The Spectral Decomposition Method: a Transparent Theory for Losses in Segmented Waveguides

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An approximate theory, giving insight into the effects of device parameters on loss in segmented waveguides, is presented. Computational results on low and high loss structures, and structures showing anomalous length dependence of the loss are discussed.

Keywords: guided wave optics, segmented waveguides, gratings, theory.

Introduction

Segmented waveguides (SWs) have raised a lot of interest due to their potential as devices for quasi-phase matching in second harmonic generation (SHG)[1], for tuning the size of the modal field [2] and for sensing [3,4]. In the latter use is made of the dependence of the loss on the index of the segments. Optimization of SWs for any of these or other purposes involves many parameters, related to both the basic structure of the waveguide (WG) and its segmentation, i.e., the distribution of transitions along the propagation direction. In order to be able to get insight into the effects of these device parameters we have developed the spectral decomposition method (SDM). This 2D method, which is correct only in the limit of low losses, leads to analytical expressions for the relative loss in terms of the segmentation and the parameters describing the basic structure.

The rest of the paper is organized as follows: below the main features of the SDM are introduced. This is followed by a section in which results are shown, discussed and compared with that of the finite difference beam propagation method (BPM) [5] and the mode expansion method (MEM)[6]. The paper ends with conclusions.

Theory

Below analytical expressions will be given for modal losses in a 2D SW for an arbitrary segmentation. The polarization is assumed to be TE, TM polarization can be treated along the same lines. A time-dependence $\exp(iwt)$ is assumed but suppressed. The wave equation to be solved is given by (see also figure 1):

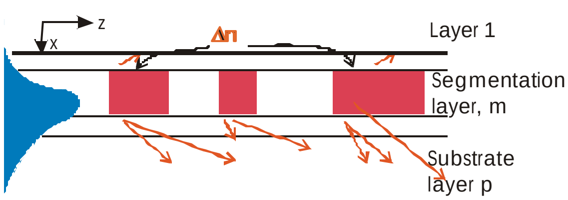

Figure 1. *Schematic picture of the considered waveguide, with segmentation in one of the layers.*

$$\{\partial_{xx} + \partial_{zz} + k_0^2 e(x,z)\} E_y = 0 . \qquad (1)$$

Here $k_0 (\equiv 2p / l)$ is the wavenumber and $e(x,z)(\equiv n^2(x,z))$ is the relative permittivity. In the below it is assumed that we may write

$$e(x,z) = e_b(x) + h(x) g(z) \Delta e , \qquad (2)$$

where e_b describes the so-called basic structure, assumed to be mono-modal, Δe describes the change in permittivity, which is here assumed to occur in one layer, layer *m*, for which $h(x) = 1$; $h(x) = 0$ in the other layers. The segmentation is described by $g(z)$, which may be of any form. It is assumed that the segmentation occurs for, say $z \geq 0$, and that for $z < 0$ the incoming field is that of the fundamental mode, corresponding to the basic structure, given by:

$$E_0 = e_0(x) \exp(-i b_0 z) , \qquad (3)$$

with propagation constant $b_0 \equiv k_0 N_0$, where N_0 is the modal index. It is assumed that the fundamental mode propagates undepleted along z. The total field is written as

$$E_y = E_0 + E_r, \tag{4}$$

where E_r is the field excited by the perturbation. Below we will concentrate on solving the radiative part of it. Substituting (4) into (1) leads to:

$$\{\partial_{xx} + \partial_{zz} + k_0^2 e_b\}E_r = A(E_0 + E_r) \approx AE_0;\; A \equiv -g(z)h(x)k_0^2 \Delta e\,. \tag{5}$$

Here we used the modal field equation for E_0, and for the second equality that the effect of modulation is weak. In the SDM (5) is solved by taking the Fourier transform of (5) with respect to z, and solving the resulting equation for each value of the spatial frequency, k_z, corresponding to radiation modes.

A lengthy but rather straightforward derivation [3] leads to the following expression for the power, radiated into the substrate and the cladding, P_p and P_1, respectively:

$$P_p + P_1 = \int_{-K}^{K} |G_0 H_p|^2 \; dk_z + \int_{-K_1}^{K_1} |G_0 H_1|^2 \; dk_z \equiv \int_{-K}^{K} H\,|G_0|^2 \; dk_z\,, \tag{6}$$

with $K_1 \equiv k_0 n_1$, $K \equiv k_0 n_p$ and $H \equiv |H_1|^2 + |H_p|^2$. The relative loss, h, is given by:

$$h = (P_p + P_1)/P_0\,, \tag{7}$$

with P_0 the power of the incoming fundamental mode. The characteristic functions, $H_{1/p}$, introduced in (6) are given by:

$$|H_p|^2 \equiv |a_p|\,|t_{mp}(T_1 + S_1)/\{2D(k_z^2 - b_0^2)\}|^2 /(2k_0 Z_0) \tag{8}$$

and

$$|H_1|^2 \equiv |a_1|\,|t_{m1}(T_2 + S_2)/\{2D(k_z^2 - b_0^2)\}|^2 /(2k_0 Z_0)\,, \tag{9}$$

with $H_{1/p} = 0$ if $|k_z| > k_0 n_{1/p}$. In the above the following notations have been used:

$a_l \equiv \sqrt{k_z^2 - k_0^2 n_l^2}$, $l = 1, p, m$; $D \equiv \exp(a_m d_m) - r_{m1} r_{mp} \exp(-a_m d_m)$; $Z_0 \equiv \sqrt{m_0 / e_0}$;

$T_1 \equiv \partial_x e_{0,m}(0)(1 + r_{m1})/a_m - e_{0,m}(0)(1 - r_{m1})$; $T_2 \equiv \partial_x e_{0,m}(d_m)(1 + r_{mp})/a_m + e_{0,m}(d_m)(1 - r_{mp})$;

$S_1 = \{\exp(a_m d_m) - r_{m1}\exp(-a_m d_m)\}e_{0,m}(d_m) - \{\exp(a_m d_m) + r_{m1}\exp(-a_m d_m)\}\partial_x e_{0,m}(d_m)/a_m$

$S_2 = -\{\exp(a_m d_m) - r_{mp}\exp(-a_m d_m)\}e_{0,m}(0) - \{\exp(a_m d_m) + r_{mp}\exp(-a_m d_m)\}\partial_x e_{0,m}(0)/a_m$.

d_m is the thickness of segmentation layer m, reflection and transmission coefficients are denoted by r and t, respectively and should be evaluated for k_z. The field of the incoming zero-order mode in layer m is denoted by $e_{0,m}(x)$, and we have used a local coordinate system for layer m ($x = 0 - d_m$). The function G_0 corresponds to the Fourier transform of $g(z)$ as follows:

$$G_0(k_z) \equiv k_0^2 \Delta e \int_0^{\infty} g(z)\exp\{i(k_z - b_0)z\}dz/\sqrt{2p} \tag{10}$$

Results and discussion

To illustrate the above we consider a three-layer structure, with refractive indices of 1, 1.8 and 1.457. The wavelength is chosen to be $l = 1mm$. The central layer is segmented, and its thicknesses are chosen to be either $d_2 = 500nm$ or $d_2 = 591nm$, corresponding to modal indices of $N_0 = 1.6795$ and $N_0 = 1.7034$, respectively. The thickness $d_2 = 591nm$ corresponds to a special situation (see below) for which the first order mode is just below cut-off, i.e., $|D| \sim 0$ at $k_z \sim< k_0 n_3$.

The characteristic functions H as a function of $n_{eff}\,(\equiv k_z / k_0)$ are given, for both considered thicknesses, in figure 2. As follows from the above H is an even function of n_{eff}. For the relatively simple (three-layer) structure the functions H are also simple showing only a few maxima and minima. Choosing $n_p \geq n_1$, the large values at $|n_{eff}| <\sim n_p$ (note here p=3) are typical, and are

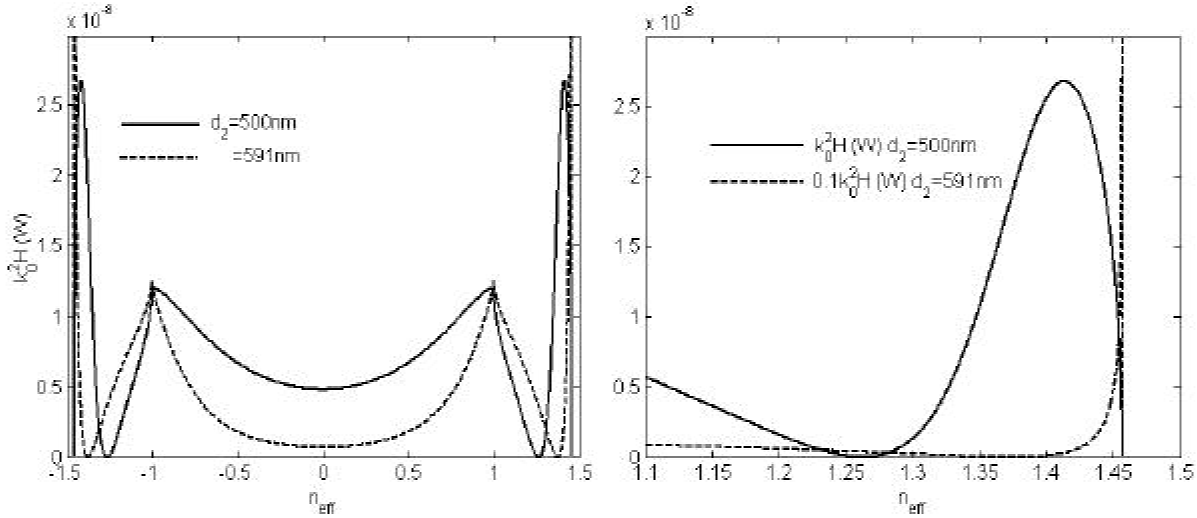

Figure 2. Characteristic functions H, for the two considered thicknesses and a modal power of $P_0 = 1W/m$ *(left hand side). The right hand side shows a detail.*

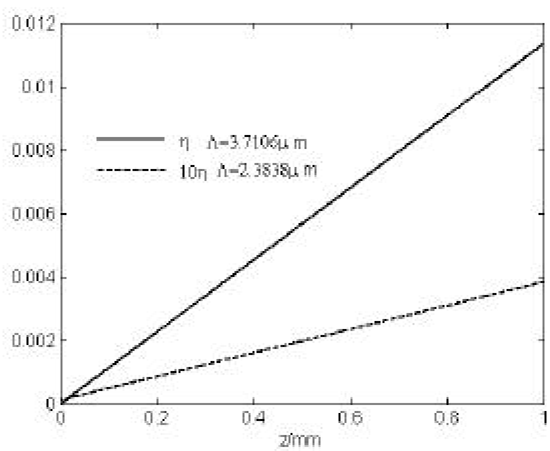

Figure 3. *Relative loss as a function of z for two different grating periods (see text,* $\Delta e = 0.01$*).*

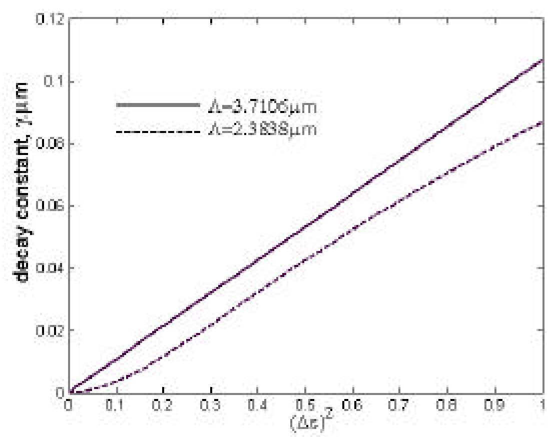

Figure 4. *Decay constant,* g*, as a function of* $(\Delta e)^2$ *for two different grating periods (see text).*

related to the term $(k_z^2 - b_0^2)^2$ occurring in the denominator of (8). At $|n_{eff}| = n_p$ usually $H = 0$, as there $a_p = 0$, unless there is a resonance (i.e., $D = 0$) exactly at that position. Then it can be shown [3] that $H \propto 1/\sqrt{K^2 - k_z^2}$. This explains the strong enhancement of H at $|n_{eff}| <\sim n_3$ in figure 2, for $d_2 = 591nm$, for which the resonance (i.e. $|D| \sim 0$), occurs just below cut-off.

In this paper we consider gratings with segments of equal lengths (duty cycle 0.5; period Λ). From (10) it can be shown that then $|G_0^2|$ consists of a central peak at $n_{eff} = N_0$ and peaks at

$n_{eff} = N_0 + (2m+1)\,l/\Lambda$, where m is an integer. The weight of the latter peaks is proportional to $1/(2m+1)^2$. All peaks narrow if the length of the grating is increased.

The above features in H can be utilized to design low-loss or, as desired for SW sensors, high-loss SWs. Examples thereof are given in figure 3, showing the relative loss, h, as a function of z according to the SDM for the cases of indicated grating periods, both calculations used $\Delta e = 0.01$. The considered grating periods correspond to a first-order ($m = -1$) peak of $|G_0^2|$ coinciding with the peak in H at $n_{eff} = 1.41$ and with the zero of H at $n_{eff} = 1.26$, leading, as expected, to high and low loss SWs, respectively.

If the index contrast increases the large difference between the losses for the two grating periods remains until $\Delta e \approx 0.3$. This can be seen from the MEM calculations given in figure 4, showing the decay constant g, defined by $P_0(z) = P_0(0)\exp(-gz)$.

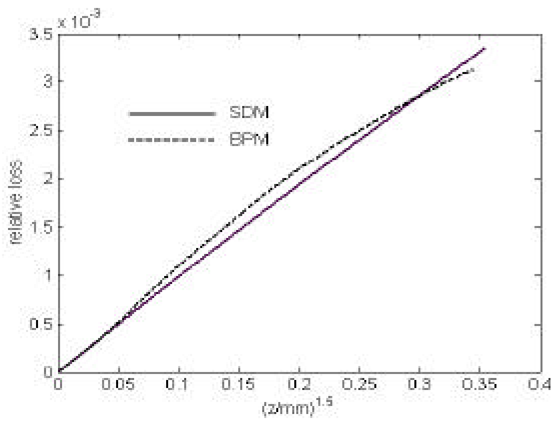

Figure 5. *Anomalous length dependence in the SW, with* $d_2 = 591nm$ *and* $\Delta e = 0.0036$.

If there is a resonance at cut-off and if also the 1st order peak of $|G_0^2|$ is also positioned exactly with $n_{eff} = n_p$ it can be shown with the SDM [3] that $h \propto z^{1.5}$. Hereby the relatively small effect of the other peaks in $|G_0^2|$ has been neglected. The above behaviour is similar to the anomalous length dependence of Cerenkov SHG [7,8]. Figure 5 shows the approximate $h \propto z^{1.5}$ dependence, according to both SDM and BPM, for the case that $d_2 = 591nm$ and a grating period of $\Lambda = 4.0577\,mm$, corresponding to a 1st order peak in $|G_0^2|$ at $n_{eff} = n_3$. This remarkable result is slightly counterintuitive, but well understandable in the light of the above: on increasing the length of a grating the k_z-region of the excited radiation modes (i.e., peaks of $|G_0^2|$) always narrows; if the excitation to these modes (i.e., H) in the considered region is peaked the relative loss will increase faster than linearly with the length.

Conclusions

An approximate theory, leading to analytical expressions for the loss in SWs has been presented. The theory opens the way to study the effect of device parameters on all kind of phenomena that may play a role in SWs

Acknowledgments

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[1] J. Webjorn, F. Laurell, G. Arvidsson, *J. Lightw. Technol.* **7**, 1597-1600, 1989.
[2] Z. Weisman, A. Hardy, *J. Lightw. Technol.* **11**, 1831-1838, 1993.
[3] S.B. Gaal, thesis, Twente, 2002. H.J.W.M.Hoekstra, S.B. Gaal, P.V. Lambeck, to be published.
[4] J. van Lith, P.V. Lambeck, H.J.W.M. Hoekstra, R.G. Heideman, submitted to ECIO '03.
[5] H.J.W.M. Hoekstra, *Opt. Quant. Electron.* **29**, 157-171, 1997.
[6] OlympIOs; *C2V software*, version 5.1.12
[7] J. Ctyroky, L. Kotacka, *Opt. Quant. Electron.* **32**, 799-818, 2000.
[8] H.J.W.M. Hoekstra, J. Ctyroky, L. Kotacka, *J. Lightw. Technol.* January 2003.