method proposed by Benonysson et al. [45] is used). Further Mixed Linear Integer Programming (MILP) is utilized to optimize the production and network to minimize the cost [57–59]. Vandermeulen et al. [60] gives an extensive review on DH for further reading.

5 Smart Buildings in DH Network

Buildings are traditionally controlled to achieve a good indoor environment. This is done independently from the surrounding energy systems. When introducing the dependency of building control on its surroundings, a decision has to be made, whether the building is in charge to meet the demand of e.g. the energy system (indirect control), or the external system can control the building system by direct control. The indirect method can be achieved by broadcasting prices that change according to the energy system demands as described below. In both schemas, direct and indirect control, the ability to control the many heating-, cooling- and ventilation devices has to be in place in the involved buildings. The below introduced Model Predictive Control (MPC) demonstrates such an enabling technology for simple buildings. In general, smart control of buildings require a central monitoring and control system.

In this section, we describe the interaction of smart buildings with DHN as an appetiser to this extensive topic. First, we describe the motivations and values of smart buildings in DHNs. Next, we introduce MPC as a technology to deliver smart control of thermal dynamics for buildings. We round up the section by introducing a hierarchical control setup, where buildings work together in the DHN to deliver flexibility.

5.1 Value Propositions by Smart Building to District Heating

There are numerous reasons for controlling buildings in smart ways; many of them targets the efficiency of the building itself, others target the interaction with the surrounding neighbourhood and infrastructures. In this section, we give a single example of a control technology, MPC, that enables both, internal and external smart solutions.

A traditional, non-smart building can be controlled by different means, manual control by the occupants, individual control on each heating and cooling device (radiators and floor heating) and also individual apartment control in multi-family buildings. Advanced buildings, such as office buildings and schools, are controlled by central installations that connect all the controllers and collect data centrally. Such systems are often controlled by skilled professionals and perform better compared to individual control. However, literature shows clearly that there are huge improvements to the performance of the latter type of buildings.

In recent years, the objectives for control are increasingly complex. On top of a request for the primary goal of optimal indoor climate conditions, energy efficiency with related CO_2-footprints and economical efficiency are the most dominating of many target objectives of control. Later years, flexibility is added to the list of objectives for the control. Control strategies that can handle these diverse demands call for rather advanced control units in buildings, smart buildings, that are able to e.g. shift heat demand in time and power, compute advanced demand patterns that compensate for temporal variations and other things.

Many methods and solutions have been proposed in the literature. One of these methods is MPC, which is a well-established and developed method for building climate control [61]. Its popularity is due to its simplicity and natural way of incorporating constraints and accounting for disturbances (such as the weather) in the optimal control problem. And not at least, the methods can be implemented in almost any hardware with computational abilities that are necessary to do essential predictive computations. Basically, these computations are very similar to the modelling and forecasting tasks applied to whole district heating systems and described in this book, because buildings are similarly affected by climate conditions [62]. However, the number of controllable units may be extremely high compared to district heating systems and so is the number of sensors involved.

Below, a simple building system is described as an example of the MPC methodology.

5.1.1 Building Control by MPC-Methodology

We give a brief example of how to model and represent a building in a DH network to describe the necessary states and properties.

Consider the following system of stochastic differential equations with observations taken at times t_k

$$\mathrm{d}\boldsymbol{x}(t) = f(\boldsymbol{x}(t), u(t), \boldsymbol{d}(t))\mathrm{d}t + g(\boldsymbol{x}(t))\mathrm{d}\boldsymbol{\omega}(t)\,, \tag{27a}$$

$$\boldsymbol{y}_k = h(\boldsymbol{x}(t_k)) + \boldsymbol{v}_k, \quad \boldsymbol{v}_k \sim N(0, R) \tag{27b}$$

where $\boldsymbol{x}$, u, and $\boldsymbol{d}$ are the system states, controllable input, and non-controllable input respectively. f is the deterministic dynamics, g is the diffusion function, $\boldsymbol{\omega}$ is a standard Brownian motion, and $\boldsymbol{v}_k$ is the observation noise. The above model formulation is an example of a grey-box model that includes physical dynamics in f and describes stochastic elements by the Brownian motion that are too complex to otherwise model. Thilker et al. [63] modelled the thermal dynamics of a Danish school building in a DH network using a hydraulic heating system with thermostatic controlled radiators. Such a model enables an MPC to control the thermal dynamics and perform tasks such as peak shaving or load balancing. They find the following states useful to include in a grey-box model

- The indoor air temperature, $T_i(t)$. This is typically the variable that is important to maintain a comfortable indoor climate.
- The temperature of the building envelope, $T_w(t)$. This contains important information about the insulation-level and how much heat is stored in the walls.
- The flow of the water in the space heating system, $\Phi(t)$; this varies as the thermostats in the radiators open and closes.
- The temperature of the radiators in the building, $T_h(t)$, is the component that delivers the heat in the rooms.
- The return water temperature, $T_{\text{ret}}(t)$, is important since it determines the amount of heat the building uses.

Together, the above states form a model that is sufficient in describing the important thermal dynamics in a large building in a DHN. The controllable input to the system, $u(t)$, is the set-point to the radiator thermostats in the building. The map between the difference in set-point and room air temperature and how open the valves are was modelled using a sigmoid function. Using a system with the above states, it was possible to optimise the operations of the building while shifting the loads to desired times [64].

5.2 Implementation of MPC in a Smart Buildings

On a building level, the MPC has the objective to satisfy certain constraints (e.g. a comfortable indoor air climate at all times) while at the same time minimising some objective. For economic MPC [65, 66], the objective typically is to minimise the cost of the heat consumption while only adjusting the set-point of the radiator valves (in the case of a building as described above). Such an optimisation problem can be formulated as

$$J(\hat{\boldsymbol{x}}_{k|k},\hat{\boldsymbol{d}}(t)) = \min_{u_k} \sum_{i=0}^{N-1} \boldsymbol{c}_{k+i} u_{k+i} \tag{28}$$

$$s.t. \quad \hat{\boldsymbol{x}}(t_0) = \hat{\boldsymbol{x}}_{k|k} \ , \tag{29}$$

$$\hat{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), u(t), \boldsymbol{d}(t)) \ , \qquad t \in \left[t_i, t_{i+N}\right[\ , \tag{30}$$

$$\hat{\boldsymbol{y}}_{k+i} = h(\hat{\boldsymbol{x}}_{k+i}) \ , \qquad i = 1, \ldots, N \ , \tag{31}$$

$$\hat{\boldsymbol{y}}_{k+i} \leq \boldsymbol{y}_{\max,k+i} \ , \qquad i = 1, \ldots, N \ , \tag{32}$$

$$\hat{\boldsymbol{y}}_{\min,k+i} \leq \hat{\boldsymbol{y}}_{k+i} \ , \qquad i = 1, \ldots, N \ , \tag{33}$$

$$\Delta u_{\min,k+i} \leq \Delta u_{k+i} \leq \Delta u_{\max,k+i} \ , \qquad i = 0, \ldots, N-1 \ , \tag{34}$$

$$u_{\min,k+i} \leq u_{k+i} \leq u_{\max,k+i} \ , \qquad i = 0, \ldots, N-1 \ , \tag{35}$$

$$u(t) = u_i \ , \qquad t \in [t_i, t_{i+1}[\ . \tag{36}$$

In the above, N is the prediction horizon, $\{\boldsymbol{c}_{k+i}\}_{i=0}^{N-1}$ is the price signal that reflects the price of the heat. The price indicates the degree to which it is acceptable to heat at any time. This could be with respect to e.g. a CO_2-signal that carries information about the CO_2-density of the heat. The objective of the MPC is then to minimise this cost while satisfying constraints on the building states and the controllable input.

A well designed MPC is capable of optimising both the thermal comfort of the indoor environment and the energy usage [67]. However, the above mentioned MPC solves only the problem of optimal control on a local building level. The building does not interact with other nearby buildings and does not necessarily align their collective heat load. The heat load of an entire district may consequently be unfavourable for the district heating company. Hence, an obvious question to pose is: How should the heat load of the building stock be shifted to be optimal for both the building and the network? Somehow, we want the entire heat load of a larger district to follow some reference heat load. For instance to follow the power production curve of renewable energy sources.

In the next section, we briefly describe an overall methodology that enables the application of the presented MPC of buildings to cooperate with the surrounding smart energy networks, amongst these district heating.

5.3 Hierarchical Control

We explained earlier in this book, the complexity of the energy systems are increasing and the demand for adaptability and flexibility has to follow to ensure robust user services at any time. In this example, we focus on 'flexibility' which is the ability to shift demands in time and power, according to the request by the surrounding energy systems, i.e., we might be requested to shift the heat loads of entire building stocks to match some reference heat load on a district level.

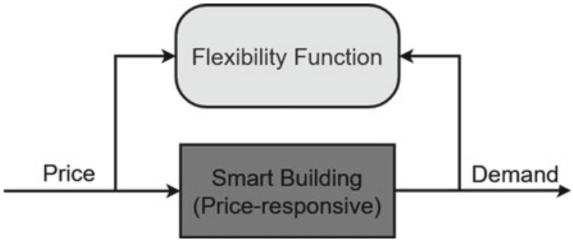


Fig. 6 The heat demand of a building can be predicted from the price signal

Madsen et al. [68] has proposed a hierarchical control approach, where the overall optimisation is distributed between involved 'layers'. One layer could be the electrical net, another the district heating system, and at the lower end the individual buildings.

The price signal is determined by a reference type of controller such that a reference heat load can be followed. This gives possibilities for peak shaving or for controlling the load such that it matches e.g. the local renewable energy production.

The goal of a hierarchical control setup is to send out a price signal to the considered building stock that gives the buildings information about the energy prices (which could reflect e.g. the CO_2-density of the available energy). A flexibility function is a model that determines the heat demand of a building (or any dynamical system) given a price signal. Figure 6 illustrates this dependence. In Junker et al. [69], a linear model is suggested, but in Junker et al. [70] it is shown that a nonlinear grey-box model using stochastic differential equations is more appropriate. By sending out a price signal to the building stock and receiving back the entire heat demand, the upper-level controller is able to shape the price signal such that the heat demand of the entire building stock follows for instance a reference heat load.

The flexibility function can be determined also for a larger DH network from which we are able to construct a two-layer control setup. The upper layer solves the problem of making the heat load of the building stock follow a reference power signal given by the district heating operators to minimise e.g. the CO_2-emissions, while the lower level consists of individual controllers for each building. The latter controllers solve the problem of keeping each building comfortably heated, i.e. maintaining the indoor temperature within the comfort constraint.

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