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The relation between working memory, number sense, and mathematics throughout primary education in children with and without mathematical difficulties

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ABSTRACT
Number sense and working memory contribute to mathematical development throughout primary school. However, it is still unclear how the contributions of each of these predictors may change across development and whether the cognitive contribution is the same for children with and without mathematical difficulties. The aim of the two studies in this paper was to shed light on these topics. In a cross-sectional design, a typically developing group of children (study 1; N = 459, Grades 1–4) and a group with mathematical difficulties (study 2; N = 61, Grades 4–6) completed a battery of number sense and working memory tests, as well as a measure of arithmetic competence. Results of study 1 indicated that number sense was important in first grade, while working memory gained importance in second grade, before predictive value of both predictors waned. Number sense and working memory supported mathematics development independently from one another from Grade 1. Analysis of task demands showed that typically developing children rely on comprehension and visualization of quantity-to-number associations in early development. Later in development, pupils rely on comparing larger numerals and working memory until automatization. Children with mathematical difficulties were less able to employ number sense during mathematical operations, and thus might remain dependent on their working memory resources during arithmetic tasks. This suggests that children with mathematical difficulties need aid to employ working memory for mathematics from an early age to be able to automatize mathematical abilities later in development.

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Mathematics is a complex, multidimensional academic skill that is difficult to master. Many children, 3–6% of the population depending on the criteria being used, experience persistent difficulties in arithmetic fluency and mathematical abilities (Shalev et al., 2000). Several cognitive functions, both *domain-specific*(pertaining to the processing of numbers) and *domain-general*(processes underlying several higher cognitive functions), have

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been implicated in both typical as well as atypical development of mathematical abilities. Precise etiological paths are unknown, however. Furthermore, the interaction between *domain-specific* and *domain-general* cognitive functions has not been studied frequently.

Two cognitive components of mathematical development regarded as particularly important contributing factors are number sense (NS) and working memory (WM). Both NS and WM predict typical variability in mathematical abilities as well as mathematical learning difficulties (e.g.,Hassinger-das et al., 2014; Szűcs et al., 2014). Unfortunately, these aspects are most often studied within their respective research traditions and are rarely combined within a single study. The field is therefore still unintegrated, even though it is highly important to consider multiple cognitive predictors of mathematical development (Szűcs et al., 2014). Moreover, in a recent review study, we found that over the course of development in general and mathematical development in particular, NS and WM may interact (Nelwan, Vissers, & Kroesbergen, 2020), further demonstrating the necessity to consider these predictors simultaneously. Not many studies, however, have taken into account several age groups of children in primary school when studying WM and NS. Moreover, studies on children with mathematical difficulties, however, show contradictory results. For example, I.C. Mammarella et al. (2013) found that verbal distraction impaired mathematical performance in children with mathematical difficulties, pointing out the contribution of a verbal working memory component, whereas a study by I. C. Mammarella et al. (2018) pointed toward the contribution of visuospatial working memory. Studies including children with mathematical difficulties can possibly yield insights that could constitute tailor-made interventions in clinical practice. The aim of the two studies presented in this paper is to further investigate the predictive value of different components of NS and visuospatialWM for mathematics, and the extent to which their influence is interdependent in both typically developing children and children with mathematical learning difficulties (MLD).

Number sense

NS has been defined as an innate and evolutionarily transmitted mechanism by which humans evaluate the number of objects. At the center of this claim lies the *approximate number system (ANS)* advocated by Dehaene and colleagues (e.g.,Dehaene, 2001). This innate mechanism allows for the estimation of magnitude without the use of language or symbols and is supposed to stand at the base of further numerical development and number-to-symbol association. The ANS is often measured using dot estimation and quantity comparison tasks, although it has been documented that findings of studies employing these tasks are inconsistent, most probably because of presentation differences inducing different strategy formation(e.g., Dietrich et al., 2015, 2019). The ANS framework was expanded by – among others – Núñez (2017), who posited that a distinction has to be made between quantity processing and quantity-to-number connection. In this view, small quantity processing is deemed to be a quality possessed by all humans and several animal species, whereas quantity-to-number connections are culturally transmitted and highly dependent on education. In a recent study (Van Hoogmoed & Kroesbergen, 2018), employing an EEG paradigm, it was found that the visual properties of a stimulus are more important than the actual number of items within that stimulus. Moreover, it was found that nonsymbolic numerosity is not activated in purely symbolic tasks. These results weaken the ANS account of development of number abilities. Furthermore, Van Hoogmoed et al. (2021) conducted a study using event related potentials, finding evidence for the automatic processing of visual properties of NS stimuli instead of the expected automatic activation of numerosity predicted by the ANS paradigm. It is therefore unlikely that the ANS is the only primary foundation of mathematical development. Studies indeed show that nonsymbolic NS is no longer a significant predictor of mathematical abilities when symbolic skills are taken into account (Fazio et al., 2014; Kolkman et al., 2013). Nevertheless, several studies have found a contribution of nonsymbolic skills, and the meta-analysis by Chen and Li (2014) confirmed this relationship, with moderate correlations.

In some studies, NS is regarded as a core set of number-related skills, both symbolic and nonsymbolic, that are required (i.e., a necessary but not sufficient attribute) to acquire more advanced skills in mathematics. Increasing knowledge of counting procedures and language abilities propel the acquisition of exact symbolic number knowledge (Reynvoet & Sasanguie, 2016). Symbolic NS is usually tapped using tasks that involve the comparison of Arabic numbers. Both the speed (Schwenk et al., 2017) and the accuracy (Landerl et al., 2004) of symbolic comparison are implicated as possible parameters bringing to light deficits underlying mathematical difficulties. Another way to measure symbolic NS is to use number line placement tasks that explicitly require the ability to map numbers in sequential order. A recent meta-analysis suggests that these components of NS are significantly correlated, even when general cognitive abilities are controlled for (Chen & Li, 2014).

A strong claim has been made by researchers advocating the importance of NS in forming a foundation for development in mathematics. In studies advocating the theory of the innate ANS module, moderate but significant relationships have been found between nonsymbolic NS and both past and future mathematical development. At the other end of the debate, nonsymbolic NS's predictive value for higher-order mathematics, learned in the later primary school period, has been questioned in recent years (e.g.,Träff, 2013). Several studies suggest that the influence of NS wanes in favor of other cognitive components like language skills or long-term memory (Geary et al., 2012). These studies put emphasis on symbolic NS as the more important predictor of mathematical development in the early primary school years (compared to nonsymbolic number sense) as well as the finding that domain-general components gain importance in later primary school years. Furthermore, cumulating evidence suggests that, in middle primary school too, the predictive value of NS measures wanes when other predictors, like phonological processing, visual WM, general executive functioning, and verbal knowledge are entered into the equation (Szűcs et al., 2014). These constructs, however, were measured in a sample of children in one age group only. This does not allow for drawing conclusions on the influences of these constructs on mathematical abilities throughout development.An exception is the study byCaviola et al. (2020), who did compare different age groups in a large sample of primary school children (Grades 2, 4, and 6). They suggest that the contribution of nonsymbolic number sense in the prediction of mathematical abilities in early, middle, and late primary school becomes insignificant when controlling for fluid intelligence. Mathematical performance was specifically associated with both symbolic comparison accuracy as well as spatial working memory. Verbal working memory was associated with mathematical performance as well. However, this association was not specific to mathematics as verbal working memory was also related to reading ability. The studies presented in this paper elaborate on these latter findings by adding a number line task to the predictor variables. Furthermore, we assessed a group of children with mathematical difficulties, providing the opportunity for comparison between this group and the group of typically developing children. Moreover, contrary to group assessment employed by most other studies, children in the studies in this paper were tested individually. This is important, especially when considering the potential influences of attentional processes during the administration of cognitive tests.

Working memory

Concurrent with the NS debate, a plethora of research has been conducted on the *domain-general* cognitive functions associated with mathematics performance and mathematical development. In recent years, several reviews and meta-analyses have appeared suggesting that WM is recruited during mathematical tasks and predicts mathematical development (e.g., Friso-van den Bos et al., 2013; Peng et al., 2016). WM can be defined as a cognitive system, limited in capacity, that allows for temporarily holding and manipulating information in the center of attention (Cowan, 2014). The most widely used model of WM is the one by Baddeley and Hitch (1974) that divides WM into several components, among which are two simple buffer components (visuospatial sketchpad and phonological loop) and a core component that was dubbed the central executive. However, a strong research tradition exists in which WM is divided into verbal and visuospatial WM (e.g., Jarvis & Gathercole, 2003; Nath & Szűcs, 2014), which adequately describes children's WM performance according to factor analyses (Jarvis & Gathercole, 2003; Swanson, 2017). This distinction has also been adopted in the two studies presented in this paper.

Correlational studies suggest a positive relation between WM and mathematical performance (e.g., Geary et al., 2004; Kroesbergen et al., 2009; Szűcs et al., 2014; for a meta-analysis, see Friso-van den Bos et al., 2013). Furthermore, longitudinal studies also show the predictive value of WM measures for development in mathematics (Best et al., 2011; Lefevre et al., 2013). In a study using a dual-task paradigm by Clearman et al. (2017), it was demonstrated that WM performance decreases when participants have to perform an arithmetic task, and vice versa. It is therefore plausible that performance of mathematics relies on WM resources. The capacity of WM increases during development, allowing children to increase the amount of information being processed and – ultimately – to acquire complex mathematical skills (Raghubar & Barnes, 2017).

Regarding the specificity of WM processes recruited in mathematical tasks, the literature is still divided. Both the verbal (e.g., Friso-van den Bos et al., 2013) and the visuospatial (e.g., Clearman et al., 2017) modality of WM have been implicated as most important. The visual component of working memory seems to be recruited while solving arithmetic problems using basic operations like addition and subtraction (Van Der Ven et al., 2013). Some authors (e.g., Wang et al., 2016) suggest that a link exists between this ability and symbolic NS, most prominently the mapping of numbers on a number line, which loads on visual information processing.

The verbal component, on the other hand, was found to be contributing to the ability to keep track of and manipulate the outcomes of arithmetic operations (Dehaene, 2001). Differences in composition of tasks measuring working memory within cited studies may account for differences in findings (Raghubar et al., 2010). For example, using a single verbal listening recall task will yield different results compared to using a comprehensive WM battery (Skagerlund & Träff, 2016; Szűcs et al., 2014). Comparing these latter studies, Skagerlund and Träff (2016) concluded there was no working memory contribution to mathematical abilites, whereas Szűcs et al. (2014) found evidence for central role of working memory. Similarly, a single listening recall task might have a different predictive value compared to an odd-one-out task, due to the differences in modalities and strategies that need to be employed performing these tasks as well as the age of the participants. For example, in a study by Alloway and Passolunghi (2011) it was found that, in 7-year-olds, verbal working memory as was measured by a listening recall task predicted number related capabilities, whereas achievements on an odd-one-out task did not. In 8-year-olds, neither the odd-one-out task nor the listening recall task predicted mathematical performance.A comprehensive battery including visual WM tasks, as was used by Szűcs et al. (2014) appears to better predict mathematical achievement. In a short review, Menon (2016) pointed out the convergence of behavioral and neuro-imaging data showing that WM processes and their neural substrates change dynamically over development. Menon highlighted the visuospatial component of WM being of prime importance in the development of arithmetic abilities during primary school. This suggests that visuospatial working memory is important for the representation and manipulation of quantity information in short-term memory.In the current papertoo, we focused on visuospatial WM, because of its high predictive value for tests of general mathematics such as national curriculum tests as well as the possibility that visuospatial WM contribution to mathematical ability changes during the primary school years (e.g., Friso-van den Bos et al., 2013). Furthermore visuospatial WM appears to strongly predict arithmetic ability as well(Foley et al., 2017).

WM can be construed of as a mental workspace in the center of attention, in which the present stimulus interacts with long-term memory (e.g., Logie & Cowan, 2015). When a mathematical problem is being solved, this mental workspace is needed to compare the size of numbers, visualize quantities, strategy selection, keeping track of progress, etcetera (e.g., Imbo & Vandierendonck, 2007). It has been demonstrated that both number line estimation, which is a number sense task, and which is generally thought of as a supportive tool for arithmetic problem solving, and arithmetic performance itself are predicted by visuospatial abilities, amongst which visuospatial WM, and that the relation between number line estimation and arithmetic is partially accounted for by these abilities (Simms et al., 2016). Visuospatial working memory thus explains part of the relation between number sense and arithmetic. In the current paper, we take this finding a step further and propose that WM can influence the relationship between NS and mathematics: In children with weaker WM capacity, the relationship between NS and mathematics may be weaker, because these children may not have the WM resources needed to utilize their NS abilities during arithmetic problem solving. This may result in the relation between number line estimation and arithmetic performance being weaker for children with lower scores on WM tasks.

On a neuronal level, too, WM and NS seem to be linked as fMRI studies suggest an integrated network of brain activity during mathematical activities (Arsalidou & Taylor, 2011). Meta-analyses show that both common and distinct neural representations of number are made in the brain when processing both symbolic as well as nonsymbolic number (Sokolowski et al., 2017). Moreover, connective tissue between these areas in this integrated network appears to strengthen heavily in the first years of formal education (Rosenberg-Lee et al., 2011). This would suggest that adequate use of NS to support mathematical problem solving is partly dependent on WM capacity, resulting in interdependency, or interaction in predictive value for mathematical performance of both cognitive skills.

Based on these findings, it is proposed that NS and WM follow an interdependent developmental pattern. Following this argument, the question of which cognitive function is most important becomes less pregnant, while the nature of this developmental pattern regarding mathematical ability would become an increasingly significant focus of study. Both the interdependency between NS and WM in predicting mathematics performance and the aforementioned developmental pattern are investigated in the first of two studies presented in this paper.

Working memory and number sense in children with mathematical difficulties

In the Netherlands, some 15% of pupils are known to experience mathematical learning difficulties (MLD) and 6–7% can be characterized as suffering from dyscalculia (Ruijssenaars et al., 2006). Typically, children are classified as MLD when performing below the level of the 25th percentile on standardized tests, while the children with dyscalculia consistently perform below the 10th percentile. In the Netherlands, the disability to profit from formal extra training in mathematics is one of the criteria for classification of dyscalculia too.

Concerning the group of children with mathematical difficulties, it is acknowledged that both NS and WM are important variables to consider when predicting mathematical development (e.g.,Desoete et al., 2012; Raghubar et al., 2010; Sasanguie et al., 2013; Toll et al., 2011).Toll and colleagues concluded that children with mathematical difficulties experience problems with the retrieval of math facts and keeping this information active during a manipulation procedure with numbers. It is highly plausible that WM constitutes this procedure. WM, being a *domain-general* cognitive function, is not recruited specifically in mathematical learning. It rather is an ability involved in a vast array of cognitive tasks and impaired in neurobiological developmental disorders like ADHD (Tarle et al., 2019). It remains unclear whether WM plays an equally important role in mathematical development in typically developing children as in children with MLD, and to what extent WM remains to play a role in the development of mathematical skills in the later primary school years. Furthermore, despite the fact that it has been shown that the contribution of visual WM wanes over the course of development (Van de Weijer-Bergsma et al., 2015), it is not clear whether the same holds true for children with mathematical difficulties. The studies described in the present paper provide an opportunity to shed light on this issue.

Regarding the role of NS, Desoete et al. (2012) showed that children with mathematical difficulties later in development start out with lower symbolic and nonsymbolic comparison abilities. More recently and in a similar vein, Malone, Burgoyne, & Hulme (2020) found that numerosity judgments and knowledge of Arabic numerals may be useful in identifying children at risk of mathematical difficulties.

Age and mathematical prowess of the participants have a positive effect on the magnitude of effect sizes of both WM and NS in the prediction of mathematical abilities(Klesczewski et al., 2018).Young children with mathematical difficulties tend to have smaller WM span than children without difficulties in mathematical development. In the study by Klesczewski and colleagues, this discrepancy remained present at follow-up in fifth grade, with the exception of visual WM. The visual WM gap between children with and without mathematical difficulties decreased over time. Findings by Friso-van den Bos et al. (2013) seem to show a similar pattern. Thus, it seems possible for children with mathematical difficulties to catch up with their peers regarding visual WM. This makes visual WM an interesting construct when studying WM-mathematics relationships in differing age groups. The studies presented in this paper add to the existing literature in this field by comparing the contribution of both NS and WM, providing the possibility to assess the joint influence of both predictors in children with mathematical difficulties in later years of primary education.

Objectives of the two studies

To summarize, previous research has demonstrated that both WM and NS are correlated with mathematical achievement and statistically predict mathematical development throughout the primary school years. The developmental interdependence of these cognitive functions is rooted in the concurrent development of structures within the cerebral cortex of the brain, specifically the prefrontal cortex, the parietal cortex, and their connective tissue. At the level of cognitive processes, too, there seems to be an interplay between NS and WM, as they allow children to appreciate and understand quantity-to-number relationships and keep track of mathematical operations, respectively. Both aspects are required to perform, understand, learn, and automatize mathematical procedures effectively. However, according to previous studies (e.g., Sasanguie et al., 2013; Van de Weijer-Bergsma et al., 2015), it is suggested that the relative importance of WM and NS changes during the development of elementary mathematical skills (i.e., arithmetic). Furthermore, several studies (e.g., Toll et al., 2016) have shown correlations between WM and NS. We therefore hypothesize that WM and NS will show interactions in predicting mathematical ability in different age groups in primary school. However, few studies have included samples from different age groups, and outcome measures have varied considerably between studies. It is therefore unclear whether this pattern fits natural development or is an artifact of study-specific variables. The first study presented in this paper aims to extend the knowledge on this subject by addressing the question of to what extent NS and WM predict arithmetic achievement at different stages of development and to what extent they interact with one another directly. It is expected that the contribution of NS and WM will be less in older children and that in first graders the contribution of NS is highest. The contribution of WM is expected to become more prominent, before waning in older children. Furthermore, it is expected that children with strong WM capacities will be better suited to keep their NS abilities online while performing arithmetic. Considering the theory of an intertwined development, WM is therefore expected to moderate the relationship between NS and arithmetic ability, strengthening the predictive value of NS over time.

A second objective, addressed in the second study, is to compare data about the contributions of NS and WM to arithmetic performance between typically developing children and children with mathematical difficulties. It is important to compare these groups, for results can have both theoretical and clinical implications; knowledge yielded from this comparison can lead the way to practical interventions tailored to different age groups and levels of arithmetic ability. Furthermore, it can shed light on the contribution and theoretical embedding of NS and WM regarding arithmetic and – ultimately – mathematics. Our hypothesis on this subject is that children with math difficulties will remain dependent on recruiting WM resources when arithmetic procedures fail to be automatized.

Study 1: The contribution of number sense and working memory to arithmetic ability during the primary school years

Material and methods

Participants

A total of 467 typically developing children from grades 1 through 4 of primary schools in The Netherlands participated in the study (51.6% girls; *M* = 8.45 years old, *SD* = 1.19 years). There were no special needs classes in this sample, and children with learning disorders were excluded from the study as all children attained at least average levels in both reading and mathematics, assessed with standardized curriculum-based test used in the Netherlands. Schools were selected based on convenience, mostly in the hometowns of the undergraduate students collecting the data.

Instruments

The Dutch assessment battery for number sense (DANS; Friso-van den Bos et al., 2015) was administered to all children. The battery contains three different tasks – nonsymbolic comparison (dot comparison), symbolic comparison (number comparison), and number lines – that together adequately reflect the different aspects of NS.

In the dot comparison task, measuring nonsymbolic NS and based on Gebuis and Reynvoet (2012), children were asked to estimate whether the dots in patterns presented on the left or the right (ranging from 1 to 100) of the computer screen were the highest number by pressing a key in a corresponding location. The task consisted of four practice items and 90 test items. Items remained on the computer screen until a response was given by the participant. Test items were controlled for convex hull and diameter, surface area, and density of the dots. Ratios between the numbers of dots were 0.63, 0.75, and 0.83. The percentage of correctly answered items was used as the test score. Good internal consistency has been found (Cronbach's α = 0.84) by Kolkman et al. (2013).

The number comparison task, measuring symbolic NS, was comparable to the dot comparison task, but the children had to respond to larger (two-digit) numbers.

Controlling for visual features did not apply to this task. The task consisted of four practice items and 90 test items. Items remained on the computer screen until a response was given by the participant.Percentage of correct scores were used in the analyses. Internal consistency is less than satisfactory (Cronbach's α = 0.61), as was found by Kolkman et al. (2013).

In the number line task, children were required to place a mark on a horizontal number line according to a given Arabic numeral that was read out loud by the experimenter. Practice items consisted of 0 and 100, on which the children received feedback, and during the test trials, children did not receive feedback on the numbers they placed on the number line. The children indicated the marked position using their fingers, while the experimenter operated the mouse to enter the child's responses. The task consisted of two practice items and 26 test items. This task taps symbolic NS but adds a mapping component that requires visuospatial abilities as well (Gilligan et al., 2019). Moreover, the efficient use of strategies also plays a role in performance on a number line estimation task (Peeters et al., 2016). Linear fit scores were computed, fitting the estimates of the children on a linear curve (Kroesbergen & van Dijk, 2015). The resulting fit scores indicated to which extent children ordered and spaced the numbers proportionally to their numerical size on the number line, with larger fit scores indicating more accurate ordering of and spacing between numbers. Internal consistency was satisfactory (Cronbach's α = 0.73), found by Kolkman et al. (2013).

Furthermore, the odd-one-out task, adopted from the AWMA (Alloway, 2007), was used to measure WM. In this task, children view three shapes, encased in a grid, presented in a row. The children are first asked to point out each odd-one-out shape, and then the location of the odd-one-out shapes have to be recalled in a specific order after each trial. Item length increased from one to five shapes and corresponding locations, and an item was only marked correct when all the locations had been pointed out in their correct order. The task terminated when three items of the same length were answered incorrectly, and the length was increased when four items of the same length were answered correctly. This is a complex span task tapping both the visuospatial sketchpad as well as the central executive (updating) component of WM, which is thought to be highly important in mathematical achievement (Lee & Bull, 2016). Test–retest reliability is .81 (Alloway et al., 2016).

Additionally, the arithmetic tempo test (De Vos, 1992) was used to measure memorized knowledge of basic facts to measure children's arithmeticability. This is a paper-and-pencil task consisting of four columns, each representing the basic arithmetic operations (addition, subtraction, multiplication, and division), and a fifth column with mixed operations. The test includes both single-digit and double-digit problems. Children were required to complete as many items as possible in a column within 1 minute, before moving to the next column. The number of correctly completed items was used as the score. Evers et al. (2009) found good test–retest reliability for this test (Cronbach's α = 0.91).

Procedure

The study took place in the spring of 2014. Parents received written information on the study, and we obtained their written consent, in accordance with the Declaration of

Helsinki, before starting the assessments and training. All tasks were administered on a laptop computer, except for the arithmetic tempo test. The latter task took place in a classroom setting. For all other tasks, children were taken to a quiet room inside the school with few distractions and instructed by a trained undergraduate student. A session lasted approximately 20 minutes, and the child was praised using predetermined phrases targeting effort and progress through the tasks, but not the accuracy of the response.

Analytical strategy

Zero-order and partial correlations were inspected to assess whether NS remained correlated to arithmetic performance when controlling for WM, and vice versa. Distributions of predictor variables were inspected for skewness, and an outlier analysis was conducted. Bayesian analyses were conducted to assess the strength of the hypothesis of positive correlation between the predictor variables. To assess the robustness of these correlations, a bootstrapping procedure was followed, similar to the procedure used by Szűcs et al. (2014). The bootstrap procedure determined empirical 95% confidence intervals for the correlations. In this way, there is no reliance on any assumptions regarding the distribution of the data. To start with, 100,000 bootstrap samples were taken with replacement, after which a correlation coefficient was computed for every sample. Confidence limits were determined at 2.5% and 97.5% centiles. Correlations were considered robust when their confidence interval did not include zero.

In this typically developing group we performed Bayesian multiple regression analyses to assess the relative contribution of WM and NS to the prediction of arithmetic ability within the group as a whole, as well as each of the four grades individually. To address the question of to what extent WM influences the contribution of NS to the development of arithmeticability, a moderation term was added to these models.

Results

Descriptive statistics, zero-order, and partial correlations

Descriptive statistics of demographics and studied variables in the four grades can be found in Table 1. Zero-order correlations between all predictive variables are shown in Table 2. In grade 1, there were correlations that yielded strong and decisive evidence against the alternative hypothesis of no correlation between most predictor variables. The WM task correlated with the symbolic and number line tasks. The number line task was correlated with the symbolic NS task. Finally, the symbolic task correlated with the nonsymbolic task. Partial correlations showed that only the number line task remained correlated with arithmeticability when controlling for the other predictor variables. Ultimately, a weak (r = .33) correlation was found. No skewed distributions or outliers were found.

In grade 2, there were correlations between symbolic NS, nonsymbolic NS, and the number line task that yielded strong and decisive evidence against the alternative hypothesis of no correlation. Partial correlations showed that when controlling for the other variables, both the WM task and the symbolic NS task remained correlated to arithmetic ability. The distributions of both the number line task and the symbolic NS

Table 1. Descriptive statistics of the dependent and predictor measures by grade.

Grade 1		*M*	*SD*
n = 116	Gender (%female)	50.00	
	Age (months)	82.97	4.64
	ArithmeticAbility (total score)	20.07	6.53
	Nonsymbolic NS (%correct)	60	6
	Symbolic NS (%correct)	81	16
	Number Line (linear fit)	.69	.20
	Visual WM (% correct)	68	7
Grade 2			
n = 117	Gender (%female)	49.60	
	Age (months)	96.09	5.71
	ArithmeticAbility (total score)	46.20	19.41
	Nonsymbolic NS (%correct)	62	5
	Symbolic NS (%correct)	93	8
	Number Line (linear fit)	.89	.10
	Visual WM (%correct)	72	7
Grade 3			
n = 113	Gender (%female)	56.60	
	Age (months)	107.73	5.18
	ArithmeticAbility (total score)	80.85	22.45
	Nonsymbolic NS (%correct)	64	6
	Symbolic NS (%correct)	95	6
	Number Line (linear fit)	.94	.05
	Visual WM (%correct)	76	8
Grade 4			
n = 113	Gender (%female)	47.90	
	Age (months)	118.69	5.38
	ArithmeticAbility (total score)	96.28	23.90
	Nonsymbolic NS (%correct)	65	6
	Symbolic NS (%correct)	95	7
	Number Line (linear fit)	.95	.04
	Visual WM (%correct)	76	8

Table 2. Zero-order (Below diagonal) and relevant partial and bootstrap (Right column) correlations of predictive variables in the typically developing groups.

		1	2	3	4	5
Grade 1	1. Visuospatial WM	–				.17
	2. Symbolic NS	.33***	–			.11
	3. Nonsymbolic NS	.23	.43***	–		
	4. Number Line	.31**	.50***	.26	–	.33*-
	5. Arithmeticability	.33**	.35***	.19	.46**	
Grade 2	Visuospatial WM	–				.32*
	Symbolic NS	.10	–			.31*
	Nonsymbolic NS	.09	.24*	–		
	Number Line	.18	.34***	.23	−.22	–
	Arithmeticability	.32**	.31**	.15		
Grade 3	Visuospatial WM	–				.29*
	Symbolic NS	.09	–			
	Nonsymbolic NS	.14	.29**	–		
	Number Line	.09	.19	.10	−.23	–
	Arithmeticability	.29*	.14	.22		
Grade 4	Visuospatial WM	–				
	Symbolic NS	.11	–			
	Nonsymbolic NS	.31**	.18	–		
	Number Line	.24	.23*	.26*	−.21	–
	Arithmeticability	.20	-.11	.15		

* BF > 10, **BF > 30, ***BF > 100

task were skewed. One outlier was removed in the number line task and four outliers in the symbolic NS task.

In grade 3, correlations waned. Only the WM task remained correlated with arithmeticability. Again, the number line task and the symbolic NS task showed skewed distributions. One and three outliers were removed, respectively.

In grade 4, some correlations between predictor variables again reappeared, but none of the predictor variables were correlated to arithmeticability to the extent that the resultant Bayes factors were worth not more than a bare mention. The number line task and the symbolic NS task showed skewed distributions. Five and two outliers were removed from the analysis.

To assess the robustness of the partial correlations found in the previous analyses, we used a bootstrapping procedure with 1,000 samples to verify that the relevant partial correlations remained. All partial correlations of variables correlated with arithmeticability proved robust in this procedure.

Regression analyses

Regression analyses were performed for each grade, with the arithmetic tempo test as dependent variable, and all NS variables and the WM variable as predictors. Results of the analyses are presented in Table 3.We choose to report the models with the highest posterior model probability (i.e., the probability of the model being the "true" model given the data). In grade 1, the predictors explained 23% of the variance. The best fitting model included the number line placement task and the WM task, with the former as the most prominent predictor. This model yielded positive evidence against the empty model. Thus, the analysis demonstrated that the mapping component of NS is particularly important in this grade, constituted by WM ability. In grade 2, the predictors explained 15% of the variance in arithmetic tempo test scores. A model comprised of symbolic NS and WM yielded the most probable model given the data. The Bayes factor showed strong evidence for the validity of this model. In grade 3, 12.6% of variance was explained by the model. A model including number line placement, WM, and to a lesser degree also nonsymbolic NS yielded the highest probability. The Bayes factor showed positive evidence for this model compared to the empty model. Finally, in Grade 4, only

Table 3. Predicting Arithmeticability per grade and the MLD Group: best fitting models, posterior model probabilities, and bayes factors.

Grade Grade 1	Predictor	*B*	β	*PMP*	*BF*
	Number Line (Mapping)	12.07	.36		
	Visual WM	14.87	.16	.20	7.00
Grade 2					
	Symbolic NS	69.92	.29		
	Visual WM	87.14	.30	.39	18.19
Grade 3					
	Nonsymbolic NS	59.25	.15		
	Number Line (Mapping)	80.41	.19		
	Visual WM	71.68	.26	.20	4.77
Grade 4					
	Number Line (Mapping)	110.83	.18		
	Visual WM	47.73	.17	.10	2.17
MLD	Visual WM	59.95	.29	.16	2.07

Table 4. Partial correlations of predictive variables (1. Visuospatial WM, 2. Symbolic NS, 3. Nonsymbolic NS, 4. Number line) and arithmetic ability in the typically developing group (all grades total).

	1	2	3	4
Arithmetic ability	.28**	.08	.12*	.37***

* BF > 10, **BF > 30, ***BF > 100

7.2% of variance was accounted for by the regression model. Given the data, WM and number line placement showed to be most important. However, the Bayes factor remained low.

Moderation analyses

In the total group of typically developing children, partial correlations showed that both nonsymbolicNS and number line placement as well as WM were correlated to arithmeticability when controlling for other relevant variables (Table 4). To show the moderation of WM on the contribution of nonsymbolic as well as number line placement to arithmetic abilities, a moderator analysis was performed with age as a predictor. This analysis showed that the interaction term added to the prediction, but only marginally (Table 5).

Moderation analyses were also performed for each grade separately, using only the NS tasks that correlated significantly with arithmetic performance.This was done because we expected the predictor variables to wax and wane over the course of the primary school years. For grade 1, we took the WM task and number line task (being robustly correlated to arithmeticability) to examine the main effects and the interaction between these two variables in predicting arithmeticability. Bayesian analysis showed that the most likely model given the data was the model in which WM and NS predict arithmeticability independently. The interaction term did not contribute to the posterior model probability. Positive evidence was found for the model in which both NS and WM are entered a predictors, without the interaction term. For grade 2, the outcomes of the symbolic task and the WM task were examined. Here, too, no contribution from the interaction term was found in predicting arithmeticability. The model with WM and NS as independent predictors yielded positive evidence against the empty model. In the moderator analysis for grade 3, the number line task and WM task were examined. Again, no contribution of the interaction term was found. Positive evidence against the empty model was found for independent predictive value of NS and WM. In grade 4, the model with the interaction term yielded the highest probability given the data. The Bayes factor was low, however. The results are shown in Table 6.

Table 5. Moderator analysis: WM on NS in the Total Group typically developing children, corrected for age.

	B	β	*PMP*	*BF*
Age	1.49	.60		
NS	48.75	.21		
WM	83.19	.19		
WMxNS	200.87	.08	.73	2.76

Table 6. Moderator analyses:WM on NS in the four grades and the MLD Group: Best fitting models, Bayes factors, and posterior model probabilities of the most likely model given the data.

Model	Group	*B*	β	*PMP*	*BF*
WM+NS	Grade 1	20.56	.47	.44	8.80
WM+NS	Grade 2	126.702	.31	.53	12.32
WM+NS	Grade 3	86.798	.11	.38	6.75
WM+NS+ NSxWM	Grade 4	1206.704	1.67	.38	1.84
WM	MLD	615.739	.78	.28	4.29

Only the NS measures that correlated significantly to arithmetic performance were included in the analyses.

Conclusion

The aim of Study 1 was to investigate the relative predictive value of NS variables and the WM variable for arithmetic performance in grades 1–4, as well as the interdependency between predictor variables. Taken together, these results suggest that, in typical development, the contributions of both NS and WM to arithmeticability fluctuate over time, adding to previous research with different samples and age groups presenting similar conclusions (e.g.,Sasanguie et al., 2013; Van de Weijer-Bergsma et al., 2015). In grade 1, the most important predictor is number line mapping and implicit strategy formation, for example, the use of self-created benchmarks (Peeters et al., 2016), suggesting that this ability is highly important in early mathematical development. Understanding the number sequence and ordinality principle is paramount in this developmental stage. In grade 2, visual WM and symbolic NS predict arithmeticability together. The symbolic NS task taps the ability to quickly identify and compare larger Arabic numerals, and it therefore differs from the number line task in the sense that it requires less visuospatial abilities and strategy use, but the task does impose more time pressure as well as a need to identify multi-digit numbers and is therefore more prone to automatization difficulties. Our data suggest a switch in predictive value from the mapping/strategic component to a comparison/identification component and visual mental operation regarding arithmetic ability. In grade 3, number line placement regains its importance over symbolic NS but appears to contribute to a lesser degree than visual WM. In grade 4, no predictors remain that yielded evidence against the null hypothesis. It should be noted that Bayes factors are generally low, so strong conclusions are not warranted. The general trend seems to be, however, that NS and WM explain less variance in arithmeticability over the course of development, which means that other factors (e.g., processing speed; Rose et al., 2011, or language abilities; Vukovic & Lesaux, 2013) gain in importance over the primary school years.

Regarding the interaction between NS and WM, the models pointed out small probabilities of the interaction terms within the four grades. It was much more likely that WM and NS might have predicted arithmetic abilities independently. This is contrary to our predictions. However, since this has been an explorative analysis, further research is needed to test this hypothesis.It might be that in predicting mathematical achievement at a given point in time, no interaction can be found, whereas in predicting development (i.e., growth of mathematical abilities) it could. Longitudinal studies should be conducted to test this hypothesis. It should be mentioned that in grade 4 an interaction effect was found, but this effect was very small and should not be interpreted without caution. The moderator analysis conducted on the total group of typically developing children did show results that had been anticipated; a strong moderation was found. Given the positive regression coefficient, this might imply that developing WM supports the efficient use of NS abilities in performing arithmetic operations. However, even though strong results were obtained in this analysis, this result should be interpreted with caution, because the data used was cross-sectional.

Considering the data on typically developing groups of children, one of the questions that arises is whether children with mathematical learning difficulties in late primary school show the same structure of predictive value of both WM and NS in arithmetic abilities, or whether they exhibit a different cognitive pattern. This question will be investigated in the next section.

Study 2: The contribution of number sense and working memory to arithmeticability in children with mathematical and attentional problems

Method

Participants

Children with scores below the 25th percentile on a standardized mathematics test, used in the Netherlands to monitor progress in math aptitude, were recruited from mainstreamschools in The Netherlands, primarily in the Rotterdam region (see Table 7 for descriptive statistics on demographics and relevant variables). The standardized mathematics test measures general ability regarding mathematics, testing both arithmetic abilities, as well as knowledge on more advanced mathematical concepts. From the outcomes of this test it is not possible to assess the specific strengths and weaknesses within or between subjects. However, all participants showed arithmetic difficulties (below 25th percentile) when tested with an arithmetictempo test.General cognitive skills were not tested prior to selection of participants. Teachers selected these pupils based on the inclusion and exclusion criteria that were given to them. Children were eligible when they were in grades 4–6 (9–12 years old; M = 10.79; SD = 0.99) at the time of testing and were rated by their teachers as having above-average (above the 80th percentile) attentional difficulties as measured by a standardized questionnaire (Scholte & van der Ploeg, 2005). Children with below-average scores (below the 25th percentile) on reading tests and/or having known psychiatric disorders other than ADHD were excluded from participation.

Table 7. Descriptive statistics of the MLD Group.

MLD		*M*	*SD*
N = 61	Gender (%female)	41.00	
	Age (months)	129.51	11.82
	ArithmeticAbility (total score)	86.49	25.98
	Nonsymbolic NS (%correct)	73	7
	Symbolic NS (%correct)	96	4
	Number Line (linear fit)	.84	.07
	Visual WM (%correct)	72	12

Instruments

There were five main variables of interest in this study: visuospatial WM, nonsymbolic comparison, symbolic comparison, number line placement, and arithmeticability. The NS variables and arithmeticability were measured using the same tasks (DANS and arithmetic tempo test, respectively) as in our first study, as described in the section above, but with fewer items in the comparison tasks (30 instead of 90 items per task). The WM task, however, was different. We used a different task because this second study was conducted later in time, and a new test that had been validated recently was available to us. Moreover, this test was more easily administered. The test was a visuospatial WM task called “Lion Game” (Van de Weijer-Bergsma et al., 2015).

The Lion Game is a visuospatial complex span task, in which children have to remember the locations of colored lions. Children are presented with a 4-by-4 matrix, containing 16 cells with green shrubbery. Each trial consists of the presentation of eight lions of different colors (red, blue, green, yellow, and purple), consecutively presented in different cells for 2000 ms. Children have to remember the last exact location at which a lion of a particular color has presented itself. They use the mouse to click on that location after the sequence has ended. The task consists of five levels of four items. WM load is manipulated by the number of colors children have to remember and update. No cutoff rules are applied (Van de Weijer-Bergsma et al., 2015). The proportion of correct responses is collected. The reliability and validity of the Lion Game has been studied by Van de Weijer-Bergsma et al. (2015). Good internal consistency (Cronbach’s $\alpha = 0.87$), satisfactory test–retest reliability (Cronbach’s $\alpha = 0.71$), and good concurrent and predictive validity have been found.

Procedure

Measurements took place in January–February 2015. All tests, except for the paper-and-pencil arithmetic fluency task, were administered on a laptop computer in the respective schools of the children in the presence of trained graduate students. Parents received written information on the study, and we obtained their written consent, in accordance with the Declaration of Helsinki, before starting the assessments and training. The study was approved by the ethics committee of the Faculty of Social and BehavioralScience, Utrecht University (FETC14-022).

Analytical strategy

As with our typically developing group, we conducted the same bootstrapping procedure for correlations described above, assessed the distributions of the predictor variables for skewness, searched for and removed outliers, and used Bayesian regression and moderation analyses. To compare the typically developing group in Study 1 with the group of children with math difficulties regarding the contribution of NS and WM to arithmeticability, slopes of the regression analyses were compared by adding an interaction term between the dummy variable Group and the relevant predictor variables.To be able to run this analysis, groups were matched on grade, so that children in grade 4 were selected and matched. We chose not to match on arithmeticability because it cannot be assumed that the level of arithmeticability of a younger child in the typically developing group has the same significance regarding our predictor variables as the same level of ability of an older child in the MLD group. Interaction terms between group (MLD vs. non-MLD) and visual WM were computed. Results on the visual WM tasks were converted to *z*-scores to compute the main affects to avoid collinearity.

Results

Zero-order, partial, and bootstrap correlations

In the MLD group, no correlations between predictive variables were found that yielded Bayes factors above 10 (Table 8). The strongest correlation was between WM and arithmeticability, but this yielded only substantial, not strong evidence against the alternative hypothesis of no correlation. Results are summarized in Table 9. When assessed, it was found that the distribution of the number line task was highly skewed. We removed a single outlier. This might have affected the results of the regression analysis.

To assess the robustness of the partial correlation between WM and arithmeticability found in Study 1, we used a bootstrapping procedure with 1,000 samples to verify that the WM measure remained correlated to arithmeticability. This correlation proved robust in this procedure.

Regression analysis

In a next step, a Bayesian regression analysis was conducted with arithmetic performance as a dependent variable and WM variables as a single predictor, which was the best fitting model considering the data. In total, 9.2% of variance in arithmeticability was explained by the WM (Table 3). Both Bayes factor and posterior model probability were low, however.

Then an interaction term was added to the model between the number line task (showing the strongest correlation with arithmetic abilities) and WM. No interaction was found in this analysis (Table 4).

Table 8. Zero-order correlations between predictive variables in the MLD Group.

	Visuospatial WM	Symbolic NS	Nonsymbolic NS	Number Line
Symbolic NS	−.05	–		
Nonsymbolic NS	.10	.11	–	
Number line[a]	−.03	.08	.19	−.16
Arithmeticability	.33	.05	-.06	

[a]Spearman correlations are reported because of skewness of the distribution

Table 9.. Descriptive statistics of the MLD Grade 4 Group.

MLD Gr. 4			
N = 19	Gender (%female)	31.6	
	Age (months)	121	7.32
	ArithmeticAbility (total score)	78.95	20.26
	Nonsymbolic NS (%correct)	.72	.07
	Symbolic NS (%correct)	.96	.04
	Number Line (linear fit)	.85	.17
	Visual WM (%correct)	.73	.12

Comparison between MLD group and typical development

To assess the differences between the MLD and typically developing groups, the slopes of the regression equations were compared to assess whether WM was a more robust predictor of arithmetic ability in the MLD group compared to the typically developing group. Descriptive statistics on this sample can be viewed in Table 8. Although the slope of the equation in the MLD group appeared to be steeper on inspection, testing the hypothesis yielded very low probabilities of the proposed model with inclusion of group membership as a predictor. According to our data, there were no differences between our groups regarding involvement of WM as a predictor of arithmetic abilities. Results are shown in Table 10.

Conclusion

From these data we can conclude that in children with MLD, contrary to typically developing children, visual WM still contributes to arithmeticability during middle and late primary school, whereas NS does not. This is only partly congruent with a study by Träff (2013) that did find a significant contribution of NS. It should be noted that regression equations should be interpreted with caution, because of the skewness of the distribution in one of the variables.

When compared directly to typically developing fourth-graders, no differences in predictive value could be reported. This might be due to insufficient power to detect these subtle differences, because we could only use children in grade 4 to match the groups on age. Furthermore, the contribution of visual WM to the explanation of variability in arithmeticability in this group of children with MLD is only small. It is therefore difficult to obtain high probabilities for this model in comparison analyses. I. C. Mammarella et al. (2018) showed that visuospatial WM specifically differentiated between children with MLD and typically developing peers. Szűcs et al. (2014) arrived at the same conclusion. Our data fit within this framework.

In addition to this confirmation, our results might put this framework in a different perspective. Interestingly, no correlations were found between predictor variables within the group with mathematical learning difficulties. Furthermore, only low correlations were found between NS measures and arithmeticability in this group. Kolkman et al. (2013) did find significant correlations between these variables in a younger group of children, although these children did not experience mathematical difficulties. Toll et al. (2016) also found positive correlations between WM and NS in first-graders, controlled for age. They also found that low WM performance combined with low NS abilities predicted low scores in mathematics. This was also found in an older sample by Kroesbergen and van Dijk (2015). This could imply that cognitive abilities underlying

Table 10. ArithmeticAbility predicted from working memory, comparison between groups: Posterior model probabilities and Bayes factors.

Predictor	*B*	β	*PMP*	*BF*
Working Memory	4.77	.19	.20	1.24
Group Membership	7.98	.16	.06	.31
Working Memory x Group	3.79	.09	.11	.26

PMP = Posterior Model Probabilities; BF = Bayes Factor.

mathematics in this group of children with mathematical difficulties and attentional difficulties remain insufficiently integrated. This integration might be a developmental task during the primary school years. The lack of correlations could mean that children with mathematical and attentional difficulties are struggling to complete this task and to use their NS abilities in the process of performing arithmetic effectively. This way they have to depend on compensatory strategies. Another possibility could be that children with mathematical and attentional difficulties do not generalize and automatize arithmetic operations due to visual WM deficits, concurring with Geary's model (Geary, 1993). Considering the data, this explanation seems unlikely in this respect, because children in this condition performed the WM task quite well on average. A last explanation could possibly be found in task specifics: it might be that the variation within the distribution of the number line task was insufficient to detect differences between participants on their NS skills.

General discussion

In this paper, we compared the contributions of NS and WM in their prediction of arithmetic achievement over the primary school years. We did this for typically developing children and for a group of children in higher grades experiencing difficulties in performing arithmetic. The results should be seen in the light of two important questions, discussed below.

Development of NS and WM: Independent or interdependent?

Depending on the perspective taken, studies tend to emphasize either WM or NS as a primary marker in the developmental process of mathematical ability. While Szűcs et al. (2014), for example, posed an executive memory-centric model of mathematical ability, this seems not to be the case throughout the development of mathematical skills, given our data. Even though our first study confirmed that nonsymbolic number sense does not seem to contribute to the prediction of mathematical abilities in different grades of primary school, symbolic comparison does seem to play an important role throughout development. Our findings concur with the study by Caviola et al. (2020)who showed that specifically symbolic NS as well as WM relate to mathematical abilities in primary school. Moreover, the first study presented in this paper takes these results further by demonstrating that number line estimation also plays an important role in addition to symbolic comparison. Also adding to the existing literature, our findings show that even though the children in this paper were assessed individually, comparable results were obtained regarding the contribution of NS and WM in predicting mathematical achievement. This might point out that attentional functions do not influence these assessments. This might be a focus of further research.

Most importantly, the first study in this paper shed light on a developmental component indicating that over time, aspects of NS and WM vary in their contribution to mathematics performance.We found WM to be the superior predictor in grades 2 and 3. However, in grades 1 and 4, WM played a minor role in predicting arithmetic prowess, when controlling for NS. This leads to a model that features a braided contribution of cognitive factors to mathematical development in which NS and WM conjointly form the backbone of mathematical development during the primary school years. Our data concur with the study conducted by Van de Weijer-Bergsma et al. (2015), who showed that the roles of different components of working memory for mathematics performance changed across developmental time, putting a developmental aspect to the forefront. We opted to adopt a similar line of reasoning, but to also account for the role of number sense in exploring predictors of mathematics performance.This leads us to hypothesize that in the first year of primary school the load of solving arithmetic problems on WM is not that high, because of the concrete and manageable nature of the tasks at hand. In grade 1 arithmetic problems rarely require numbers beyond 10 and are therefore possible to tackle with, for example, counting fingers, taxing working memory to a lesser extent. When problem difficulty increases and the tasks become more abstract, the load on WM increases too. In grades 2 and 3 the cognitive burden of arithmetic increases until, by grade 4, basic operations are largely automatized and no longer stress WM resources.

The various aspects of NS appeared to shift in contribution to arithmetic performance from grade 1 to 3. Number line placement contributed most in grades 1 and 3, suggesting that the symbolic and mapping aspects of NS are the most relevant to arithmetic performance in these grades. One should note that the comparison task showed a ceiling effect in the higher grades, which might partly explain the small correlation found in the present study. We could have used the reaction times as a variable, and these could have been a valid measure for NS if we were able to control for basic processing speed. We did the analyses on reaction times as well and saw that over the years, contribution of reaction times increased tremendously. This fed the idea that processing speed increases in predictive value as arithmetic fluency increased and basic operations were getting more and more automatized. Moreover, reaction times can be misleading, especially in the group of children experiencing mathematical difficulties, as these children could potentially show low reaction times, but a high amount of errors.

The operational processes underlying mathematical abilities and development remain unclear, however, since the number line placement task has been receiving some criticism in the literature. It is unclear whether visuospatial strategy use and WM are underpinning the scores obtained in this task (Van Hoogmoed & Kroesbergen, 2018). The visual WM task and number line placement task should share a common background in visuospatial abilities (also see Simms et al., 2016). Indeed, correlations were found between visual WM and number line placement in our first study, but number line estimation performance and working memory did not show an interaction in predicting mathematical ability. The mapping component of the number line placement task therefore seems to contribute uniquely to arithmeticability to some degree. The ability to map between nonsymbolic quantity and symbolic number appears to be important in grades 1 and 3. This finding is comparable to the findings by Kolkman et al. (2013). The present study, though, seems to extend this finding to and older age range.Another possible explanation might be that the number line task taps into a more *domain-general* strategy formation ability that accounts for the contribution of the number line task to the prediction of mathematical ability (Peeters et al., 2016). The results of the present study do not allow for predictions on the employment of different strategies, however.

Unsurprisingly, symbolic number comparison was found to contribute to arithmetic-ability, whereas nonsymbolic NS did not predict mathematical ability in our sample. This is congruent with many studies (e.g., Geary et al., 2017). Surprisingly, however, symbolic comparison predicted arithmetic only in grade 2, alongside WM, with the latter being the more robust predictor. This is congruent with findings in the same age group by Sasanguie et al. (2013), however. The braided development of symbolic comparison and mapping as they pertain to mathematics has, to the best of our knowledge, not been documented yet. The underlying rationale for the switch between symbolic number comparison and mapping number onto a number line might be that in grade 2 children are required to quickly identify larger numbers, as required in the symbolic NS task but not the number line tasks, during which numbers were read to the children. Indeed, during this phase, educational methods in The Netherlands make the switch from numbers below 20 to larger numbers (SLO). In grades 1 and 3, mapping seems to be the most prominent factor, probably because children rely on number line strategies by this time to complete arithmetic problems. As Szűcs et al. (2014) suggested, NS in general does seem to play a lesser role than has been found in some studies, when other factors like WM are taken into account. However, symbolic NS does seem to play an important role in some developmental stages, as was shown by the first study presented in this paper.

In the first study, we also found that predictive variables were correlated with one another, at least in the typically developing group. However, no moderating effects were found. This is contradictory to our hypothesis that WM moderates the contribution of NS in predicting arithmeticability, and vice versa. Thus, WM does not constrain the contribution of NS to mathematics performance. Previous research has shown that in kindergarten both constructs are highly correlated (e.g., Kolkman et al., 2013; Passolunghi & Lanfranchi, 2008; Xenidou-Dervou et al., 2013). In the present study, too, significant correlations were found. However, we found no difference in predictive value of NS between children with lower and higher WM capacity in any of the grades. While it is too early to draw conclusions on the matter due to the cross-sectional nature of this study, this might imply that from the start of formal education, these two functions contribute to mathematical performance independently. It might be that the pattern we found in our data reflects stages in the development of these cognitive functions toward maturation. In short, NS and WM seem to play distinct roles in arithmetic performance, at least from the start of formal education, and are correlated. It is, however, still unclear what the causal relations would be and whether there are confounding factors. This could be an objective for further longitudinal research.

Working memory and number sense in children with mathematical difficulties: Functionally different from typical development?

Confirming our predictions, WM remains a predictor in the group of children with mathematical difficulties. This suggests that these children continue to recruit WM resources when performing arithmetic operations in the later grades of primary school while their typically developing peers have already automatized these operations. These results fit in with recent literature on WM and MLD (e.g.,Attout et al., 2014;

Kuhn et al., 2016; Maehler & Schuchardt, 2016; Szűcs et al., 2013), which proposed a WM-centered approach for explaining mathematical learning difficulties. The present study showed that this seems to hold true for children in later stages of primary school as well. The lack of predictive value of NS in this group, found in the second study presented in this paper, suggests that these abilities cannot be effectively put to use in the MLD group, and may suggest that this group has to rely on WM processes to complete the mathematical tasks. Another explanation would be that mathematical ability in this group is not curbed by NS, so that individual differences in NS neither explain nor predict mathematical achievement. In any case, WM does seem to curb mathematical abilities, but it is possible and plausible that NS still plays a role. In our second study, there was a restricted range of scores on NS tasks, most prominently in the distribution of scores on the number line placement task. This might have affected the outcomes of the regression analysis. We might either need a paradigm to measure the mapping component in a way that does not yield skewed scores (such as an open-ended number line task, Reinert et al., 2019), or a much larger sample. Whatever the underlying cause may be, this notion is contrary to influential papers by Von Aster and Shalev (2007) andButterworth (2010), who posited that NS is the core deficit in mathematical difficulties. It might still be the case that NS is most important and that children circumvent NS and rely completely on WM in completing arithmetic tasks, posing a burden on WM resources. This hypothesis, however, is not supported by the descriptive statistics in the present study. Of note is that in our group with mathematical learning difficulties, no correlations were found between cognitive predictors of arithmetic abilities. This unpredicted finding might reflect the underlying differences in brain development between typically developing children and children exhibiting mathematical deficits, with decreased interconnection of functions leading to deviating development (Menon, 2016). It is likely that children with mathematical learning difficulties develop in a functionally different manner regarding NS and WM as they pertain to mathematical abilities. This needs to be tested empirically, however.

Limitations

Our results should be seen in the light of four main limitations. Firstly, comparison between the two studies was not easy due to the small group of children that could actually be compared when controlled for age and therefore a lack of statistical power to find differences in the contribution of WM and NS to mathematical difficulties, and because of the fact that we used different WM tests in the two samples. Furthermore, the MLD group was a heterogeneous group of children with additional attentional difficulties, in the sense that the cutoff criteria for attentional difficulties were chosen liberally. This limits the possibilities for interpretation and generalization of these results.

Secondly, we did not include measures of other important predictors (executive functions or other cognitive components like language), as some authors have recommended (e.g.,Szűcs et al., 2014). Such predictors likely correlate with the variables included in the current studies and may share explained variance in arithmetic skills as included in these studies. Inclusion of such variables, although beyond the scope of the current studies, may yield different patterns of predictive power.

Moreover, we should note that, in these studies, a speeded arithmetic task was used. As arithmetic is a simple skill, from a cognitive standpoint, compared to broader mathematical abilities. These results can therefore not be readily generalized to math tasks requiring many cognitive skills. However, since these studies focused on both the relationship between two cognitive functions as well as children with mathematical problems, we felt that arithmetic fluency was a good starting point. In addition, even though in Grade 4 many of the answers to arithmetic problems presented in the test will have been drawn from long-term memory, some of the problems will not have been automatized. This means that, even in Grade 4, this speeded arithmetic test will have drawn upon WM resources. The relations between various other cognitive skills needed to perform arithmetic (like long-term memory, processing speed) is a very interesting topic for further research. More research is also needed to shed light on the relationships of cognitive functions pertaining to broader mathematical abilities.

Conclusions and implications

As hypothesized, NS and WM seem to follow a braided development in predicting arithmetic abilities over the primary school years before their influence wanes after arithmetic operations are automatized. Furthermore, children with mathematical difficulties follow a different developmental pattern and are less able to utilize their NS abilities during arithmetic operations, and thus they might remain dependent on their WM resources during mathematical tasks. The source of this deviating development might originate early in development, before NS and WM differentiate in grade 1 of primary school (Purpura et al., 2017). These hypotheses still need to be tested empirically in future studies. Longitudinal studies are of prime importance, because they can yield evidence regarding the relationship between NS and WM over the course of development. Microgenetic methods could also yield interesting results, as they might show the changes more precisely, especially regarding intervention studies or in dual-task designs. Given the heterogeneous background of mathematical difficulties, single-case experimental designs might yield interesting results as well, especially when tailoring specific interventions to remediate found weaknesses in WM and/or NS. Specifically, studies should include measures of multiple cognitive components pertinent to mathematical development. Furthermore, given the importance of visual WM, it might be sensible to include measures of basic visuospatial processing (like a figure-ground task or object comparison task) to control for these processes.

More practically, these conclusions imply that it is of paramount importance to identify at-risk children at an early age, during kindergarten. Children at risk of developing mathematical learning difficulties need aid to employ their WM for mathematics from an early age to be able to automatize mathematical abilities later in development. In addition, developing and automatizing NS abilities and arithmetic fact retrieval could take the burden of WM, so it remains important to foster NS skills independently as well. This has been demonstrated in a younger age group (Ramani et al., 2019) but could hold true for older children as well. Furthermore, our results show that mathematical difficulties have a heterogeneous cognitive background. The heterogeneity calls for the need to look closely at the cognitive profile of a child during the assessment of mathematical difficulties, as they might vary considerably between individuals. This leads to the conclusion that, on an individual level, intervention plans need to be tailor-made.

Disclosure statement

The authors declare that this research was conducted in the absence of any commercial or financial relationships that could be construed as a personal interest.

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